What Is The Equation Of The Line That Is Parallel To The Given Line And Passes Through The Point (3,

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Understanding the equation of a line in coordinate geometry is fundamental for solving numerous mathematical problems. When asked to find a line parallel to a given line that passes through a specific point, it involves concepts like slope, point-slope form, and the properties of parallel lines. This comprehensive guide will walk you through the process step-by-step, ensuring you grasp the key ideas and can perform these calculations confidently.

Understanding Lines and Their Equations

What Is a Line in Coordinate Geometry?

A line in the coordinate plane is a straight one-dimensional figure that extends infinitely in both directions. Its position is determined by its slope and intercepts. The equation of a line provides a mathematical description of its location and orientation.

Common Forms of a Line's Equation

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By + C = 0$

Where:
- m is the slope of the line
- (x_1, y_1) is a point on the line

Key Concepts for Finding Parallel Lines

What Does It Mean for Lines to Be Parallel?

Two lines are parallel if they are in the same plane and never intersect. This occurs precisely when their slopes are equal but their y-intercepts are different (unless they are the same line).

Properties of Parallel Lines

- Same slope m
- Different y-intercepts b (or different points)
- No intersection points

Why Is Slope Important?

The slope determines the steepness and direction of a line. When lines are parallel, maintaining the same slope is essential for their parallelism.

Step-by-Step Guide to Find the Equation of a Parallel Line Passing Through a Point

Suppose we are given:
- A line with an equation (say, in slope-intercept form or any form)
- A point (x_0, y_0), in this case, $(3, y)$, where y is specified or to be determined

Our goal: Find the equation of a line parallel to the given line that passes through $(3, y)$.

Step 1: Identify the Slope of the Given Line

- Convert the given line into slope-intercept form $y = mx + b$ if necessary
- Extract the slope m

Example:

Suppose the given line is $2x - 3y + 5 = 0$.
Convert to slope-intercept form:

$$
2x - 3y + 5 = 0 \Rightarrow -3y = -2x - 5 \Rightarrow y = \frac{2}{3}x + \frac{5}{3}
$$

Slope: $m = \frac{2}{3}$

Step 2: Recognize the Slope of the Parallel Line

Since parallel lines have identical slopes, the new line passing through $(3, y)$ will also have slope $m = \frac{2}{3}$.

Step 3: Use the Point-Slope Form to Write the Equation

The point-slope form of a line is:

$$
y - y_1 = m(x - x_1)
$$

where (x_1, y_1) is the point the line passes through, and m is the slope.

Applying the Point $(3, y)$:

$$
y - y_0 = \frac{2}{3}(x - 3)
$$

Note: Since y_0 is unspecified, the line's equation can be written generally as:

$$
y - y_0 = \frac{2}{3}(x - 3)
$$

which simplifies to:

$$
y = \frac{2}{3}x + \left(y_0 - 2 \right)
$$

If the specific y-coordinate of the point is provided, substitute it directly.

Step 4: Write the Equation in Slope-Intercept or Standard Form

Depending on the requirement, you can convert the point-slope form into:

- Slope-Intercept Form:
$$
y = \frac{2}{3}x + c
$$
where $c = y_0 - 2$.

- Standard Form:
$$
ax + by + c' = 0
$$
by rearranging the equation accordingly.

Handling Different Types of Given Line Equations

The process remains similar regardless of the initial form of the given line. The key steps are to determine the slope and then apply the point-slope formula.

Example 1: Given line in standard form

Suppose the given line is $4x + y = 7$ and the point is $(3, y)$.

Step 1: Find the slope:
$$
4x + y = 7 \Rightarrow y = -4x + 7
$$
Slope: $m = -4$

Step 2: Use the point $(3, y_0)$:
$$
y - y_0 = -4(x - 3)
$$

Step 3: Write the equation:

$$
y = -4x + 12 + y_0
$$

If y_0 is specified, substitute to get the explicit equation.

Special Cases and Additional Considerations

When the Point Is Not Fully Specified

If the point is only partially specified, such as $(3, y)$, and y is unknown, the general form of the equation will include y as a parameter. Alternatively, if the point has a specific y-coordinate, the equation can be fully determined.

Vertical and Horizontal Lines

- Vertical lines: Have undefined slope; equation is $x = x_0$.
- Horizontal lines: Have slope 0; equation is $y = y_0$.

For vertical lines, parallel lines are also vertical and have the same x-coordinate.

Practical Applications of Finding Parallel Lines

Understanding how to find the equation of a line parallel to a given line passing through a specific point has numerous applications, including:

- Designing roads and pathways that run parallel to existing structures
- Creating parallel programming in computer graphics
- Solving geometric problems involving parallelism in architecture
- Analyzing trends in data fitting and regression lines

Summary of the Process

1. Identify the slope m: Convert the given line to slope-intercept form or directly determine the slope from standard form.
2. Use the point and the slope: Apply the point-slope formula $y - y_1 = m(x - x_1)$.
3. Simplify: Convert to the desired form (slope-intercept or standard form).
4. Verify: Ensure the new line passes through the given point and is parallel to the original line.

Conclusion

Finding the equation of a line parallel to a given line and passing through a specific point involves understanding the core principles of slopes and line equations. By accurately determining the slope of the original line and applying the point-slope form, you can derive the new line's equation efficiently. Mastering these steps enhances your problem-solving skills in coordinate geometry and opens doors to tackling more complex geometric and algebraic challenges.

Additional Resources and Practice Problems

To solidify your understanding, consider practicing with the following:

1. Given the line $y = 3x + 2$, find the equation of the line parallel passing through $(4, 5)$.

2. Given the line $5x - 2y = 8$, find the parallel line passing through $(1, -3)$.

3. Determine the equation of the line parallel to $y = -\frac{1}{2}x + 4$ passing through $(0, 0)$.

By practicing these problems, you'll build confidence in applying the concepts of parallel lines and line equations in various contexts.

Remember: The key always lies in understanding the slope and the point through which the new line passes. When these are clear, finding the equation of a parallel line becomes a straightforward process.

Frequently Asked Questions

What is the general approach to find the equation of a line parallel to a given line passing through a specific point?

First, determine the slope of the given line, then use the point-slope form with that slope and the given point to find the new line's equation.

How do you find the slope of a line parallel to a given line?

Parallel lines have the same slope, so the slope of the new line is the same as that of the given line.

If a line passes through (3, y) and is parallel to 2x + 3y = 6, what is its equation?

First, find the slope of the given line (which is -2/3). Then, use point-slope form with point (3, y) and slope -2/3 to find the equation.

What is the point-slope form of a line passing through (3, y) with slope m?

The point-slope form is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point (3, y) and m is the slope.

Can you provide an example of finding the equation of a line parallel to 4x - y = 7 passing through (3, 2)?

Yes. First, rewrite the line in slope-intercept form: y = 4x - 7, so slope m = 4. Using point (3, 2), the equation is y - 2 = 4(x - 3), which simplifies to y = 4x - 10.

What if the given line is vertical? How do you find a parallel line passing through (3, y)?

A vertical line has an undefined slope, so its equation is x = constant. To find a parallel line passing through (3, y), the new line is also vertical: x = 3.

How does the value of y in the point (3, y) affect the equation of the parallel line?

The value of y determines the specific point through which the line passes, but the slope remains the same; you substitute y into the point-slope form accordingly.

What are common mistakes to avoid when finding the equation of a parallel line passing through a point?

Common mistakes include miscalculating the slope, confusing the slope of the original line, and incorrectly applying the point-slope form. Always verify the slope and correctly substitute the point coordinates.

Is the process of finding a parallel line passing through (3, y) different if the original line is given in standard form versus slope-intercept form?

The process is similar; you convert the original line to slope-intercept form to find the slope if necessary. Then, use the point-slope form with the known point and slope to find the new line's equation.

Additional Resources

Equation of the Line That Is Parallel to the Given Line and Passes Through the Point (3, y)

Understanding the equation of a line that is parallel to a given line and passes through a specific point is a fundamental concept in coordinate geometry. This topic is essential for students and professionals working in fields that involve graphing, algebra, or spatial reasoning. In this comprehensive article, we will explore the fundamental principles, step-by-step methods, and practical examples to find the equation of such a line, focusing on the scenario where the point is (3, y). We will also discuss various features, advantages, and potential challenges associated with this process.

Introduction to Parallel Lines and Their Equations

Before diving into the specifics of deriving the equation of a parallel line passing through a given point, it is crucial to understand the concepts of parallel lines and their equations.

What Are Parallel Lines?

Parallel lines are lines in a plane that never intersect, regardless of how far they are extended. They maintain a constant distance apart and have identical slopes. In coordinate geometry, the defining characteristic of parallel lines is that their slopes are equal.

General Equation of a Line

The most common form of a linear equation in the Cartesian plane is:

$$ y = mx + c $$

where:
- m is the slope of the line,
- c is the y-intercept (the point where the line crosses the y-axis).

Understanding the slope-intercept form makes it straightforward to analyze and compare lines, especially when determining whether lines are parallel.

Understanding the Slope and Its Role in Parallel Lines

The Significance of Slope

The slope (m) indicates the steepness and direction of a line. Two lines are parallel if and only if they have the same slope.

Determining the Slope of the Given Line

Suppose we are given the equation of the original line, for example:

$$ y = m_1 x + c_1 $$

To find the equation of a line parallel to this one passing through a point (x_0, y_0), the primary step is to identify m_1. Since the lines are parallel, the new line will have the same slope:

$$ m_2 = m_1 $$

Formulating the Equation of the Parallel Line Passing Through a Specific Point

Step 1: Identify the Slope of the Given Line

Given the original line, convert it into slope-intercept form if necessary, or directly read off the slope if it’s already in that form.

Example:
Suppose the original line is:

$$ y = 2x + 5 $$

The slope is $m_1 = 2$.

Step 2: Use the Point-Slope Form

The point-slope form of a line's equation is:

$$ y - y_0 = m (x - x_0) $$

where:
- (x_0, y_0) is the point the line passes through,
- m is the slope.

Applying to our point (3, y):
If the y-coordinate is known, say $y = y_0$, then:

$$ y - y_0 = m (x - 3) $$

This gives the equation of the line passing through $(3, y_0)$ with slope m.

Step 3: Express the Equation in Slope-Intercept Form

Expand and rearrange the point-slope form to slope-intercept form:

$$ y = m (x - 3) + y_0 $$

$$ y = m x - 3m + y_0 $$

Depending on the value of y_0, this form can be adjusted accordingly.

Handling the Case When the Point Is (3, y)

In the prompt, the point is given as (3, y), but y is unspecified. This opens two scenarios:

Scenario 1: The y-coordinate is known
If a specific y-coordinate (say, $y = y_0$) is provided, the process is straightforward as shown above.

Scenario 2: The y-coordinate is unspecified
If only the x-coordinate (3) is specified, and we seek the equation of the line passing through all points with $x=3$ and some y, then the line must be parallel to the given line, with an arbitrary y-intercept. In such a case, the equation depends on the specific point or additional constraints.

Practical Examples and Step-by-Step Solutions

Let's explore some concrete examples to clarify the process.

Example 1: Find the equation of the line parallel to $y = 3x + 2$ passing through (3, 7)

Solution:

1. Identify the slope of the given line:
$m_1 = 3$

2. Use the point-slope form:
$(x_0, y_0) = (3, 7)$

$$ y - 7 = 3 (x - 3) $$

3. Expand to slope-intercept form:
$$ y - 7 = 3x - 9 $$

$$ y = 3x - 9 + 7 $$

$$ y = 3x - 2 $$

Result:

The equation of the line parallel to $y = 3x + 2$ passing through (3, 7) is:

$$ y = 3x - 2 $$

Example 2: Find the equation of the line parallel to $y = -\frac{1}{2}x + 4$ passing through (3, y), where y is unknown

Solution:

1. Identify the slope:
$m_1 = -\frac{1}{2}$

2. Equation in point-slope form:
Since y is unspecified, the equation is:

$$ y - y_0 = -\frac{1}{2} (x - 3) $$

3. Expressing in slope-intercept form:
$$ y = -\frac{1}{2} x + \frac{3}{2} + y_0 $$

Here, y_0 can be any value, which means the line can pass through any point with $x=3$, and y depends on y_0.

Special Cases and Additional Considerations

Vertical Lines

When the original line is vertical, such as $x = a$, the slope is undefined. The parallel line will also be vertical and will have the form:

$$x = a$$

passing through the given point $(3, y)$ only if $a = 3$. If $a \neq 3$, then no such line passes through the point with the same slope.

Horizontal Lines

If the original line is horizontal, such as $y = k$, then the parallel line also has slope zero:

$$y = y_0$$

and passes through the point $(3, y)$, which must satisfy $y = y_0$.

Features and Advantages of the Method

- Simplicity: Using the point-slope form makes it straightforward to find the line's equation once the slope and a point are known.
- Flexibility: Applicable to any line, whether slope-intercept, standard form, or vertical/horizontal.
- Clarity: Provides a clear step-by-step approach that can be easily followed and adapted.

Limitations and Challenges

- Ambiguity with Unknown Coordinates: When the y-coordinate of the point is unknown, the solution becomes a family of lines rather than a single line.
- Vertical Lines: Special handling is required for vertical lines since their slope is undefined.
- Assumption of Linearity: The method assumes the line is straight; it doesn't apply to non-linear curves.

Conclusion

The process of finding the equation of a line parallel to a given line and passing through a specific point, such as (3, y), hinges on understanding the importance of slope in defining line relationships. By identifying the slope of the original line and applying the point-slope form, one can derive the new line's equation efficiently. This method is widely applicable across various types of lines, with special considerations for vertical and horizontal cases. Mastery of this technique is invaluable for students and professionals dealing with geometric problems, offering a reliable way to analyze and construct lines under given constraints.

In summary, whether dealing with known or unknown \(y \)-coordinates, the core approach remains consistent: determine the slope, leverage the point-slope form, and convert to the desired form for clarity and application. This foundational skill enhances one's ability to navigate the broader landscape of coordinate geometry with confidence and precision.

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