Use The Ratio Test To Determine Whether N=2[infinity]n!n+4 Converges Or Diverges. (a) Find The Ratio

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When analyzing infinite series, a fundamental aspect is to determine whether the series converges or diverges. One effective method for such an analysis is the Ratio Test, also known as the d'Alembert's ratio test. This test is particularly useful for series involving factorials, exponentials, or other functions with rapid growth. In this article, we will examine the series:

$$
\sum_{n=2}^{\infty} n! (n + 4)
$$

and apply the Ratio Test to ascertain whether this series converges or diverges. The initial step in this process is to find the ratio of successive terms, which will allow us to evaluate the limit necessary for the Ratio Test.

Understanding the Series and Its Terms

Before diving into the ratio calculation, it's crucial to understand the nature of the series and its individual terms.

Series Definition

The series in question is:

$$
\sum_{n=2}^{\infty} a_n
$$

where

$$
a_n = n! (n + 4)
$$

This series sums the terms starting from $n=2$ up to infinity.

Characteristics of the Terms

- Factorial Component: $n!$ grows very rapidly as n increases.
- Linear Component: $n + 4$ increases linearly with n.
- Combined Behavior: The product $n! (n + 4)$ indicates the terms grow faster than exponential functions, suggesting the series might diverge, but confirmation via the Ratio Test is necessary.

Applying the Ratio Test

The Ratio Test states:

> For a series $\sum a_n$, compute

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
$$

> If $L < 1$, the series converges absolutely.

> If $L > 1$ or $L = \infty$, the series diverges.

> If $L = 1$, the test is inconclusive.

Our goal is to compute the limit:

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
$$

Calculating the Ratio $\frac{a_{n+1}}{a_n}$

Let's explicitly write out a_{n+1}:

$$
a_{n+1} = (n+1)! \times (n+1 + 4) = (n+1)! \times (n+5)
$$

Recall $a_n = n! \times (n + 4)$.

Therefore, the ratio is:

\[

\frac{a_{n+1}}{a_n} = \frac{(n+1)! \times (n+5)}{n! \times (n+4)}
\]

Simplifying the Ratio

We can simplify the factorial part:

\[
(n+1)! = (n+1) \times n!
\]

Substituting back:

\[
\frac{a_{n+1}}{a_n} = \frac{(n+1) \times n! \times (n+5)}{n! \times (n+4)}
\]

Cancel \(n! \):

\[
\frac{a_{n+1}}{a_n} = \frac{(n+1)(n+5)}{n+4}
\]

This simplifies to:

\[
\boxed{
\frac{a_{n+1}}{a_n} = \frac{(n+1)(n+5)}{n+4}
}
\]

Evaluating the Limit as \(n \to \infty \)

To apply the Ratio Test, we need to compute:

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty}
\frac{(n+1)(n+5)}{n+4}
\]

Since all terms are positive for sufficiently large \(n \), the absolute value isn't necessary. Now, analyze the limit:

$$
L = \lim_{n \to \infty} \frac{(n+1)(n+5)}{n+4}
$$

Limit Calculation

Expand numerator:

$$
(n+1)(n+5) = n^2 + 5n + n + 5 = n^2 + 6n + 5
$$

So,

$$
L = \lim_{n \to \infty} \frac{n^2 + 6n + 5}{n + 4}
$$

Divide numerator and denominator by n:

$$
L = \lim_{n \to \infty} \frac{\frac{n^2}{n} + \frac{6n}{n} + \frac{5}{n}}{\frac{n}{n} + \frac{4}{n}} = \lim_{n \to \infty} \frac{n + 6 + \frac{5}{n}}{1 + \frac{4}{n}}
$$

As $n \to \infty$, $\frac{5}{n} \to 0$ and $\frac{4}{n} \to 0$, so:

$$
L = \frac{\infty + 6 + 0}{1 + 0} = \infty
$$

Conclusion from the Ratio Test

Since:

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty
$$

which is greater than 1, the Ratio Test indicates that the series

$$
\sum_{n=2}^{\infty} n! (n + 4)

\]

diverges.

Summary and Implications

The application of the Ratio Test has shown that the terms of the series grow too rapidly for the series to converge. Specifically, because $L \to \infty$, the series diverges. This conclusion aligns with the intuition that factorial growth dominates linear and exponential growth, leading to divergence.

Key points to remember:

- The factorial term $n!$ grows faster than any exponential.
- When combined with a linear factor $(n + 4)$, the terms a_n still tend to infinity rapidly.
- The Ratio Test effectively captures this growth by showing the ratio of successive terms tends to infinity.
- Series with factorials in the numerator tend to diverge unless factors are specifically designed to counterbalance the growth.

Additional Considerations

While the Ratio Test confirms divergence, it’s worth noting alternative tests or approaches:

- Root Test: Could also confirm divergence due to factorial growth.
- Comparison Test: Comparing with known divergent series such as $\sum n!$ confirms divergence.
- Asymptotic Analysis: The factorial growth dominates polynomial or exponential growth, ensuring divergence.

Final Remarks

In conclusion, applying the Ratio Test to the series $\sum_{n=2}^\infty n!(n+4)$ reveals that the series diverges. The critical step involved calculating the ratio of successive terms and evaluating its limit as $n \to \infty$. The result, $L = \infty$, clearly indicates divergence, emphasizing the importance of the Ratio Test in analyzing series with factorial components.

Understanding the behavior of such series is essential in advanced calculus and mathematical analysis, especially when dealing with factorials and growth rates. Always consider multiple convergence tests when analyzing series, and pay close attention to the growth rates of terms to draw accurate conclusions.

Keywords: Ratio Test, Series Convergence, Divergence, Factorials, Infinite Series, Limit, Mathematical Analysis, Series Test, N factorial, Series Analysis

Frequently Asked Questions

What is the purpose of using the Ratio Test in analyzing the series N=2 to infinity of n! / (n + 4)?

The Ratio Test helps determine whether the series converges or diverges by examining the limit of the ratio of successive terms.

How do you set up the ratio test for the series Σ (n! / (n + 4)) from n=2 to infinity?

You compute the limit as n approaches infinity of |a_{n+1} / a_n|, where a_n = n! / (n + 4).

What is the expression for the ratio a_{n+1} / a_n in this series?

a_{n+1} / a_n = [(n+1)! / (n+5)] ÷ [n! / (n+4)] = [(n+1)! / (n+5)] [(n+4) / n!].

How do you simplify the ratio (n+1)! / n!?

Since (n+1)! = (n+1) n!, the ratio simplifies to (n+1).

What is the simplified form of the ratio a_{n+1} / a_n?

The ratio simplifies to (n+1) (n+4) / (n+5).

What is the limit of the ratio as n approaches infinity?

lim_{n→∞} [(n+1)(n+4)] / (n+5) = lim_{n→∞} (n^2 + 5n + 4) / (n+5).

What is the value of the limit, and what does it imply about the convergence?

The limit is infinity, which means the series diverges according to the Ratio Test.

Based on the Ratio Test result, does the series N=2 to infinity of n! / (n + 4) converge or diverge?

It diverges because the limit of the ratio as n approaches infinity is infinity, exceeding 1.

Additional Resources

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Introduction

In the realm of mathematical analysis, especially within infinite series and sequences, convergence tests serve as essential tools to determine whether a series approaches a finite limit or diverges to infinity. Among these, the Ratio Test (also known as the d'Alembert's ratio test) stands out due to its simplicity and broad applicability, particularly for series involving factorials, exponentials, or powers.

This article thoroughly examines the series:

\[
\sum_{n=2}^{\infty} \frac{n!}{n + 4}
\]

and applies the Ratio Test to assess whether this series converges or diverges. We will explore the derivation of the ratio, interpret the result, and analyze the implications in detail, providing a comprehensive understanding suitable for students, educators, and mathematical enthusiasts alike.

The Series in Focus: A Closer Look

The series under consideration is:

\[
\sum_{n=2}^{\infty} \frac{n!}{n + 4}
\]

At first glance, the factorial in the numerator suggests rapid growth, which often hints at divergence. However, the presence of the linear denominator \(n + 4\) moderates the growth somewhat. To rigorously determine the convergence status, the Ratio Test provides an effective method.

The Ratio Test: An Overview

The Ratio Test states that for a series $\sum a_n$, if

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
$$

exists, then:

- If $L < 1$, the series converges absolutely.
- If $L > 1$, the series diverges.
- If $L = 1$, the test is inconclusive.

Applying this to our series involves calculating the limit of the ratio $\frac{a_{n+1}}{a_n}$, where

$$
a_n = \frac{n!}{n + 4}
$$

Step-by-Step Calculation of the Ratio

1. Expressing a_{n+1}:

$$
a_{n+1} = \frac{(n+1)!}{(n+1) + 4} = \frac{(n+1)!}{n + 5}
$$

2. Computing the ratio $\frac{a_{n+1}}{a_n}$:

$$
\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{n + 5}}{\frac{n!}{n + 4}} = \frac{(n+1)!}{n + 5} \times \frac{n + 4}{n!}
$$

Simplify numerator and denominator:

$$
= \frac{(n+1)!}{n!} \times \frac{n + 4}{n + 5}
$$

Recall that:

$$
(n+1)! = (n+1) \times n!
$$

So,

$$
\frac{a_{n+1}}{a_n} = (n+1) \times \frac{n + 4}{n + 5}
$$

\]

Analyzing the Limit as \(n \to \infty\)

The limit of the ratio as \(n \to \infty\) is:

\[
L = \lim_{n \to \infty} (n+1) \times \frac{n + 4}{n + 5}
\]

Rearranged as:

\[
L = \lim_{n \to \infty} (n+1) \times \left(1 + \frac{4 - 5}{n + 5}\right)
\]

But it's more straightforward to analyze directly:

\[
L = \lim_{n \to \infty} (n+1) \times \frac{n + 4}{n + 5}
\]

Note that as \(n \to \infty\):

- \(n+1 \sim n\),
- \(n+4 \sim n\),
- \(n+5 \sim n\).

Hence,

\[
L \sim n \times \frac{n}{n} = n \times 1 = n
\]

Therefore,

\[
L = \lim_{n \to \infty} (n+1) \times \frac{n + 4}{n + 5} = \lim_{n \to \infty} (n+1) \times \left(1 + \frac{-1}{n + 5}\right)
\]

But the dominant behavior is:

\[
L \sim n \quad \text{as } n \to \infty
\]

which tends to infinity.

Interpretation of the Limit

Since:

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty
$$

and the ratio exceeds 1 (in fact, tends to infinity), the Ratio Test conclusively indicates that:

$$
\boxed{\text{The series diverges.}}
$$

Deep Dive: Why Does the Series Diverge?

The critical insight here is that the factorial growth in the numerator dominates any linear denominator's moderating effect. As n increases, $n!$ grows faster than any exponential or polynomial function, leading to the terms a_n becoming unbounded. In particular, the ratio $\frac{a_{n+1}}{a_n}$ tending to infinity signals that each successive term grows even larger relative to the previous one, preventing the series from converging.

This behavior aligns with the general understanding that factorials grow super-exponentially, and series involving factorials often diverge unless mitigated by factorial in the denominator or other damping factors.

Additional Observations and Confirmations

While the Ratio Test provides a clear verdict, it's instructive to verify through alternative methods or heuristic reasoning:

- Comparison Test: Since $a_n = \frac{n!}{n+4}$, compare to $b_n = n!$. The terms a_n grow roughly as $n!$, which diverges when summed from $n=2$ to infinity, confirming divergence.

- Root Test: The root test, examining $\lim_{n \to \infty} \sqrt[n]{a_n}$, would also tend to infinity, indicating divergence.

- Behavior of Terms: For large n, a_n behaves approximately as $n!$ divided by a linear term, which still diverges as the factorial dominates.

Summary and Implications

Applying the Ratio Test to the series $\sum_{n=2}^\infty \frac{n!}{n+4}$, we find that:

- The ratio $\frac{a_{n+1}}{a_n}$ increases without bound as $n \to \infty$.

- The limit L exceeds 1, specifically tending to infinity.
- The series diverges due to the super-exponential growth of factorials.

This conclusion aligns with the broader principles of series convergence, emphasizing that factorial growth in the numerator generally precludes convergence unless offset by comparable factorial or exponential factors in the denominator—conditions not met in this case.

Final Remarks

The Ratio Test remains a powerful tool in the mathematician's arsenal for analyzing series, especially those involving factorials or exponential functions. In this scenario, it succinctly demonstrates the divergence of the series $\sum_{n=2}^\infty \frac{n!}{n + 4}$, providing clarity and rigor to the convergence question.

Understanding the behavior of such series has broader implications in fields such as combinatorics, probability theory, and mathematical modeling, where factorial terms frequently appear. Recognizing divergence early helps prevent misapplications of series summation and guides the formulation of convergent alternatives or the need for different analytical approaches.

In conclusion: Using the Ratio Test, the series $\sum_{n=2}^\infty \frac{n!}{n + 4}$ diverges due to the unbounded growth of the ratio $\frac{a_{n+1}}{a_n}$ as n approaches infinity.

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