What Is An Equation Of The Line That Passes Through The Point (5 , 1) (5,1) And Is Parallel To The

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Understanding the equation of a line that passes through a specific point and runs parallel to another line is a fundamental concept in coordinate geometry. When given a point such as (5, 1) and a line to which the new line must be parallel, the goal is to determine an explicit algebraic expression—called the equation of the line—that satisfies these conditions.

The phrase "parallel to the" naturally leads to the question: parallel to what? Typically, in problems like this, the line in question is specified or implied, such as "parallel to the line $y = 2x + 3$" or some other known line. This article will explore the general principles involved, assuming the reference line is known, and will guide you through the process of deriving the equation of the desired line.

Understanding Parallel Lines and Their Properties

The Concept of Parallel Lines

Parallel lines are lines in a plane that are always equidistant from each other and never intersect, regardless of how far they are extended. In coordinate geometry, the key characteristic of parallel lines is that they share the same slope but have different y-intercepts.

Mathematical Representation of Parallel Lines

- If a line has the equation $y = mx + b$, where m is the slope and b is the y-intercept,
- Any line parallel to it will have the same slope m but a different y-intercept b'.

This property allows us to find the equation of a line parallel to a given line once we know its slope.

Determining the Equation of a Line Passing Through a Point and Parallel to a Given Line

Step 1: Identify the Slope of the Given Line

Suppose the line to which the new line must be parallel is given by:

- $y = m_1x + b_1$

The slope of this line is m_1.

Step 2: Use the Slope for the New Line

Since parallel lines share the same slope:

- The slope of the new line, m_2, is equal to m_1.

Step 3: Use the Point to Find the Equation

Given that the new line passes through the point $(x_0, y_0) = (5, 1)$:

- The general form of the line is $y = m_2x + b_2$

Plugging in the point (5, 1):

- $1 = m_2 5 + b_2$

Solving for b_2:

- $b_2 = 1 - 5m_2$

Therefore, the equation of the new line is:

- $y = m_2x + (1 - 5m_2)$

This formula gives all lines passing through (5, 1) and parallel to the original line $y = m_1x + b_1$.

Applying the Method: Examples

Example 1: Parallel to y = 2x + 3

- The slope of the original line, m_1, is 2.
- The new line must also have slope $m_2 = 2$.
- Using the point (5, 1):

$b_2 = 1 - 5\ 2 = 1 - 10 = -9$

- Equation of the new line:

$y = 2x - 9$

Example 2: Parallel to y = -0.5x + 4

- Slope: $m_1 = -0.5$
- Same slope: $m_2 = -0.5$
- Using point (5, 1):

$b_2 = 1 - 5\ (-0.5) = 1 + 2.5 = 3.5$

- Equation:

$y = -0.5x + 3.5$

General Form of the Equation of the Line

The form:

- $y = m\ x + b$

or, rearranged into standard form:

- $m\ x - y + c = 0$

where c is a constant determined by rearranging.

Given the point (5, 1) and the slope m, the equation can be written as:

- $y - y_0 = m(x - x_0)$

or in point-slope form:

- $y - 1 = m(x - 5)$

which can then be transformed into slope-intercept or standard form as needed.

Special Cases and Additional Considerations

Vertical Lines

If the original line is vertical, e.g., $x = a$, then:

- Any line parallel to it is also vertical, with equation $x = a$.
- If the new line passes through (5, 1), its equation is simply $x = 5$.

Horizontal Lines

If the original line is horizontal, e.g., $y = c$, then:

- Any line parallel to it is also horizontal with the same y-value.
- Passing through (5, 1), the line is $y = 1$.

Summary and Key Points

- To find the equation of a line passing through a specific point and parallel to a given line, determine the slope of the given line.
- Use the point-slope form to derive the new line's equation.
- The key property of parallel lines is equal slopes.
- The specific point (5, 1) provides the necessary data to find the y-intercept once the slope is known.
- Special cases include vertical and horizontal lines, which require different approaches.

Conclusion

Understanding the relationship between parallel lines and their equations is essential in coordinate geometry. By identifying the slope of the reference line and applying the point through which the new line passes, you can derive the equation of the line that satisfies both conditions. Whether dealing with standard lines, vertical, or horizontal lines, the principles remain consistent, allowing for precise and versatile problem-solving in geometric contexts.

This approach can be extended to more complex problems involving multiple lines and intersections, serving as a foundational skill in mathematics, physics, engineering, and related fields.

Frequently Asked Questions

What is the general form of the equation of a line passing through the point (5, 1) parallel to a given line?

The general form is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point (5, 1) and m is the slope of the line parallel to the given line.

How do I find the equation of a line passing through (5, 1) and parallel to y = 2x + 3?

Since the lines are parallel, they have the same slope m = 2. Using point-slope form: y - 1 = 2(x - 5). Simplifying gives y = 2x - 9.

What is the slope of a line passing through (5, 1) and parallel to y = -4x + 7?

The slope of the parallel line is the same as the given line, which is -4.

Can you give an example of the equation of a line passing through (5, 1) and parallel to y = 3x - 2?

Yes. Since the slope is 3, the line passing through (5, 1) is y - 1 = 3(x - 5), which simplifies to y = 3x - 14.

What steps are involved in writing the equation of a line through (5, 1)

parallel to a known line?

First, identify the slope of the known line. Then, use the point-slope form with the point (5, 1) and that slope. Finally, simplify to slope-intercept form if desired.

If a line passes through (5, 1) and is parallel to $y = -1/2x + 4$, what is its equation?

The slope of the parallel line is -1/2. Using point-slope form: $y - 1 = -1/2(x - 5)$, which simplifies to $y = -1/2x + 9/2$.

Additional Resources

Understanding the Equation of a Line Passing Through a Given Point and Parallel to Another Line

When exploring the fascinating world of coordinate geometry, a fundamental concept is understanding how to determine the equation of a line given specific characteristics. One common problem involves finding the equation of a line that passes through a particular point and is parallel to another line. This task not only reinforces the principles of slope and intercepts but also deepens comprehension of how geometric relationships translate into algebraic expressions.

In this detailed guide, we will examine the question: What is an equation of the line that passes through the point (5, 1) and is parallel to a given line? For clarity and completeness, we will break down each component, explore the underlying principles, and work through illustrative examples.

Fundamental Concepts in Line Equations and Parallelism

Before delving into solving specific problems, it's crucial to understand some foundational concepts:

1. The Equation of a Line in Various Forms

- Slope-Intercept Form ($y = mx + b$):
Represents a line where m is the slope, and b is the y-intercept.
Example: $y = 2x + 3$

- Point-Slope Form ($y - y_1 = m(x - x_1)$):

Used when a point (x_1, y_1) and slope m are known.
Example: For point (4, 5) and slope 3, $y - 5 = 3(x - 4)$.

- Standard Form ($Ax + By = C$):
A general form where A, B, and C are constants.
Example: $2x - 3y = 6$

2. The Concept of Slope and Its Role in Parallel Lines

- Slope (m):
Measures the steepness of a line, calculated as (change in y)/(change in x).
For two points (x_1, y_1) and (x_2, y_2):
$m = (y_2 - y_1) / (x_2 - x_1)$

- Parallel Lines:
Lines that have the same slope but different y-intercepts.
- Key property: If two lines are parallel, their slopes are equal.

Problem Breakdown: Finding a Line Through a Point and Parallel to Another Line

Let's analyze the problem step-by-step:

> Given:
> - A point (5, 1) through which the line passes
> - The line is parallel to another line (which we will define or assume)

> Question:
> - Find the equation of the line passing through (5, 1) and parallel to the given line.

Note: Since the specific line to which the new line is parallel isn't provided in the prompt, we will consider common scenarios:

- Scenario A: The original line's equation is known, e.g., $y = 2x + 3$.

- Scenario B: The line's slope is known but the equation isn't specified.

- Scenario C: The line is described verbally, e.g., "parallel to the line passing through (0, 0) and (4, 8)."

In each case, the core principle remains the same: find the slope of the given line, then use that slope and the point (5, 1) to form the new line's equation.

Determining the Slope of the Given Line

1. When the Equation of the Given Line Is Known

Suppose the line parallel to which our new line is to be constructed has the equation:

- $y = 2x + 3$

Here, the slope (m) of the line is directly visible:

- Slope (m): 2

2. When Only Two Points on the Given Line Are Known

Suppose the original line passes through points (x_1, y_1) and (x_2, y_2). Its slope is:

- $m = (y_2 - y_1) / (x_2 - x_1)$

For example, if the line passes through (0, 0) and (4, 8):

- $m = (8 - 0) / (4 - 0) = 8/4 = 2$

The slope, again, is 2, confirming the parallel line will also have slope 2.

Constructing the Equation of the Parallel Line

Once the slope m of the original line is known, the next step is to find the equation of the line passing through (5, 1).

1. Using Point-Slope Form

The point-slope form is:

$$
y - y_1 = m (x - x_1)
$$

Plugging in:

- $x_1 = 5$

- $y_1 = 1$

- m = (found from previous step)

The equation becomes:

$$
y - 1 = m (x - 5)
$$

2. Converting to Slope-Intercept Form (Optional)

Expanding the above:

$$
y - 1 = m x - 5 m
$$

$$
y = m x - 5 m + 1
$$

This is the equation of the line passing through (5, 1) and parallel to the original line.

3. Expressing the Final Equation

Depending on the context, you may prefer the slope-intercept form:

$$
y = m x + c
$$

Where:

$$
c = - 5 m + 1
$$

This gives a complete description of the line.

Worked Example: Finding the Equation of a Line Passing Through (5,1) and Parallel to y = 2x + 3

Let's walk through a concrete example:

Given: The line $y = 2x + 3$

Step 1: Determine the slope of the given line:

- $m = 2$

Step 2: Use point-slope form with point (5, 1):

$$
y - 1 = 2 (x - 5)
$$

Step 3: Expand:

$$
y - 1 = 2x - 10
$$

Step 4: Solve for y:

$$
y = 2x - 10 + 1
$$

\]
\[
y = 2x - 9
\]

Result: The equation of the line passing through (5, 1) and parallel to y = 2x + 3 is:

\[
\boxed{y = 2x - 9}
\]

Additional Considerations and Variations

While the above example assumes a specific given line, real-world problems often involve different scenarios. Here are some variations and considerations:

1. Parallel to a Line in Standard Form

Suppose the original line is expressed as:

\[
3x - 4y = 7
\]

To find its slope:

\[
3x - 4y = 7 \Rightarrow -4y = -3x + 7
\]

\[
y = \frac{3}{4}x - \frac{7}{4}
\]

- Slope m = 3/4

The parallel line passing through (5, 1):

\[
y - 1 = \frac{3}{4}(x - 5)
\]

Expanding:

\[
y - 1 = \frac{3}{4}x - \frac{15}{4}
\]

\[
y = \frac{3}{4}x - \frac{15}{4} + 1
\]

\[
y = \frac{3}{4}x - \frac{15}{4} + \frac{4}{4}
\]

\[
y = \frac{3}{4}x - \frac{11}{4}
\]

Final equation:

\[
\boxed{y = \frac{3}{4}x - \frac{11}{4}}
\]

---

2. Horizontal and Vertical Lines

- Horizontal lines: Have zero slope (m=0).
Example: y = c

- Vertical lines: Have undefined slope.
Example: x = a constant

If the given line is horizontal, e.g., y = 4, then the parallel line passing through (5, 1) is also y = 4.

If the given line is vertical, e.g., x = 7, then the parallel line passing through (5, 1) is x = 5.

Special Cases and Additional Insights

1. When the Slope Is Zero or Undefined

- Zero slope (horizontal line): Equation is y = constant.

- The parallel line through (x_0, y_0) will also be $y = y_0$.

- Undefined slope (vertical line): Equation is x = constant.

- The parallel line through (x_0, y_0) is $x = x_0$.

2. Multiple

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