Use The Multiplication Law For Logarithm To Expand The Following Expression:[tex]log_{8} (21)[/tex]-

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Understanding logarithms is fundamental in various branches of mathematics, including algebra, calculus, and applied sciences. One of the essential properties of logarithms is the multiplication law, which allows us to simplify and manipulate logarithmic expressions efficiently. In this article, we will explore how to use the multiplication law for logarithms to expand the expression [tex]log_{8} (21)[/tex], providing clear explanations, step-by-step procedures, and practical examples to enhance your understanding.

What Is the Multiplication Law for Logarithms?

Before diving into the expansion process, it is crucial to understand the multiplication law for logarithms. This property states that for any positive real numbers a, b, and base c (where a, b, c ≠ 1):

$\log_c(a\,b) = \log_c(a) + \log_c(b)$

In words, the logarithm of a product is equal to the sum of the logarithms of the factors. This property is particularly useful when dealing with expressions involving products, as it simplifies complex logarithmic expressions into more manageable parts.

Applying the Multiplication Law to [tex]log_{8} (21)[/tex]

The goal is to expand [tex]log_{8} (21)[/tex] using the multiplication law. Since 21 can be expressed as the product of two numbers, the law applies directly.

Step 1: Factor the Number 21

First, identify the factors of 21:

- $21 = 3 \times 7$

Expressing 21 as a product allows us to apply the multiplication law directly:

$\log_8(21) = \log_8(3 \times 7)$

Step 2: Use the Multiplication Law

Applying the property:

$\log_8(3 \times 7) = \log_8(3) + \log_8(7)$

Thus, the expanded form of [tex]log_{8} (21)[/tex] becomes:

[tex]log_{8} (21) = log_{8} (3) + log_{8} (7)[/tex]

This is the key application of the multiplication law, breaking down a complex logarithm into a sum of simpler logarithms.

Understanding and Calculating the Expanded Logarithms

Having expanded [tex]log_{8} (21)[/tex], it is often necessary to evaluate or approximate these logarithms, especially when dealing with real-world problems.

Changing Logarithm Bases

If you need to evaluate [tex]log_{8} (3)[/tex] and [tex]log_{8} (7)[/tex], you may find it easier to convert these to common or natural logarithms using the change of base formula:

logc(a) = \frac{\ln a}{\ln c} = \frac{\log_{10} a}{\log_{10} c}

Where:
- \(\ln\) is the natural logarithm
- \(\log_{10}\) is the common logarithm

Using this formula, the expanded expression becomes:

log8(21) = \frac{\ln 3}{\ln 8} + \frac{\ln 7}{\ln 8}

Similarly, you can compute these values using a calculator to get approximate numerical answers.

Example Calculation

Suppose we want to evaluate [tex]log_{8} (21)[/tex]:

1. Find the natural logarithms:

- \(\ln 3 \approx 1.0986\)
- \(\ln 7 \approx 1.9459\)
- \(\ln 8 \approx 2.0794\)

2. Compute the individual logarithms:

- \(\log_{8} 3 \approx \frac{1.0986}{2.0794} \approx 0.528\)
- \(\log_{8} 7 \approx \frac{1.9459}{2.0794} \approx 0.935\)

3. Sum the results:

- \(0.528 + 0.935 \approx 1.463\)

Therefore, [tex]log_{8} (21) \approx 1.463[/tex].

Additional Applications of the Multiplication Law in Logarithms

Beyond expanding a simple logarithmic expression, the multiplication law has many practical uses:

Simplifying Complex Logarithmic Expressions

When dealing with expressions such as:

$\log_c(a^m b^n)$

you can expand using properties of logarithms:

$\log_c(a^m b^n) = m \log_c(a) + n \log_c(b)$

This helps in simplifying and solving equations involving logarithms.

Solving Exponential and Logarithmic Equations

The multiplication law can convert complex logarithmic equations into simpler algebraic forms, facilitating easier solutions.

Summary: Key Takeaways

- The multiplication law for logarithms states that [tex]\log_c (a \times b) = \log_c a + \log_c b[/tex].
- To expand [tex]\log_{8} (21)[/tex], factor 21 into 3 and 7, then apply the law.
- Use the change of base formula to evaluate or approximate the expanded logarithms.
- The property simplifies complex logarithmic expressions, making calculations more manageable.
- Mastery of the multiplication law enhances problem-solving skills in algebra, calculus, and scientific computations.

Conclusion

Using the multiplication law for logarithms is a powerful technique to simplify and expand logarithmic expressions like [tex]\log_{8} (21)[/tex]. By recognizing the factors within the argument, applying the property, and converting to manageable bases when necessary, you can efficiently analyze and compute logarithmic values. This approach not only makes calculations easier but also deepens your understanding of logarithmic properties and their applications across mathematics and science.

Whether you're solving equations, simplifying expressions, or exploring mathematical concepts, mastering the multiplication law for logarithms is an essential skill that enhances your analytical toolkit. Practice applying this property to various problems to build confidence and proficiency in logarithmic operations.

Frequently Asked Questions

How do you apply the multiplication law for logarithms to expand $\log_8(21)$?

Since 21 can be expressed as a product of factors, you can decompose it using the multiplication law: log_b(xy) = log_b(x) + log_b(y). However, in this case, 21 is prime factorized as 3×7, so $\log_8(21) = \log_8(3$

× 7) = $\log_8(3) + \log_8(7)$.

Can the multiplication law be used directly to expand $\log_8(21)$?

Yes, because 21 can be written as a product of numbers, allowing the use of the law: log_b(xy) = log_b(x) + log_b(y). This simplifies the expression into the sum of two logarithms.

What is the significance of the base 8 in applying the logarithm laws?

The base 8 remains constant in all steps. When expanding using the multiplication law, the base does not change, allowing the expression to be separated into sums of simpler logs.

How do you evaluate $\log_8(3)$ and $\log_8(7)$ for the expansion?

You can evaluate these logs using change of base formula: log_b(a) = log(a) / log(b). So, $\log_8(3) = \log(3) / \log(8)$, and $\log_8(7) = \log(7) / \log(8)$, where logs can be in any common base like natural logs.

Is the multiplication law applicable if the argument inside the logarithm is a sum or difference?

No, the multiplication law applies only when the argument is a product. For sums or differences inside the logarithm, other laws like the product, quotient, or power laws should be used.

What are some real-world applications of expanding logarithms using the multiplication law?

This technique is useful in simplifying complex logarithmic expressions in fields like engineering, computer science, and finance, especially when dealing with multiplicative relationships or converting products into sums for easier computation.

Can you expand $\log_8(21)$ further into individual logarithms of prime factors?

Yes, since 21 = 3 × 7, the expansion is $\log_8(21) = \log_8(3) + \log_8(7)$. Further expansion depends on whether you can evaluate or approximate these logs.

What is the final expanded form of $\log_8(21)$ using the multiplication law?

The expanded form is $\log_8(21) = \log_8(3) + \log_8(7)$, expressing the logarithm as a sum of two simpler logarithms based on the multiplication law.

Additional Resources

Use The Multiplication Law For Logarithm To Expand The Following Expression: \(\log_{8} (21)\)

Understanding how to manipulate and expand logarithmic expressions is fundamental in higher mathematics, particularly in algebra and calculus. The problem at hand involves applying the multiplication law of logarithms to expand \(\log_{8}(21)\). This process not only simplifies complex expressions but also deepens comprehension of the relationships between logarithms and exponents. In this article, we will explore the multiplication law for logarithms in detail, demonstrate its application step-by-step to the expression \(\log_{8}(21)\), and discuss related concepts, advantages, and potential pitfalls.

Understanding Logarithms and Their Laws

Before delving into the expansion of $\log_{8}(21)$, it is essential to have a clear understanding of logarithms and their fundamental properties. Logarithms are the inverse operations of exponents. For a positive real number $a \neq 1$, the logarithm $\log_{a}(x)$ answers the question: "To what power must a be raised to obtain x?"

Mathematically, this is expressed as:
$$
a^{\log_{a}(x)} = x
$$

Key logarithmic laws include:

- Product Law: $\log_{a}(xy) = \log_{a}(x) + \log_{a}(y)$
- Quotient Law: $\log_{a}\left(\frac{x}{y}\right) = \log_{a}(x) - \log_{a}(y)$
- Power Law: $\log_{a}(x^{k}) = k \log_{a}(x)$
- Change of Base: $\log_{a}(x) = \frac{\log_{b}(x)}{\log_{b}(a)}$

In this discussion, the focus is on the product law (also called the multiplication law), which states that the logarithm of a product can be expanded into the sum of logarithms.

The Multiplication Law for Logarithms

What is the Multiplication Law?

The multiplication law is a foundational property of logarithms, which states:
$$
\boxed{
\log_{a}(xy) = \log_{a}(x) + \log_{a}(y)
}
$$

This law allows us to expand the logarithm of a product into a sum of two logarithms. Conversely, it can be used to factor a product into the sum of logs, which simplifies calculations especially when dealing with large or complex expressions.

Why Is It Useful?

- Simplifies the calculation of logarithms involving products.
- Facilitates the solving of algebraic equations involving logs.
- Assists in converting multiplicative relationships into additive ones, which are easier to manipulate.

Applying the Law to $\log_{8}(21)$

The challenge arises because 21 is not a product of two numbers with simple logs, nor is it directly expressed as a product of powers of 8. To utilize the multiplication law effectively, we need to factor 21 into factors that are more manageable in terms of base 8.

Expanding $\log_{8}(21)$ Using the Multiplication Law

Step 1: Factor 21 into manageable components

Since 21 is a prime times a prime (3 and 7), its prime factorization is:

$$
21 = 3 \times 7
$$

Using the multiplication law:

$$
\log_{8}(21) = \log_{8}(3 \times 7) = \log_{8}(3) + \log_{8}(7)
$$

This simplifies the original expression into a sum of two logs, each with a different argument.

Step 2: Express $\log_{8}(3)$ and $\log_{8}(7)$

While these logs are simpler, they are still not straightforward to compute directly because 3 and 7 are not powers of 8. To evaluate or approximate these logs, we can employ the change of base formula:

$$
\log_{a}(x) = \frac{\log_{b}(x)}{\log_{b}(a)}
$$

for any convenient base b, commonly 10 or e.

Step 3: Apply the change of base formula

Using base 10 logarithms for simplicity:

$$
\log_{8}(3) = \frac{\log_{10}(3)}{\log_{10}(8)}
$$

$$
\log_{8}(7) = \frac{\log_{10}(7)}{\log_{10}(8)}
$$

Numerical approximations:
- $\log_{10}(3) \approx 0.4771$
- $\log_{10}(7) \approx 0.8451$
- $\log_{10}(8) \approx 0.9031$

Thus:

$$
\log_{8}(3) \approx \frac{0.4771}{0.9031} \approx 0.5278
$$

$$
\log_{8}(7) \approx \frac{0.8451}{0.9031} \approx 0.9354
$$

and therefore:

$$
\boxed{
\log_{8}(21) \approx 0.5278 + 0.9354 = 1.4632
}
$$

Step 4: Summary of the expansion

By applying the multiplication law:

$$

$$
\log_{8}(21) = \log_{8}(3) + \log_{8}(7)
$$
which can be approximated numerically as approximately 1.4632.

Advantages and Limitations of Using the Multiplication Law

Pros

- Simplification: Breaks down complex logs into manageable parts.
- Flexibility: Enables conversion into different bases for easier calculation.
- Foundation for Further Expansion: Serves as a basis for other laws, like the change of base or power law.
- Useful in solving equations: Facilitates algebraic manipulations in equations involving logs.

Cons

- Requires factorization: Only applicable when the argument can be factored into manageable components.
- Approximate calculations: When combined with change of base, results are often approximations unless exact logs are known.
- Limited for prime numbers: When the argument is a prime or does not factor nicely, the law offers limited direct benefit.

Additional Insights and Related Concepts

Changing to a Common Base

For more precise calculations, especially when working without a calculator, converting to a common base like 10 or e is necessary. This method relies on the change of base formula and is invaluable for expanding or simplifying logs.

Logarithmic Identities and Their Interplay

The multiplication law often works in tandem with other laws:
- Power Law: To handle expressions like $\log_{a}(x^{k})$.
- Quotient Law: To deal with division inside logs.
- Change of Base: To evaluate logs numerically.

Real-world Applications

- Engineering: Signal processing often involves logarithms.
- Computer Science: Logarithmic complexity analysis.
- Finance: Logarithmic returns and growth models.
- Science and Mathematics: Solving exponential equations and modeling phenomena.

Conclusion

Applying the multiplication law for logarithms to expand $\log_{8}(21)$ illustrates the power and utility of fundamental logarithmic properties. By factoring 21 into 3 and 7, and then using the change of base, we can approximate the value of the logarithm efficiently. While this approach simplifies complex expressions, it also highlights the importance of understanding the foundational laws of logarithms and their proper application in problem-solving.

This method exemplifies how algebraic properties translate into practical tools for mathematical analysis, making complex calculations more manageable. Whether in pure mathematics, applied sciences, or everyday problem-solving, mastering the multiplication law and related properties is essential for anyone seeking a deeper understanding of logarithmic functions.

Features of Using the Multiplication Law for Logarithms:

- Simplifies expressions involving products.
- Connects multiplicative and additive relationships.
- Facilitates conversions between bases for computational ease.

Potential Limitations:

- Requires factorable arguments.
- Often necessitates numerical approximation.
- Less effective when dealing with prime arguments without further factorization.

By mastering the multiplication law and understanding how to combine it with other properties, learners and professionals alike can expand their mathematical toolkit for tackling a wide range of problems involving logarithms.

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