We Want To Determine If The Sequence 68n Is Monotonic. Using The Difference Test We Get That Sn 1sn=

We Want To Determine If The Sequence 68n Is Monotonic. Using The Difference Test We Get That Sn 1sn= has become a foundational question in mathematical analysis, especially when exploring the behavior of sequences. Understanding whether a sequence is monotonic—either entirely non-increasing or non-decreasing—helps in establishing limits, convergence, and other important properties in calculus and analysis. This article delves deeply into the process of determining the monotonicity of the sequence $a_n = 68n$ by employing the difference test, exploring the concepts involved, and illustrating the process step by step.

Understanding Monotonic Sequences

What Is a Monotonic Sequence?

A sequence $\{a_n\}$ is called monotonically increasing (or non-decreasing) if each term is greater than or equal to the previous one:

$$
a_{n+1} \geq a_n \quad \text{for all } n.
$$

Similarly, it is monotonically decreasing (or non-increasing) if each term is less than or equal to the previous:

$$
a_{n+1} \leq a_n \quad \text{for all } n.
$$

A sequence that is either entirely non-decreasing or non-increasing is called monotonic.

Why Is Monotonicity Important?

Monotonic sequences are crucial because they often simplify the process of finding limits and analyzing convergence. For example:
- Every bounded monotonic sequence converges (by the Monotone Convergence Theorem).
- Monotonic sequences provide straightforward insights into the behavior of functions and series.

The Sequence $a_n = 68n$

Definition and Intuition

The sequence in question is:

$$
a_n = 68n,
$$

where n is a natural number (or positive integer). Since 68 is a positive constant, the sequence essentially scales the natural numbers by 68.

Intuitive understanding: As n increases, a_n increases linearly because multiplying by a positive number preserves the order. Therefore, it's natural to suspect that the sequence is increasing.

Applying the Difference Test to Determine Monotonicity

The Difference Test Explained

The difference test involves examining the difference between consecutive terms:

$$
\Delta a_n = a_{n+1} - a_n.
$$

- If $\Delta a_n \geq 0$ for all n, then the sequence is non-decreasing.
- If $\Delta a_n \leq 0$ for all n, then the sequence is non-increasing.
- If the sign of Δa_n changes, the sequence is not monotonic.

Calculating the Difference for $a_n = 68n$

Let's compute:

$$
a_{n+1} - a_n = 68(n+1) - 68n = 68n + 68 - 68n = 68.
$$

Since this difference is constant and equal to 68 (a positive number), it indicates that for every n,

$$
a_{n+1} - a_n = 68 > 0.
$$

Interpretation: The sequence increases by 68 units each step, confirming it is increasing.

Conclusion on Monotonicity

Is the Sequence Monotonically Increasing?

Given:

$$
a_{n+1} - a_n = 68 > 0,
$$

for all n, the sequence $\{a_n\} = 68n$ is monotonically increasing.

Is the Sequence Monotonically Decreasing?

Since the difference is positive, the sequence cannot be decreasing.

Additional Properties of the Sequence $a_n = 68n$

Unboundedness and Limit

- The sequence $\{a_n\}$ is unbounded above because as $n \to \infty$,
$$
a_n = 68n \to \infty.
$$
- It does not have a finite limit, but it diverges to infinity.

Implications of Monotonicity

Since the sequence is increasing and unbounded, it diverges to infinity. This is a typical behavior for sequences like $a_n = c n$, where $c > 0$.

Summary and Key Takeaways

- The difference test involves calculating $\Delta a_n = a_{n+1} - a_n$.
- For $a_n = 68n$, $\Delta a_n = 68$ for all n.
- Since $\Delta a_n > 0$ everywhere, the sequence is monotonically increasing.
- The sequence diverges to infinity as n approaches infinity.
- Monotonicity helps in understanding the convergence or divergence of sequences, and in this case, confirms divergence due to unbounded growth.

Further Applications of the Difference Test

The difference test is not limited to simple sequences like $a_n = 68n$. It applies broadly to:
- Polynomial sequences
- Exponential sequences
- Sequences defined by recursive relations
- More complex functions where the general term involves variables or parameters

Example: For the sequence $b_n = \frac{1}{n}$,
$$
\Delta b_n = \frac{1}{n+1} - \frac{1}{n} = -\frac{1}{n(n+1)} < 0,
$$
which indicates the sequence is decreasing.

Example: For the sequence $c_n = (-1)^n$,
$$
\Delta c_n = (-1)^{n+1} - (-1)^n,
$$
which alternates between positive and negative, indicating non-monotonic

behavior.

Conclusion

In summary, by applying the difference test to the sequence $a_n = 68n$, we found that the difference between consecutive terms is always positive, confirming that the sequence is monotonically increasing. This analysis illustrates the power of the difference test as a straightforward method for establishing the monotonicity of sequences. Recognizing whether a sequence is monotonic is essential for understanding its long-term behavior, convergence properties, and for applying theorems like the Monotone Convergence Theorem. The sequence $68n$, with its clear and consistent increase, exemplifies a simple but fundamental case of a monotonic sequence that diverges to infinity.

Remember: Whenever analyzing sequences, always consider their differences. It provides a quick and reliable way to determine whether the sequence is increasing, decreasing, or non-monotonic.

Frequently Asked Questions

How can we determine if the sequence 68n is monotonic using the difference test?

To determine if 68n is monotonic, we analyze the difference between consecutive terms, specifically calculating $S_{n+1} - S_n$, and check if this difference is always non-negative or non-positive.

What does the difference test tell us about the monotonicity of the sequence 68n?

The difference test involves computing the difference between successive terms; if the difference is consistently positive, the sequence is increasing; if negative, decreasing; if it changes sign, the sequence is not monotonic.

How do we interpret the expression $S_{n+1} - S_n = 1 - s_n$ in the context of the sequence 68n?

This expression indicates that the change between consecutive terms depends on the value of s_n; analyzing the sign of $1 - s_n$ helps determine whether the sequence is increasing or decreasing at each step.

If the difference $S_{n+1} - S_n = 1 - s_n$ is positive for all n, what can we conclude about the sequence 68n?

If $1 - s_n > 0$ for all n, then the sequence is increasing (monotonically non-decreasing). Conversely, if it's negative for all n, the sequence is decreasing.

What additional information do we need to fully determine the monotonicity of the sequence 68n?

We need to know the explicit form or behavior of s_n to analyze the sign of 1 – s_n across all n, which in turn helps us conclude whether the sequence is monotonic.

Can the difference test be conclusive for the monotonicity of the sequence 68n without knowing s_n?

No, without specific information about s_n, the difference test alone cannot conclusively determine the monotonicity; we must analyze the actual values or formula of s_n to make a definitive conclusion.

Additional Resources

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Introduction

In the fascinating world of sequences and series, understanding the behavior of a sequence — whether it consistently increases, decreases, or fluctuates — is fundamental. Among the various types of sequences studied in mathematics, monotonic sequences hold particular importance due to their predictable nature. Today, we explore a specific sequence defined by the term 68n, where n is a natural number, and delve into whether this sequence exhibits monotonic behavior. To do this, we employ a common analytical tool known as the difference test, which allows us to determine the nature of the sequence's progression by examining the differences between successive terms. This article will provide a comprehensive, yet accessible, examination of the sequence 68n, illustrating how the difference test applies and what conclusions we can draw about its monotonicity.

Understanding the Sequence 68n

Before diving into the analysis, it’s essential to clarify what the sequence 68n represents. The notation suggests that each term in the sequence, denoted as S_n, is obtained by multiplying the index n by 68. Formally, we can write:

$$S_n = 68n$$

where n belongs to the set of natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$.

This sequence begins as:

- $S_1 = 68 \times 1 = 68$
- $S_2 = 68 \times 2 = 136$
- $S_3 = 68 \times 3 = 204$
- $S_4 = 68 \times 4 = 272$

and so on. It forms a simple linear sequence with a constant rate of increase, making it a prime candidate for monotonic analysis.

The Concept of Monotonic Sequences

A sequence is called monotonically increasing if each term is greater than or equal to the previous one, i.e.,

$$ S_{n+1} \geq S_n \quad \text{for all } n $$

Similarly, it is monotonically decreasing if each term is less than or equal to the previous one:

$$ S_{n+1} \leq S_n \quad \text{for all } n $$

If the sequence satisfies either of these conditions throughout, it is considered monotonic. This property is significant because monotonic sequences tend to have limits or exhibit predictable behavior, which is vital in various mathematical and real-world applications.

Applying the Difference Test

The difference test is a straightforward and effective method to analyze the monotonicity of a sequence. It involves examining the difference between successive terms:

$$ \Delta S_n = S_{n+1} - S_n $$

- If $\Delta S_n \geq 0$ for all n, the sequence is monotonically increasing.
- If $\Delta S_n \leq 0$ for all n, the sequence is monotonically decreasing.
- If the sign of ΔS_n varies, the sequence is not monotonic.

This test is particularly convenient for sequences defined by algebraic expressions because the difference simplifies to a function of n.

Calculating the Difference for $S_n = 68n$

Let's compute the difference $S_{n+1} - S_n$:

$$
\begin{aligned}
S_{n+1} &= 68(n+1) = 68n + 68 \\
S_n &= 68n
\end{aligned}
$$

Therefore,

$$
\Delta S_n = S_{n+1} - S_n = (68n + 68) - 68n = 68
$$

This calculation reveals a crucial insight: the difference between consecutive terms is a constant, 68, regardless of n.

Interpretation of the Difference and Monotonicity

Since $\Delta S_n = 68$ for all n, which is a positive constant, the sequence exhibits the following characteristics:

- For every n, $S_{n+1} - S_n = 68 > 0$.
- This means that each subsequent term is greater than the previous one by a fixed amount.

Consequently, the sequence $S_n = 68n$ is strictly increasing for all natural numbers n. The sequence does not fluctuate or decrease at any point; rather, it advances steadily upward, aligning with the definition of a monotonically increasing sequence.

Broader Implications and Significance

The analysis of this particular sequence exemplifies a broader principle: linear sequences with positive coefficients are inherently monotonic. Specifically:

1. Linear sequences with positive slopes are increasing: Since the common difference ΔS_n is positive, the sequence ascends without exception.
2. Constant difference signifies uniform growth: The fixed increment of 68 between terms reflects a steady rate of increase, which simplifies the analysis and ensures monotonicity.

In practical terms, such sequences model phenomena with consistent growth rates, like savings accumulating at a fixed rate or time-dependent measurements increasing uniformly. Recognizing their monotonic nature is vital in fields ranging from economics to physics, where predicting trends and behaviors depends on understanding the sequence’s progression.

Visualizing the Sequence

To further solidify the understanding, imagine plotting the sequence $S_n = 68n$ on a graph:

- The x-axis represents the term index n.
- The y-axis shows the value of the term S_n.

Given the linear nature, the graph would be a straight line with a slope of 68, starting at $S_1 = 68$ when $n=1$. The positive slope visually confirms the sequence's monotonic increase.

Summary of Key Findings

- The sequence $S_n = 68n$ is linear and defined over the natural

numbers.
- The difference between successive terms is a constant: \(\Delta S_n = 68 \).
- Since this difference is positive for all \(n \), the sequence is strictly increasing.
- Therefore, the sequence is monotonically increasing.

This straightforward application of the difference test illustrates that sequences with positive constant differences are inherently monotonic and increasing.

Final Thoughts: Why Monotonicity Matters

Understanding whether a sequence is monotonic is not just a theoretical exercise; it has practical implications. Monotonic sequences are predictable and often easier to analyze, especially when considering limits or convergence in calculus, or modeling real-world phenomena where consistent growth or decline is assumed.

In the case of the sequence \(68n \), knowing it is strictly increasing reassures us that as \(n \) grows larger, the sequence's terms will continue to grow without any dips or fluctuations. This property can be essential in scenarios such as:

- Setting expectations in financial models.
- Planning in engineering where certain parameters increase linearly.
- Analyzing time-series data with steady growth.

Ultimately, the difference test provides a simple yet powerful tool for mathematicians and scientists alike to determine the behavior of sequences and series, offering clarity amid potentially complex data.

Conclusion

Through the application of the difference test, we've established that the sequence \(S_n = 68n \) is strictly increasing and therefore monotonic. The constant positive difference between successive terms confirms the sequence's steady ascent. This analysis exemplifies how fundamental mathematical tools can elucidate the nature of sequences, offering insights that extend well beyond pure mathematics into practical and scientific domains. Understanding such properties helps in modeling, prediction, and analysis across diverse fields, reinforcing the importance of foundational concepts like monotonicity in the broader landscape of mathematical reasoning.

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