What Is The Equation Of A Line Parallel To The Line That Passes Through Thepoints (1, 8) And (3, 14)

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Understanding the equation of lines is fundamental in algebra and coordinate geometry. When dealing with lines, one common problem is finding the equation of a line that is parallel to a given line and passes through a specific point. This article explores how to determine the equation of a line parallel to the line passing through the points (1, 8) and (3, 14). We will delve into the concepts of slope, parallel lines, and the point-slope form of a line’s equation, providing detailed explanations, step-by-step methods, and practical examples to enhance your understanding.

Understanding the Basics: Lines, Slopes, and Equations

Before tackling the specific problem, it’s essential to review some fundamental concepts in coordinate geometry.

What Is a Line in the Coordinate Plane?

A line in the coordinate plane is a set of points that extends infinitely in both directions, following a straight path. It can be described mathematically using an equation that relates its x and y coordinates.

What Is the Slope of a Line?

The slope (m) of a line measures its steepness and is defined as the ratio of the vertical change to the horizontal change between any two points on the line:

$$ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} $$

This ratio reflects how much y increases or decreases per unit increase in x.

Equation of a Line in Different Forms

- Point-Slope Form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line.
- Slope-Intercept Form: $y = mx + b$, where b is the y-intercept.
- Standard Form: $Ax + By = C$, where A, B, and C are constants.

Understanding these forms allows flexibility in solving various line equations problems.

Step-by-Step Solution to Find the Equation of a Parallel Line

Given the points $(1,8)$ and $(3,14)$, our goal is to find the equation of a line parallel to the line passing through these points, which also passes through a specified point (or generally, to understand the process).

Step 1: Find the Slope of the Original Line

Using the two points $(x_1, y_1) = (1, 8)$ and $(x_2, y_2) = (3, 14)$:

$$
m = \frac{14 - 8}{3 - 1} = \frac{6}{2} = 3
$$

The slope of the original line is 3.

Step 2: Understand the Concept of Parallel Lines

Parallel lines are lines that have the same slope but different y-intercepts. Therefore, any line parallel to the original line will also have a slope of 3.

Step 3: Write the Equation of the Parallel Line

To find the equation of the line parallel to the original and passing through a specific point, say, (x_0, y_0), we use the point-slope form:

$$
y - y_0 = m(x - x_0)
$$

Suppose the line passes through the point (x_0, y_0). For example, if the point is $(5, 10)$, then:

$$
y - 10 = 3(x - 5)
$$

Expanding:

$$
y - 10 = 3x - 15
$$

$$
y = 3x - 15 + 10
$$

$$
y = 3x - 5
$$

This is the equation of a line parallel to the original that passes through $(5, 10)$.

Step 4: Generalize for Any Point

If the point through which the parallel line passes is any point (x_0, y_0), then the equation is:

$$
y = 3x + c
$$

where c is the y-intercept, which can be found by substituting the point coordinates:

$$
y_0 = 3x_0 + c \Rightarrow c = y_0 - 3x_0
$$

Thus, the general equation of the line parallel to the original passing through (x_0, y_0) is:

$$
y = 3x + (y_0 - 3x_0)
$$

Step 5: Example with a Specific Point

Suppose you want the parallel line passing through $(2, 5)$:

$$
c = 5 - 3 \times 2 = 5 - 6 = -1
$$

Equation:

$$
y = 3x - 1
$$

Summary of the Process

- Find the slope of the original line.
- Use the same slope for the parallel line.
- Use a known point for the parallel line to find the specific equation.
- Write the equation in the slope-intercept form.

Additional Considerations: Equations of Lines and Their Applications

Understanding how to find the equation of a line parallel to a given line has practical applications in various fields, including engineering, architecture, and physics. Let’s explore some related concepts and extensions.

Parallel Lines and Their Properties

- Equal Slopes: Parallel lines always have the same slope.
- Distinct Y-Intercepts: The lines do not intersect, so their y-intercepts are different unless they are the same line.
- Distance Between Parallel Lines: The distance between two parallel lines can be calculated using the formula:

$$
d = \frac{|c_2 - c_1|}{\sqrt{m^2 + 1}}
$$

where c_1 and c_2 are the y-intercepts of the two lines in slope-intercept form.

Perpendicular Lines and Their Equations

In contrast to parallel lines, perpendicular lines have slopes that are negative reciprocals:

$$
m_{perpendicular} = -\frac{1}{m_{original}}
$$

This concept is useful when solving problems involving perpendicularity, such as in coordinate geometry and vector calculus.

Practical Examples and Practice Problems

To solidify your understanding, consider the following practice problems:

1. Find the equation of a line parallel to the line passing through $(2, 3)$ and $(4, 7)$, passing through the point $(5, 2)$.
2. Determine the equation of a line perpendicular to the original line passing through $(0, 0)$.
3. Given a line $y = 2x + 5$, find the equation of a line parallel passing through $(-1, 4)$.

Solutions:

1. Slope of original line:

$$
m = \frac{7 - 3}{4 - 2} = \frac{4}{2} = 2
$$

Parallel line passing through $(5, 2)$:

$$
y - 2 = 2(x - 5) \Rightarrow y = 2x - 10 + 2 = 2x - 8
$$

2. Perpendicular line to $y = 2x + 5$:

$$
m_{perp} = -\frac{1}{2}
$$

Through $(0, 0)$:

$$
y - 0 = -\frac{1}{2}(x - 0) \Rightarrow y = -\frac{1}{2}x
$$

3. Parallel to $y = 2x + 5$ passing through $(-1, 4)$:

$$
y - 4 = 2(x + 1) \Rightarrow y = 2x + 2 + 4 = 2x + 6
$$

Conclusion: Mastering the Equation of Parallel Lines

Determining the equation of a line parallel to another and passing through a specific point is a fundamental skill in algebra and geometry. The key steps involve calculating the original line’s slope, recognizing that parallel lines share this slope, and then applying the point-slope form to find the specific equation. Whether working through theoretical problems or applied scenarios, these concepts form the backbone of understanding line equations in the coordinate plane.

By practicing these methods and understanding the properties of parallel lines, students and professionals can efficiently analyze geometric relationships, solve complex problems, and apply these principles in real-world contexts. Remember, the core idea hinges on the invariance of the slope for parallel lines, making it a powerful and straightforward tool in the mathematician’s toolkit.

Frequently Asked Questions

How do I find the slope of the line passing through the points (1, 8) and (3, 14)?

To find the slope, use the formula $(y_2 - y_1) / (x_2 - x_1)$. So, (14 - 8) / (3 - 1) = 6 / 2 = 3.

What is the slope of a line parallel to the one passing through (1, 8) and (3, 14)?

A line parallel to the given line will have the same slope, which is 3.

How do I write the equation of a line parallel to the one passing through (1, 8) and (3, 14)?

Use the point-slope form with the point (1, 8) and the slope 3: $y - 8 = 3(x - 1)$. Simplify to get the equation.

What is the equation of the line parallel to the given line passing through the point (1, 8)?

The equation is $y - 8 = 3(x - 1)$, which simplifies to $y = 3x + 5$.

Can I find the equation of the parallel line passing through any point?

Yes, using the point-slope form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is the point.

What is the general form of the equation of a line parallel to a given line?

It has the same slope as the original line but can have any intercept: $y = mx + b$, where m is the original line's slope.

How do I verify that two lines are parallel?

Check if they have the same slope. If the slopes are equal, the lines are parallel.

What is the significance of the points (1, 8) and (3, 14) in finding the parallel line?

They help determine the slope of the original line, which is then used to find the equation of any parallel line.

Are there different equations for lines parallel to the same line passing through different points?

Yes, each point will produce a different line parallel to the original, with the same slope but different y-intercepts.

Additional Resources

What Is The Equation Of A Line Parallel To The Line That Passes Through The Points (1, 8) And (3, 14)

Introduction

In the realm of coordinate geometry, the concept of parallel lines is fundamental. It serves as a cornerstone for understanding more complex geometric and algebraic principles. The question "What is the equation of a line parallel to the line that passes through the points (1, 8) and (3, 14)?" is more than a simple algebraic problem; it embodies the essential qualities of slope, intercepts, and the geometric relations between lines. This investigative exploration aims to dissect this problem thoroughly, offering a detailed, scholarly perspective suitable for academic review and practical application alike.

Understanding the Basics: Lines, Slopes, and Parallelism

Before delving into the specific problem, it is crucial to establish a solid understanding of the fundamental concepts involved.

The Equation of a Line

The most common form for a line in the Cartesian plane is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line,
- b is the y-intercept.

Alternatively, the point-slope form is often employed when a point and slope are known:

$$ y - y_1 = m(x - x_1) $$

Slope: The Rate of Change

The slope m of a line signifies its steepness and direction. It is calculated as:

$$ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} $$

for two distinct points (x_1, y_1) and (x_2, y_2).

Parallel Lines and Equal Slopes

Two lines are parallel if and only if they have identical slopes. That is:

$$ m_1 = m_2 $$

This property forms the core of our investigation, as we seek the equation of any line parallel to a given line.

Step-By-Step Analysis of the Given Line

The first task involves determining the equation of the original line passing through the points $(1, 8)$ and $(3, 14)$.

Calculating the Slope

Applying the slope formula:

$$
m = \frac{14 - 8}{3 - 1} = \frac{6}{2} = 3
$$

This indicates the original line has a slope of 3.

Equation of the Original Line

Using point-slope form with point $(1, 8)$:

$$
y - 8 = 3(x - 1)
$$

Simplifying to slope-intercept form:

$$
y - 8 = 3x - 3 \implies y = 3x + 5
$$

Hence, the original line's equation is:

$$ y = 3x + 5 $$

This equation will serve as the reference for identifying parallel lines.

The Core Question: Equation of a Line Parallel to the Original

Given that any line parallel to $y = 3x + 5$ must share the same slope, the general form of such lines is:

$$
y = 3x + c
$$

where c is any real number representing the y-intercept of the new line.

This leads to an infinite set of possible lines, each parallel to the original, distinguished only by their y-intercept.

Deep Dive: Characteristics of Parallel Lines

Understanding the properties of these lines deepens our comprehension.

Geometric Interpretation

- All lines with the same slope $m = 3$ are equidistantly spaced in terms of their angle with the x-axis.
- They do not intersect each other unless they are coincident (sharing the same y-intercept).

Algebraic Implications

- The difference in their y-intercepts c determines their vertical displacement.
- As c varies, the lines shift vertically but maintain the same inclination.

Practical Applications and Variations

In practical contexts, one might need to find the equation of a parallel line that passes through a specific point (x_0, y_0). This is a common problem in coordinate geometry and has direct implications in fields such as engineering, physics, and computer graphics.

Finding a Parallel Line Through a Given Point

Suppose the problem specifies a point (x_0, y_0) through which the new line must pass. The steps are:

1. Use the known slope $m = 3$.
2. Plug into the point-slope form:

$$
y - y_0 = 3(x - x_0)
$$

3. Convert to slope-intercept form:

$$
y = 3x + (y_0 - 3x_0)
$$

The intercept $c = y_0 - 3x_0$ is determined directly from the given point.

Case Study: Constructing Specific Parallel Lines

To illustrate, consider two example points:

- Point A: $(2, 10)$
- Point B: $(-1, 2)$

Using the point-slope form:

- For point A:

$$
y - 10 = 3(x - 2) \implies y = 3x + 4
$$

- For point B:

$$
y - 2 = 3(x + 1) \implies y = 3x + 5
$$

Notice that the lines passing through these points are:

- $y = 3x + 4$
- $y = 3x + 5$

which are parallel, differing only in their y-intercepts.

Significance in Broader Mathematical Context

The analysis of lines parallel to a given line extends beyond simple algebra. It plays a crucial role in:

- Linear Programming: Constraints often involve lines and their parallels.
- Geometry and Design: Architectural plans frequently employ parallel lines.
- Physics: Trajectories with identical slopes, representing constant velocity or acceleration vectors.
- Computer Graphics: Rendering parallel lines for visual effects.

Understanding the fundamental principles governing these lines enhances problem-solving skills across disciplines.

Common Misconceptions and Clarifications

While the concept of parallel lines appears straightforward, common pitfalls include:

- Confusing slope with intercept: Parallel lines share the same slope but can have different y-intercepts.
- Assuming the same points: Different points will generate different lines; the key is the slope.
- Neglecting the domain: The parallel lines are infinite in extent, but specific applications may restrict their domain.

Clarifying these misconceptions ensures accurate problem-solving and precise mathematical reasoning.

Conclusion

The question "What is the equation of a line parallel to the line that passes through the points (1, 8) and (3, 14)?" reveals the elegance and consistency of algebraic geometry. The original line, passing through these points, has the equation $y = 3x + 5$. Any line parallel to it must share the same slope $m = 3$, with the general form:

$$
\boxed{
y = 3x + c
}
$$

where c is any real number, reflecting the infinite possibilities of parallel lines in the plane.

This exploration demonstrates how a simple question encapsulates core principles of slope, intercepts, and geometric relationships. It also highlights the importance of understanding these concepts for applications across mathematics, science, and engineering.

By mastering the properties of parallel lines and their equations, students and professionals alike can approach complex problems with confidence, leveraging foundational knowledge to unlock a wide array of practical and theoretical insights.

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About the Author

[Author Name], Ph.D., specializes in mathematical education and applied algebra. With over a decade of experience in academic research and instruction, they have contributed extensively to the dissemination of mathematical concepts through articles, workshops, and online platforms. Their focus lies in making complex topics accessible and relevant to diverse audiences.

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