What Is The Equation, In Point-slope Form, Of The Line That Is Parallel To The Given Line And Passes

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Understanding how to find the equation of a line that is both parallel to a given line and passes through a specific point is a fundamental skill in algebra and analytic geometry. This process involves grasping the concepts of slope, point-slope form, and the properties of parallel lines. Whether you're a student preparing for an exam or a professional needing a quick refresher, mastering this topic will enhance your problem-solving toolkit. In this article, we'll explore the step-by-step approach to derive the equation of such a line, including definitions, formulas, examples, and tips to ensure clarity and confidence in your calculations.

Understanding the Basics: Lines, Slopes, and Forms

What Is a Line's Equation?

A line in a coordinate plane can be described mathematically by an equation that relates the x and y coordinates of any point on the line. The most common forms include:

- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

Each form has its advantages depending on what information you have about the line.

The Concept of Slope

The slope (m) of a line measures its steepness and is calculated as the ratio of the change in y to the change in x between two points:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

For lines to be parallel, they must have identical slopes.

Point-slope Form of a Line

The point-slope form is particularly useful when you know a point on the line and its slope:

$$ y - y_1 = m(x - x_1) $$

where:
- (x_1, y_1) is a point on the line
- m is the slope of the line

This form directly relates the known point to the slope, making it convenient for deriving the equation of a parallel line.

Determining the Equation of a Parallel Line in Point-slope Form

Step 1: Identify the Slope of the Given Line

The first step is to find the slope (m) of the original line. The form of the original line's equation will guide you:

- If the line is in slope-intercept form (y = mx + b), the slope is immediately visible as m.

- If the line is in standard form (Ax + By = C), solve for y:

$$ y = -\frac{A}{B}x + \frac{C}{B} $$

and identify the slope as $-\frac{A}{B}$.

- If the line is given as a graph, select two points on the line and compute the slope.

Step 2: Recognize Parallel Lines Have Equal Slopes

Since parallel lines never intersect and maintain the same steepness, the slope of the new line must be identical to that of the original line.

$$ m_{new} = m_{original} $$

Step 3: Use the Given Point to Write the Equation in Point-slope Form

Suppose you are given a point (x_1, y_1) through which the new line passes. The point-slope form becomes:

$$ y - y_1 = m(x - x_1) $$

where m is the slope of the original line.

Step 4: Write the Final Equation

Insert the known point coordinates and the slope into the point-slope formula, resulting in the equation of the desired line.

Example:

Given the line $y = 2x + 3$ and the point $(4, 1)$, find the equation of the line parallel to the given line passing through this point.

- Slope of the given line: $m = 2$
- Point on the new line: $(4, 1)$
- Equation in point-slope form:

$$y - 1 = 2(x - 4)$$

This is the required equation of the line in point-slope form.

Additional Tips and Common Mistakes

Tips for Success

- Always identify the slope correctly, especially when converting from different forms.
- When given the line in standard form, rewrite it into slope-intercept form for easy slope extraction.
- Ensure the point used is on the new line and correctly substituted.
- Keep track of signs (+/-) when substituting points and calculating.

Common Mistakes to Avoid

- Using the slope from the original line but forgetting to confirm it is parallel (i.e., same slope).
- Mixing up x and y coordinates of the point.
- Forgetting to simplify the point-slope form or converting it into other forms if needed.
- Assuming the given line is in slope-intercept form when it might be in a different form.

Examples for Practice

Example 1:

Given the line $y - 3 = -\frac{1}{2}(x + 4)$, find the equation of a line parallel to it passing through $(0, 0)$.

Solution:

- Slope of the given line: $m = -\frac{1}{2}$
- Point: $(0, 0)$
- Equation:

$$y - 0 = -\frac{1}{2}(x - 0)$$

$$y = -\frac{1}{2}x$$

Example 2:

Find the point-slope form of the line parallel to $3x - 4y = 7$ passing through $(2, -1)$.

Solution:

- Convert to slope-intercept form:

$$3x - 4y = 7 \Rightarrow -4y = -3x + 7 \Rightarrow y = \frac{3}{4}x - \frac{7}{4}$$

- Slope: $m = \frac{3}{4}$
- Point: $(2, -1)$
- Equation:

$$y - (-1) = \frac{3}{4}(x - 2)$$

$$ y + 1 = \frac{3}{4}(x - 2) $$

Conclusion

Mastering the process of finding the equation, in point-slope form, of a line parallel to a given line passing through a specific point is essential for solving many geometry problems. The key steps involve identifying the original line's slope, recognizing that parallel lines share the same slope, and then applying the point-slope formula with the given point. With practice, you will become proficient at quickly deriving these equations and applying them to real-world and academic problems.

Remember:
- Always verify the slope before proceeding.
- Use the point-slope form for clarity and ease.
- Practice with diverse examples to solidify your understanding.

By following these guidelines, you'll enhance your algebraic skills and deepen your understanding of the geometric relationships between lines in a coordinate plane.

Frequently Asked Questions

What is the general form of the point-slope equation of a line that is parallel to a given line?

The point-slope form of a line parallel to a given line has the same slope as the original line and passes through a specific point. It is written as $y - y_1 = m(x - x_1)$, where m is the slope of the original line, and (x_1, y_1) is a point on the new line.

How do you find the equation in point-slope form of a line parallel to y = 2x + 3 passing through (4, 5)?

Since the original line has slope 2, the parallel line also has slope 2. Using point (4, 5), the equation is $y - 5 = 2(x - 4)$.

What is the significance of the slope in writing the point-slope form of a parallel line?

The slope determines the slope of the parallel line. Since parallel lines have identical slopes, using the original line's slope ensures the new line is parallel.

Can the point-slope form be converted to slope-intercept form after finding the line parallel to a given line?

Yes, after writing the point-slope form, you can simplify and rearrange it to slope-intercept form ($y = mx + b$) for easier interpretation.

If the original line is given in standard form, how do you find the point-slope form of a parallel line passing through a specific point?

First, convert the standard form to slope-intercept form to find the slope. Then, use that slope and the given point in the point-slope formula $y - y_1 = m(x - x_1)$.

What are common mistakes to avoid when writing the equation of a parallel line in point-slope form?

Common mistakes include using the wrong slope, mixing points from different lines, or incorrectly substituting values into the formula. Always verify the slope matches the original line and that the point lies on the new line.

How do you determine the point-slope equation of a line parallel to 3x - 4y = 7 passing through (1, -2)?

First, rewrite the original line in slope-intercept form: $y = (3/4)x - 7/4$, so the slope is 3/4. Then, using point (1, -2), the equation is $y - (-2) = (3/4)(x - 1)$, or $y + 2 = (3/4)(x - 1)$.

Is the point-slope form of a line unique for a given point and slope?

Yes, for a specific point and slope, the point-slope form $y - y_1 = m(x - x_1)$ uniquely defines the line passing through that point with that slope.

Additional Resources

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Introduction

In the realm of algebra and analytic geometry, understanding how to formulate the equations of lines is fundamental. When dealing with lines that are parallel to a given line and pass through a specific point, the point-slope form becomes an essential tool. This article provides a comprehensive exploration of how to find the equation, in point-slope form, of a line parallel to a given line and passing through a designated point.

Understanding the Core Concepts

Before delving into the step-by-step process, it’s crucial to grasp the key concepts involved:

1. Line Equations and Forms

- Standard Form: $Ax + By = C$
- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Each form has its advantages depending on the problem context. The point-slope form is particularly useful when a specific point and slope are known.

2. Parallel Lines

- Definition: Two lines are parallel if they lie in the same plane and do not intersect; their slopes are equal.
- Implication: To find a line parallel to a given line, the new line must have the same slope as the original.

3. The Role of the Point

- The problem specifies that the new line passes through a certain point (x_1, y_1).
- This point acts as the anchor for the new line, which, combined with the slope, defines the equation.

Step-by-Step Approach to Finding the Equation in Point-Slope Form

Let's break down the process into clear, actionable steps.

Step 1: Identify the Slope of the Given Line

Suppose the given line is expressed in slope-intercept form:

$$
y = m_1 x + b
$$

- The slope of this line is m_1.

Alternatively, if the line is in standard form:

$$
A x + B y = C
$$

- The slope m_1 is calculated as:

$$
m_1 = - \frac{A}{B}
$$

Note: Always check the form of the given line to accurately determine the slope.

Step 2: Confirm the Slope for the Parallel Line

- Since parallel lines share the same slope, the slope of the new line m_2 is:

$$
m_2 = m_1
$$

- This is the key to ensuring the new line is parallel.

Step 3: Use the Given Point

- The point through which the new line passes is (x_1, y_1).

- This point will serve as the reference point in the point-slope form.

Step 4: Write the Equation in Point-Slope Form

- The general point-slope form is:

$$
y - y_1 = m (x - x_1)
$$

- Substitute $m = m_2 = m_1$:

$$
y - y_1 = m_1 (x - x_1)
$$

This is the desired equation.

Practical Examples and Applications

Applying these steps in real problems helps solidify understanding.

Example 1: Straightforward Scenario

Given line: $y = 2x + 3$

Point: $(4, 7)$

Step 1: The slope of the given line is $m_1 = 2$.

Step 2: The slope of the parallel line is also 2.

Step 3: Using the point $(4, 7)$, the equation becomes:

$$
y - 7 = 2(x - 4)
$$

Result: The point-slope form of the parallel line passing through $(4, 7)$ is:

$$
y - 7 = 2(x - 4)
$$

Example 2: Line in Standard Form

Given line: $3x - 4y = 8$

Point: $(1, 2)$

Step 1: Compute the slope:

$$
m_1 = - \frac{A}{B} = - \frac{3}{-4} = \frac{3}{4}
$$

Step 2: Parallel line slope:

$$
m_2 = \frac{3}{4}
$$

Step 3: Write the equation:

$$
y - 2 = \frac{3}{4}(x - 1)
$$

Result: The line in point-slope form:

$$
y - 2 = \frac{3}{4}(x - 1)
$$

Deeper Insights: Connecting to Other Forms

While point-slope form is straightforward when the slope and point are known, sometimes it's necessary to convert between different forms for clarity or further applications.

Converting to Slope-Intercept Form

From the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

- Expand:

$$
y = m(x - x_1) + y_1
$$

- Distribute m:

$$
y = m x - m x_1 + y_1
$$

- Simplify to slope-intercept form:

$$
y = m x + (- m x_1 + y_1)
$$

where $- m x_1 + y_1$ is the y-intercept.

Converting to Standard Form

- Rearrange:

$$
y - y_1 = m(x - x_1)
$$

- Expand:

\[

$$
y - y_1 = m x - m x_1
$$

- Bring all terms to one side:

$$
m x - y + (- m x_1 + y_1) = 0
$$

or equivalently,

$$
m x - y = m x_1 - y_1
$$

which is in standard form.

Special Cases and Considerations

While the process seems straightforward, certain special cases require attention.

1. Vertical Lines

- A vertical line has an undefined slope, represented as:

$$
x = x_1
$$

- The parallel line to a vertical line is also vertical, passing through the same x-coordinate as the point:

$$
x = x_1
$$

- In point-slope form, vertical lines can be expressed as $x = x_1$, which does not fit the typical $y - y_1 = m(x - x_1)$ form due to the undefined slope.

2. Horizontal Lines

- For a horizontal line, the slope is zero:

$$
y = y_1
$$

- The equation of a line parallel to it passing through (x_1, y_1) is simply:

$$
y = y_1
$$

- In point-slope form, this reduces to:

$$
y - y_1 = 0 \times (x - x_1) \Rightarrow y - y_1 = 0
$$

Practical Tips for Finding the Equation in Point-Slope Form

- Always verify the slope of the original line, especially if it's in standard form.
- When dealing with vertical lines, switch to the appropriate form and remember that point-slope form may not be applicable.
- When given the point and the original line, double-check the calculations of the slope before proceeding.
- Use the point-slope form for quick line equations when the point and slope are known, especially for graphing or further calculations.

Summary

To find the equation, in point-slope form, of a line that is parallel to a given line and passes through a specified point, follow these essential steps:

1. Determine the slope of the given line:

- Convert the line into slope-intercept or standard form, then compute the slope.
- Remember: for standard form $Ax + By = C$, the slope is $-A/B$.

2. Identify the slope of the new line:

- Since the lines are parallel, the slope remains the same.

3. Use the point-slope form:

$$
y - y_1 = m (x - x_1)
$$

- Plug in the point \((x_1, y_1) \) and the slope \(m \).

4. Simplify or convert as needed:

- The point-slope form is flexible; convert to slope-intercept or standard form if required.

Final Thoughts

Mastering the process of finding the equation in point-slope form of a line parallel to a given line passing through a specific point is vital for students and professionals working in algebra, geometry, and related fields. It enhances problem-solving efficiency and deepens understanding of geometric relationships. Whether dealing with straightforward lines or more complex scenarios involving vertical or horizontal lines, the principles remain consistent and reliable.

By internalizing these steps and concepts, you will be well-equipped to handle a wide array of problems involving parallel lines and their equations, reinforcing your overall mathematical proficiency and geometric intuition.

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