# Volume Of A Solid RevolutionThe Region Between The Graphs Of Y = X^2 And Y = 3x Isrotated Around The

**Volume Of A Solid RevolutionThe Region Between The Graphs Of Y = X^2 And Y = 3x Isrotated Around The** is a fundamental concept in calculus that involves calculating the volume of a three-dimensional object formed by rotating a specific region around an axis. This type of problem combines the principles of integration with geometric visualization, allowing students and professionals to understand how to find the volume of complex shapes generated by revolution. When dealing with the region between the graphs of $y = x^2$ and $y = 3x$, understanding the process of setting up and evaluating the integral is essential for solving such problems accurately and efficiently.

## Understanding the Region Between y = x^2 and y = 3x

### Identifying the Intersection Points

Before calculating the volume, it is crucial to understand the region bounded by the two curves. The first step involves finding the points where $y = x^2$ and $y = 3x$ intersect:

- Set the equations equal: $x^2 = 3x$
- Solve for x: $x^2 - 3x = 0$
- Factor: $x(x - 3) = 0$
- Solutions: $x = 0$ and $x = 3$

These intersection points, $x = 0$ and $x = 3$, define the interval over which the region exists.

### Sketching the Region

Visualizing the region helps in understanding the shape and the axis of revolution. The parabola $y = x^2$ opens upward, while the line $y = 3x$ is a straight line passing through the origin with a slope of 3. Between $x = 0$ and $x = 3$, $y = 3x$ lies above $y = x^2$, creating a bounded region that can be revolved to generate a solid.

## Choosing the Axis of Rotation

The axis around which the region is revolved significantly affects the method of calculating volume. Common axes include:

- **Revolving around the x-axis**: The most typical scenario, often involving washers or disks.

- **Revolving around the y-axis**: Suitable when integrating with respect to y or when the region is better expressed in terms of y.

For this article, we will focus on revolution around the x-axis, as it is a common and illustrative case.

# Setting Up the Integral for Volume

## Using the Disk/Washer Method

When the region is revolved around the x-axis, the volume V can be found by integrating the cross-sectional areas:

- In the disk method, the radius of each disk is determined by the distance from the axis of rotation to the curve.
- In the washer method, the region between two curves creates a hollow disk with an inner and outer radius.

Since $y = 3x$ is above $y = x^2$ in the interval, the outer radius is the line $y = 3x$ and the inner radius is $y = x^2$.

## Formulating the Integral

The volume of the solid obtained by revolving the bounded region around the x-axis is given by:

$$V = \pi \int_{a}^{b} \left[ R_{\text{outer}}^{2} - R_{\text{inner}}^{2} \right] dx$$

where:

- $R_{outer}$ = y-value of the line $y = 3x$
- $R_{inner}$ = y-value of the parabola $y = x^2$

Substituting:

$$R_{\text{outer}} = 3x$$

$$R_{\text{inner}} = x^2$$

and the limits $a = 0$, $b = 3$:

$$V = \pi \int_{0}^{3} \left[ (3x)^2 - (x^2)^2 \right] dx$$

which simplifies to:

$$V = \pi \int_{0}^{3} \left[ 9x^2 - x^4 \right] dx$$

# Calculating the Integral

## Performing the Integration

Integrate term-by-term:

$$
V = \pi \left[ \int_{0}^{3} 9x^2 dx - \int_{0}^{3} x^4 dx \right]
$$

Compute each integral:

$$
\int 9x^2 dx = 9 \times \frac{x^3}{3} = 3x^3
$$

$$
\int x^4 dx = \frac{x^5}{5}
$$

Evaluate at the limits:

$$
V = \pi \left[ 3x^3 \Big|_{0}^{3} - \frac{x^5}{5} \Big|_{0}^{3} \right]
$$

Calculate:

$$
3 \times (3)^3 - 0 = 3 \times 27 = 81
$$

$$
\frac{(3)^5}{5} - 0 = \frac{243}{5}
$$

Subtract:

$$
V = \pi \left( 81 - \frac{243}{5} \right)
$$

Expressing with common denominator:

$$
V = \pi \left( \frac{405}{5} - \frac{243}{5} \right) = \pi \times \frac{162}{5}
$$

Final volume:

$$
V = \frac{162\pi}{5}
$$

which is approximately 101.787 cubic units.

# Alternative Method: Shell Method

While the disk/washer method is straightforward for rotation around the x-axis, the shell method provides an alternative approach, especially useful when revolving around the y-axis or when the region is better expressed in terms of y.

## Using the Shell Method

The shell method involves integrating cylindrical shells:

$$
V = 2\pi \int_{a}^{b} \text{radius} \times \text{height} \, dx

In this case:

- Radius = y (distance from y-axis)
- Height = difference between the x-values of the curves at a given y

Express the x-values from the equations:

$$
y = x^2 \Rightarrow x = \sqrt{y}
$$

$$
y = 3x \Rightarrow x = \frac{y}{3}
$$

Between x = 0 and x = 3, y ranges from 0 to 9 (since at x=3, y=9). For a given y in [0, 9], the x-values are between $\sqrt{y}$ and $y/3$. The height of the shell at y is:

$$
h(y) = \frac{y}{3} - \sqrt{y}
$$

The volume integral becomes:

$$
V = 2\pi \int_{0}^{9} y \times h(y) dy
$$

which can be computed accordingly.

# Applications of Volume of Revolution

Understanding how to compute the volume of solids of revolution has numerous applications in engineering, physics, and manufacturing:

- Designing objects with rotational symmetry, such as bottles or vases
- Calculating the volume of irregularly shaped objects for material estimation
- Modeling physical phenomena like the distribution of mass in rotating bodies
- Analyzing volume change in processes involving rotation and slicing

Mastery of these techniques enables precise calculations essential for real-world problem-solving.

# Summary and Key Takeaways

- The region between $y = x^2$ and $y = 3x$ from x=0 to x=3 can be revolved around an axis to generate a solid.
- The volume is calculated using the disk/washer method by integrating the difference of

squares of the outer and inner radii.

- The integral simplifies to $\frac{162\pi}{5}$, approximately 101.787 cubic units.
- Alternative methods like the shell method provide flexibility based on the axis of rotation and the region's shape.
- Understanding these concepts is fundamental for advanced applications in calculus and related fields.

By mastering the process of setting up and evaluating integrals for volumes of revolution, students and professionals can analyze complex shapes with confidence, opening doors to innovative solutions in science and engineering.

Remember: Always verify the region boundaries, choose the most suitable method, and carefully set up your integral for accurate results.

# Frequently Asked Questions

## What is the method to find the volume of a solid formed by rotating the region between $y = x^2$ and $y = 3x$ around a specified axis?

The method typically involves setting up an integral using the disk/washer method or the shell method, depending on the axis of rotation, to compute the volume of the solid formed by rotating the region between $y = x^2$ and $y = 3x$.

## How do you determine the limits of integration when calculating the volume of the revolution between $y = x^2$ and $y = 3x$?

The limits are found by solving $y = x^2$ and $y = 3x$ for their points of intersection, which give the x-values where the region starts and ends.

## What is the significance of choosing the axis of rotation in calculating the volume of the solid between $y = x^2$ and $y = 3x$?

The axis of rotation determines whether you use the disk/washer or shell method and affects how you set up the integral, as it impacts the radius functions and the limits of integration.

## Can the volume of the solid be found by integrating with

## respect to x or y? How do you decide which variable to integrate with respect to?

Yes, the volume can be found by integrating with respect to x or y. The choice depends on the axis of rotation and the orientation of the region; typically, you choose the variable that simplifies the integral setup.

## What is the typical process to set up the integral for the volume when rotating around the x-axis for the region between y = x^2 and y = 3x?

You set up the integral using the washer method, where the radius functions are derived from the y-values, and integrate with respect to x over the intersection points of the curves.

## How do you verify that the limits of integration are correct when calculating the volume of the rotated region?

You verify by solving the intersection points of y = x^2 and y = 3x to find the correct x-values, and ensure the integral covers the entire region between these points.

## What challenges might arise when calculating the volume of the solid formed by rotating y = x^2 and y = 3x, and how can they be addressed?

Challenges include setting up the correct bounds and choosing the appropriate method (disk or shell). These can be addressed by carefully analyzing the region, solving for intersection points, and selecting the method that simplifies the integral.

## Is it possible to find the volume by rotating around the y-axis instead of the x-axis? How does this change the setup?

Yes, rotating around the y-axis is possible. It requires expressing x as a function of y and setting up the integral accordingly, often using the shell method with respect to y.

## What are common applications of calculating volumes by rotating regions between curves like y = x^2 and y = 3x?

These calculations are useful in engineering, physics, and manufacturing for determining the volume of objects with curved surfaces, designing containers, and solving problems involving rotational symmetry.

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Introduction

The exploration of volumes generated by the rotation of regions bounded by curves is a fundamental topic in integral calculus, bridging geometric intuition with analytical rigor. Among the myriad of problems in this domain, calculating the volume of a solid of revolution formed by rotating a specific region around a given axis stands out for its blend of conceptual depth and practical application.

In this comprehensive investigation, we delve into the problem of determining the volume of the solid obtained by revolving the region bounded between the curves $Y = X^2$ and $Y = 3X$ around a specified axis. The problem not only exemplifies the classical techniques of integration but also highlights the importance of understanding the geometric relationships and the careful setup of integrals.

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Context and Significance

The problem of calculating the volume of a solid of revolution is a cornerstone in multivariable calculus, with applications spanning engineering, physics, and even biological sciences. It enables practitioners to model real-world objects, from manufactured parts to biological structures, by understanding how areas transform into three-dimensional bodies through rotation.

Specifically, the region between the parabola $Y = X^2$ and the line $Y = 3X$ presents an interesting case study because:

- The intersection points define the limits of integration and are essential for setting up the problem.
- The region's shape leads to different methods of calculation depending on the axis of revolution.
- It offers insight into the comparison between disk/washer methods and cylindrical shells.

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Defining the Region: Initial Analysis

Before approaching the volume calculation, it is crucial to understand the geometric properties of the bounded region.

The Curves:

- $Y = X^2$: a standard parabola opening upwards.
- $Y = 3X$: a straight line with a positive slope.

Intersection Points:

To find the points where the curves intersect:

Set $X^2 = 3X$:

$X^2 - 3X = 0$
$X(X - 3) = 0$

Thus, $X = 0$ or $X = 3$.

Corresponding Y-values:

- At $X=0$: $Y= 0$
- At $X=3$: $Y= 3\times3=9$

Therefore, the region is bounded between $X=0$ and $X=3$, with the parabola and line crossing at these points.

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Choosing the Axis of Revolution

The axis of rotation significantly influences the approach:

- Revolution around the x-axis: involves slicing perpendicular to the x-axis, leading to disk or washer methods.
- Revolution around the y-axis: involves slicing perpendicular to the y-axis, potentially more complex due to the functions' forms.
- Revolution around other axes: such as y=constant lines, adds additional layers of complexity.

For clarity and illustrative purposes, this article will consider the revolution around the x-axis, a common and instructive case.

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Mathematical Framework and Methodology

The problem necessitates setting up integrals based on the chosen method:

- Disk/Washer Method: suited when the region is revolved around an axis parallel to the coordinate axes, with the shape of cross-sections being disks or washers.
- Cylindrical Shells Method: advantageous when dealing with regions where the vertical or horizontal slices are more manageable, especially if the region extends along the axis of revolution.

Given the region between the curves and the axis of revolution:

For revolution around the x-axis:

- The washer method is typically employed.
- Cross-sectional disks are perpendicular to the x-axis.
- The outer and inner radii depend on the functions' positions relative to the x-axis.

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Step-by-Step Calculation

1. Setup Using the Washer Method

- For each X in [0,3], the vertical cross-section is a washer with:

- Outer radius: the distance from the x-axis to the upper boundary of the region.
- Inner radius: the distance from the x-axis to the lower boundary.

- Since the region is between $Y= X^2$ and $Y=3X$, the following applies:

- The line $Y=3X$ is above the parabola $Y= X^2$ between $X=0$ and $X=3$.
- The outer radius (from x-axis to the line): $R\_outer = 3X$
- The inner radius (from x-axis to the parabola): $R\_inner = X^2$

- The volume element dV (a washer) is:

$dV = \pi [R\_outer]^2 - \pi [R\_inner]^2 dx = \pi ( (3X)^2 - (X^2)^2 ) dx$

- Integrate from X=0 to X=3:

$V = \int_0^3 \pi [ (3X)^2 - (X^2)^2 ] dx$

2. Simplification of the Integral

- Compute the integrand:

$(3X)^2 = 9X^2$
$(X^2)^2 = X^4$

- Therefore:

$V = \pi \int_0^3 [ 9X^2 - X^4 ] dx$

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Evaluation of the Integral

Calculate:

$V = \pi [ \int_0^3 9X^2 dx - \int_0^3 X^4 dx ]$

- First integral:

$\int 9X^2 dx = 9 (X^3 / 3) = 3X^3$

- Second integral:

$\int X^4 dx = X^5 / 5$

- Applying limits:

$V = \pi [ 3X^3 |_0^3 - X^5 / 5 |_0^3 ]$

$= \pi [ 3(3)^3 - (3)^5 / 5 - 0 + 0 ]$
$= \pi [ 3 27 - 243 / 5 ]$
$= \pi [ 81 - 48.6 ]$ (since $243/5 = 48.6$)

- Final calculation:

$V \approx \pi (32.4) \approx 32.4 \pi$

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Interpretation and Implications

The calculated volume, approximately $32.4\pi$ cubic units, signifies the three-dimensional space generated when the specified region between the parabola and the line is revolved about the x-axis. This value reflects the geometric complexity of the region, emphasizing the importance of precise integral setup and the appropriate choice of method.

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Extending the Analysis: Alternative Axes and Methods

While the above calculation focuses on rotation around the x-axis, other axes of revolution can be considered:

Revolution around the y-axis:

- Requires expressing the functions as X in terms of Y, leading to different integral bounds and potentially the shell method being more advantageous.

Revolution around lines other than the axes:

- For example, around y=constant lines, which may involve shifting or changing variables.

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Practical Applications and Broader Context

Understanding the volume of such solids has practical relevance:

- Engineering: designing rotational parts where the shape is derived from bounded regions.
- Manufacturing: estimating material usage for objects formed by rotating planar regions.
- Physics: modeling rotational bodies in dynamics and thermodynamics.

Moreover, the problem underscores the importance of:

- Accurately determining the intersection points for setting bounds.
- Choosing the most efficient method based on the region's shape and the axis of rotation.
- Simplifying integrals through algebraic manipulation for computational feasibility.

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Concluding Reflections

The investigation of the volume of a solid revolution formed by rotating the region between $Y = X^2$ and $Y = 3X$ around the x-axis exemplifies the synergy between geometric intuition and calculus. It demonstrates the critical importance of careful region analysis, strategic method selection, and precise integral setup.

This problem not only reinforces foundational calculus concepts but also opens avenues for further exploration, such as considering different axes of rotation, employing numerical methods for complex regions, and applying these techniques to real-world engineering and scientific challenges.

Ultimately, mastering such problems enhances mathematical literacy and provides a robust toolkit for tackling three-dimensional modeling problems across diverse disciplines.

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