What Is The Area Of The Region In The First Quadrant Enclosed By The Graphs Of Y = Sin(2x) And Y = X

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Understanding the area enclosed between two curves is a fundamental concept in integral calculus. In this article, we explore the problem of finding the area of the region in the first quadrant bounded by the graphs of the functions Y = Sin(2x) and Y = X. This problem combines key mathematical concepts such as solving equations, finding points of intersection, and evaluating definite integrals, making it a rich topic for learners and enthusiasts alike.

Introduction to the Problem

The goal is to determine the exact area of the region in the first quadrant where the curves Y = Sin(2x) and Y = X intersect and enclose a bounded region. Specifically, we want to:

- Identify the points of intersection between the two curves.
- Determine which curve is on top and which is on the bottom within this region.
- Set up the appropriate integral(s) to compute the area.
- Evaluate the integral(s) to find the precise area.

Understanding these steps lays the foundation for solving many similar problems in calculus involving the area between curves.

Understanding the Graphs Involved

Before delving into calculations, it’s essential to understand the behavior of the functions involved:

Graph of Y = Sin(2x)

- The sine function is periodic with a period of π (since the argument is

2x).
- Within the first quadrant ($x > 0$, $y > 0$), Sin(2x) oscillates between 0 and 1.
- The key points on this graph are at:
- $x = 0$, $y = 0$
- $x = \pi/4$, $y = \text{Sin}(\pi/2) = 1$ (maximum point)
- $x = \pi/2$, $y = \text{Sin}(\pi) = 0$

Graph of Y = X

- The line $y = x$ is a straight line passing through the origin with a slope of 1.
- In the first quadrant, it increases from (0,0) to infinity.

The intersection points between these two graphs occur where $Y = \text{Sin}(2x)$ and $Y = X$ are equal.

Finding the Points of Intersection

To find the points where the curves intersect, set:

$$
\sin(2x) = x
$$

This transcendental equation cannot be solved algebraically in closed form, so numerical methods are used to approximate solutions.

Approximating the Intersection Points

- At $x = 0$:
$$
\sin(0) = 0 \Rightarrow 0 = 0
$$
So, (0, 0) is an intersection point.

- Next, look for solutions in $(0, \pi/2)$:
Since $\sin(2x) \leq 1$, and x increases from 0 upward, the intersection will occur where the sine curve intersects the line $y = x$.

- To find the second point, analyze the behavior:
- At $x = 0.5$:
$$
\sin(1) \approx 0.84, \quad y = 0.5

\]
Since 0.84 > 0.5, the sine is above the line.
- At x = 0.8:
\[
\sin(1.6) \approx 0.9996, \quad y=0.8
\]
Still, sine > line.
- At x = 1:
\[
\sin(2) \approx 0.9093, \quad y=1
\]
Now, sine < line.

The crossing point is between x=0.8 and x=1. Use linear interpolation:
- At x = 0.9:
\[
\sin(1.8) \approx 0.978, \quad y=0.9
\]
sine > line.
- At x=0.95:
\[
\sin(1.9) \approx 0.945, \quad y=0.95
\]
sine < line.

So, between x=0.9 and x=0.95, the functions cross again. Approximate the intersection at about x ≈ 0.92.

Using more precise numerical methods (such as Newton-Raphson or graphing calculators), the approximate second intersection point is:

\[
x \approx 0.905
\]

Corresponding y-value:

\[
y \approx \sin(2 \times 0.905) \approx \sin(1.81) \approx 0.97
\]

Thus, the two intersection points in the first quadrant are approximately:

\[
\boxed{
(0, 0) \quad \text{and} \quad (0.905, 0.97)
}
\]

Determining Which Curve Is on Top

In the interval between 0 and approximately 0.905, compare the functions:

- At x = 0.5:
$$
\sin(1) \approx 0.84, \quad y=x=0.5
$$
sine > line.

- At x = 0.9:
$$
\sin(1.8) \approx 0.98, \quad y=0.9
$$
sine > line.

- At x = 1:
$$
\sin(2) \approx 0.909, \quad y=1
$$
sine < line.

Therefore, from x=0 to x ≈ 0.905, the curve y = Sin(2x) lies above y = x. After x ≈ 0.905, the line y = x becomes above the sine curve, but since the intersection occurs at x ≈ 0.905, the region of interest is between these points.

Setting Up the Integral for Area Calculation

The area enclosed between two curves is given by the integral of the difference between the top and bottom functions over the interval of intersection:

$$
\text{Area} = \int_{a}^{b} [\text{top curve} - \text{bottom curve}] \, dx
$$

In this case:

- For $0 \leq x \leq x_{1}$, where $x_{1} \approx 0.905$, the sine curve is on top.
- The line y = x is on the bottom.

Thus,

\[

\text{Area} \approx \int_{0}^{x_{1}} [\sin(2x) - x] \, dx
\]

Calculating the Integral

The integral:

\[
A = \int_{0}^{x_{1}} [\sin(2x) - x] \, dx
\]

can be separated into two parts:

\[
A = \int_{0}^{x_{1}} \sin(2x) \, dx - \int_{0}^{x_{1}} x \, dx
\]

First integral:

\[
\int \sin(2x) \, dx
\]

Using substitution:

\[
u = 2x \Rightarrow du= 2 dx \Rightarrow dx = \frac{du}{2}
\]

So,

\[
\int \sin(2x) \, dx = \frac{1}{2} \int \sin u \, du = -\frac{1}{2} \cos u + C
= -\frac{1}{2} \cos(2x) + C
\]

Second integral:

\[
\int x \, dx = \frac{x^{2}}{2} + C
\]

Putting it all together:

\[
A = \left[-\frac{1}{2} \cos(2x)\right]_0^{x_{1}} -
\left[\frac{x^{2}}{2}\right]_0^{x_{1}}

\]

Substitute the limits:

\[
A = \left(-\frac{1}{2} \cos(2 x_{1}) + \frac{1}{2} \cos(0)\right) -
\left(\frac{x_{1}^{2}}{2} - 0\right)
\]

Recall that \(\cos(0) = 1\), so:

\[
A = -\frac{1}{2} \cos(2 x_{1}) + \frac{1}{2} - \frac{x_{1}^{2}}{2}
\]

Using the approximate value \(x_{1} \approx 0.905\):

\[
2 x_{1} \approx 1.81
\]

Calculate:

\[
\cos(1.81) \approx -0.239
\]

Now, plug in the numbers:

\[
A \approx -\frac{1}{2} \times (-0.239) + \frac{1}{2} - \frac{(0.905)^2}{2}
\]

\[
A \approx 0.1195 + 0.5 - \frac{0.819}{2}
\]

\[

Frequently Asked Questions

How do I find the points of intersection between the graphs of y = sin(2x) and y = x in the first quadrant?

To find the intersection points, set sin(2x) = x and solve for x in the first quadrant (x ≥ 0). Since this equation cannot be solved algebraically easily, numerical methods or graphing tools are typically used to approximate the solutions within the interval from 0 to where sin(2x) and x intersect.

What is the general approach to calculating the area enclosed between y = sin(2x) and y = x in the first quadrant?

The general approach involves: 1) finding the intersection points of the two curves, 2) determining which curve is on top within the interval, and 3) integrating the difference of the functions over that interval to find the total enclosed area.

How do I determine which function is on top between the intersection points?

You evaluate both functions at a test point within the interval. If sin(2x) > x at that point, then sin(2x) is on top; otherwise, x is on top. Typically, near zero, sin(2x) starts above x, but you need to verify at the intersection points.

What is the integral setup to find the enclosed area between y = sin(2x) and y = x?

The area A is given by A = ∫[a to b] |sin(2x) - x| dx, where a and b are the intersection points. Since the top function varies, you may need to split the integral if the top function changes within the interval.

Can I approximate the area numerically if I can't find the exact intersection points?

Yes, numerical methods such as Simpson's rule or using graphing calculator tools can approximate the intersection points and the area between the curves, providing a reliable estimate when exact solutions are difficult.

What is the approximate area enclosed in the first quadrant between y = sin(2x) and y = x?

Using numerical approximation methods, the approximate enclosed area is about 0.45 square units, but the exact value requires solving for intersection points precisely and computing the definite integral.

Are there any special considerations when calculating the area between these two curves in the first quadrant?

Yes, it's important to accurately determine the intersection points, verify which curve is on top in each sub-interval, and carefully set up the integral to account for any changes in the top function. Numerical methods can aid in handling complex intersection points.

Additional Resources

Area Calculation — Exploring the Enclosed Region Between \(y = \sin(2x) \) and \(y = x \)

Introduction

When delving into the realm of calculus and analytical geometry, one of the most intriguing problems involves determining the area enclosed by the intersection of two curves. Specifically, for the functions \(y = \sin(2x) \) and \(y = x \) within the first quadrant, the question arises: What is the exact area of the region bounded by these graphs? This problem not only exemplifies fundamental calculus techniques like finding points of intersection and setting up definite integrals but also offers a rich exploration into the behavior of trigonometric and linear functions in tandem.

In this detailed exploration, we will analyze the problem step-by-step, providing a comprehensive understanding of how to approach, formulate, and evaluate the area in question. Think of this as an expert review, where each mathematical detail is meticulously examined to reveal the elegance and precision involved in such calculations.

Understanding the Functions and Their Behavior

Before delving into the calculations, it's crucial to understand the nature of the functions involved.

The Function \(y = \sin(2x) \)

- Periodicity: The sine function \(\sin(\theta) \) has a period of \(2\pi \). For \(\sin(2x) \), the period becomes \(\pi \) because of the coefficient \(2 \) inside the argument:

\[
\text{Period} = \frac{2\pi}{2} = \pi
\]

- Amplitude: The maximum and minimum values are \(1 \) and \(-1 \), respectively, ensuring the graph oscillates between these bounds.

- Behavior in the First Quadrant: For \(x \in [0, \frac{\pi}{2}] \), \(\sin(2x) \) rises from \(0 \) at \(x=0 \) to \(1 \) at \(x =

$\frac{\pi}{4}$, then decreases back to 0 at $x = \frac{\pi}{2}$.

The Function $y = x$

- Linear Growth: A straight line passing through the origin with a slope of 1, increasing steadily from 0 at $x=0$.

- In the First Quadrant: Extends from 0 at the origin to higher values as x increases.

Identifying the Points of Intersection

The first crucial step is to find the points where $y = \sin(2x)$ and $y = x$ intersect within the first quadrant $(x \geq 0, y \geq 0)$.

Setting the Equations Equal

$$
\sin(2x) = x
$$

This transcendental equation generally lacks a straightforward algebraic solution, so we analyze it graphically and numerically.

Graphical Intuition

- At $x=0$, both functions are zero. So, $x=0$ is an intersection point.

- For small positive x, since $\sin(2x) \approx 2x$ for $x \to 0$, and $2x$ is greater than x, the sine curve is initially above the line.

- As x increases, $\sin(2x)$ oscillates, but within the first quadrant, it peaks at $x = \frac{\pi}{4}$, where $\sin(2x) = 1$.

- At $x = \frac{\pi}{4} \approx 0.7854$, $\sin(2x) = 1$ while $y = x \approx 0.7854$. Since $1 > 0.7854$, the sine is above the line here.

- At $x = 1$, $\sin(2) \approx 0.9093$, which is less than 1, i.e., the line $y = x$ surpasses $\sin(2x)$. So, an intersection occurs somewhere between $x=0.7854$ and $x=1$.

- At $x=1.2$, $\sin(2.4) \approx 0.6755$, which is less than 1. Since the sine is less than x at $x=1.2$, the sine curve remains below the line here.

- We deduce that the functions intersect at two points:

1. At $x=0$ (trivial solution).
2. At some $x = x_1$ between 0.7854 and 1.

- To find the precise x_1, we employ numerical methods such as the bisection method or Newton-Raphson.

Numerical Approximation of the Intersection Point

Let's approximate x_1:

- Start with $x=0.8$:

$$
\sin(2 \times 0.8) = \sin(1.6) \approx 0.9996
$$

$$
y = 0.8
$$

$$
\sin(2x) - x \approx 0.9996 - 0.8 = 0.1996 > 0
$$

- Now $x=0.9$:

$$
\sin(1.8) \approx 0.9738
$$

$$
0.9738 - 0.9 = 0.0738 > 0
$$

- $x=1.0$:

$$
\sin(2) \approx 0.9093
$$

$$
0.9093 - 1.0 = -0.0907 < 0
$$

Between 0.9 and 1.0, the function crosses zero. Let's check at $x=0.95$:

$$
\sin(1.9) \approx 0.9455

$$
0.9455 - 0.95 \approx -0.0045 < 0
$$

At $x=0.92$:

$$
\sin(1.84) \approx 0.9634
$$

$$
0.9634 - 0.92 = 0.0434 > 0
$$

Between 0.92 and 0.95, the sign changes from positive to negative. Approximate the root:

- At $x \approx 0.935$:

$$
\sin(1.87) \approx 0.9563
$$

$$
0.9563 - 0.935 \approx 0.0213 > 0
$$

- At $x=0.945$:

$$
\sin(1.89) \approx 0.9454
$$

$$
0.9454 - 0.945 \approx 0.0004 > 0
$$

- At $x=0.948$:

$$
\sin(1.896) \approx 0.9437
$$

$$
0.9437 - 0.948 \approx -0.0043 < 0
$$

Thus, the intersection point x_1 is approximately 0.947 radians.

Summary of intersection points:

- $x=0$

- $x \approx 0.947$

Determining Which Function Is On Top

Within the interval $[0, 0.947]$, the sine function $y = \sin(2x)$ is above the line $y = x$. Beyond $x \approx 0.947$, the line becomes dominant.

The region enclosed is therefore bounded between:

- $x=0$ and $x \approx 0.947$

- By the curves $y = \sin(2x)$ (above) and $y = x$ (below)

Setting Up the Integral for the Enclosed Area

The area A of the region in the first quadrant enclosed by the graphs is given by:

$$
A = \int_{x=0}^{x \approx 0.947} \left[\sin(2x) - x \right] dx
$$

This integral accounts for the vertical difference between the curves at each x, summed over the interval of intersection.

Computing the Integral

Let's evaluate:

$$
A = \int_0^{x_1} \sin(2x) dx - \int_0^{x_1} x dx
$$

where $x_1 \approx 0.947$.

First integral:

$$
\int \sin(2x) dx
$$

```
Using substitution:

\[
u = 2x \Rightarrow du = 2 dx \Rightarrow dx = \frac{du}{2}
\]

\[
\int \sin(2x) dx = \frac
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