What Is The Particular Solution To The Differential Equation Dy/dx = 2x/y With The Initial Condition

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Understanding differential equations is fundamental in mathematics, physics, engineering, and many applied sciences. Among the various types of differential equations, first-order equations are particularly significant due to their relative simplicity and extensive applications. One such equation is the differential equation:

$$
\frac{dy}{dx} = \frac{2x}{y}
$$

When solving differential equations, an essential goal is to find the general solution, which includes an arbitrary constant. However, often we are given an initial condition, such as $y(x_0) = y_0$, and we seek the particular solution that satisfies this initial condition. This article explores the process of solving the differential equation $\frac{dy}{dx} = \frac{2x}{y}$ to find its particular solution given an initial condition.

Understanding the Differential Equation

Type and Form of the Equation

The differential equation:

$$
\frac{dy}{dx} = \frac{2x}{y}
$$

is a first-order, separable differential equation. It can be rewritten as:

$$
y \, dy = 2x \, dx
$$

This form indicates that the variables y and x can be separated on opposite sides of the equation, allowing for straightforward integration.

Significance of the Equation

This particular differential equation models physical scenarios where the rate of change of a quantity y depends on both x and y in a multiplicative manner. For example, it can describe certain thermodynamic processes, growth models, or geometric problems where the relationship between variables is multiplicative.

Solving the Differential Equation

Step 1: Separate Variables

Starting from:

$$
\frac{dy}{dx} = \frac{2x}{y}
$$

Multiply both sides by y and dx:

$$
y \, dy = 2x \, dx
$$

This step separates y and x, setting the stage for integration.

Step 2: Integrate Both Sides

Integrate each side:

$$
\int y \, dy = \int 2x \, dx
$$

Calculating integrals:

$$
\frac{y^2}{2} = x^2 + C
$$

where C is the constant of integration.

Step 3: Express the General Solution

Multiply both sides by 2:

$$
y^2 = 2x^2 + 2C
$$

Let $K = 2C$, which is an arbitrary constant. The general solution is therefore:

$$
\boxed{
y^2 = 2x^2 + K
}
$$

or equivalently:

$$
y = \pm \sqrt{2x^2 + K}
$$

Finding the Particular Solution with Initial Conditions

Understanding Initial Conditions

An initial condition specifies the value of the solution at a particular point, such as:

$$
y(x_0) = y_0
$$

Given this, we can determine the specific value of K to find the particular solution.

Applying the Initial Condition

Suppose the initial condition is:

$$
y(x_0) = y_0
$$

Substitute into the general solution:

$$
y_0^2 = 2x_0^2 + K
$$

Solve for K:

$$
K = y_0^2 - 2x_0^2
$$

Thus, the particular solution is:

$$
\boxed{
y^2 = 2x^2 + y_0^2 - 2x_0^2
}
$$

or

$$
y = \pm \sqrt{2x^2 + y_0^2 - 2x_0^2}
$$

The choice of positive or negative root depends on the initial condition and the context of the problem.

Special Cases and Considerations

When Does the Solution Exist?

The square root imposes a domain restriction:

$$
2x^2 + y_0^2 - 2x_0^2 \geq 0
$$

which must be satisfied for the solution to be real-valued.

Choosing the Sign of the Root

The initial condition might specify whether y is positive or negative at $x = x_0$. This choice influences the sign in the particular solution:

- If $y_0 > 0$, choose the positive root.
- If $y_0 < 0$, choose the negative root.

Applications of the Solution

Understanding how to find particular solutions to differential equations like $\frac{dy}{dx} = \frac{2x}{y}$ has numerous applications:

- Physics: Modeling energy transfer or thermodynamic processes where variables depend on each other quadratically.
- Engineering: Calculating stress or strain in materials where relationships are quadratic.
- Biology: Describing population growth or decay under certain assumptions.

Summary: Key Takeaways

- The differential equation $\frac{dy}{dx} = \frac{2x}{y}$ is separable, allowing straightforward integration.
- Its general solution is $y^2 = 2x^2 + K$.
- To find the particular solution, substitute the initial condition $y(x_0) = y_0$ to solve for K.
- The particular solution then becomes:

$$
\boxed{
y = \pm \sqrt{2x^2 + y_0^2 - 2x_0^2}
}
$$

- Domain restrictions depend on the initial condition and the sign of the square root.
- Proper interpretation of the initial condition ensures the correct branch of the solution.

Conclusion

The process of solving the differential equation $\frac{dy}{dx} = \frac{2x}{y}$ demonstrates the power of separation of variables and algebraic manipulation in solving first-order differential equations. By applying initial conditions, we refine the general solution into a particular solution that precisely models the specific scenario at hand. Whether in theoretical mathematics or applied sciences, mastering these techniques ensures a robust understanding of how variables interact dynamically within various systems.

Keywords for SEO Optimization:

- Differential equations
- Particular solution
- Initial condition
- First-order differential equations
- Separable differential equations
- Solving $dy/dx = 2x/y$
- General solution
- Applying initial conditions
- Mathematical modeling
- Differential equation solutions

Frequently Asked Questions

What is the general approach to find the particular solution for the differential equation dy/dx = 2x/y with given initial conditions?

The typical approach involves separating variables, integrating both sides, and then applying the initial condition to solve for the constant of integration.

How do you solve the differential equation dy/dx = 2x/y to find its particular solution?

Rearrange as y dy = 2x dx, integrate both sides to get $(1/2) y^2 = x^2 + C$, then solve for y to find $y = \pm\sqrt{2x^2 + 2C}$.

What role does the initial condition play in determining the particular solution of dy/dx = 2x/y?

The initial condition provides a specific point (x_0, y_0) which allows you to solve for the constant of integration C, resulting in a unique particular solution.

Can you provide an example of finding the particular solution

given an initial condition?

Yes. For example, if $y(1) = 3$, then substitute $x=1$ and $y=3$ into $(1/2) y^2 = x^2 + C$ to find $C = (1/2)9 - 1 = 4.5 - 1 = 3.5$. The particular solution is $y = \pm\sqrt{(2x^2 + 23.5)}$.

What is the explicit form of the particular solution to dy/dx = 2x/y with initial condition $y(x_0) = y_0$?

The particular solution is $y = \pm\sqrt{(x^2 + C)}$, where C is determined by substituting x_0 and y_0 into the equation: $C = (1/2) y_0^2 - x_0^2$.

Are there any special considerations when solving for the particular solution of dy/dx = 2x/y?

Yes. Since the solution involves a square root, ensure the expression under the root is non-negative, and consider the sign (positive or negative root) based on initial conditions.

What is the significance of the constant of integration in the particular solution?

The constant of integration accounts for the family of solutions to the differential equation, and the initial condition helps select the specific solution within this family.

Is the solution to dy/dx = 2x/y unique for a given initial condition?

Yes, once the initial condition is specified, it determines the constant C, leading to a unique particular solution.

How does the initial condition $y(x_0) = y_0$ influence the graph of the particular solution?

It shifts and shapes the solution curve to pass through the point (x_0, y_0), effectively selecting a specific curve from the family of solutions.

Additional Resources

What Is The Particular Solution To The Differential Equation Dy/dx = 2x/y With The Initial Condition

Understanding differential equations is fundamental in fields ranging from physics and engineering to economics and biology. They describe how a quantity changes concerning another and often encapsulate complex real-world phenomena. One common task when solving differential equations is finding a particular solution that satisfies specific initial conditions, making the solution uniquely tailored to a particular scenario.

In this article, we delve into the differential equation:

$$\frac{dy}{dx} = \frac{2x}{y}$$

and explore how to find its particular solution given an initial condition, such as $y(x_0) = y_0$. We will break down the process step-by-step, providing clarity on the mathematical techniques involved, including separation of variables, integration, and applying initial conditions to determine constants.

Understanding the Differential Equation

Before jumping into the solution, it's essential to interpret what the differential equation represents and its general nature.

The Form of the Equation

The equation:

$$\frac{dy}{dx} = \frac{2x}{y}$$

is a first-order, nonlinear differential equation. Its structure suggests that it relates the rate of change of y with respect to x directly to the ratio of $2x$ and y.

Physical or Geometric Interpretation

While the equation is abstract, such relationships can describe phenomena like:

- The relationship between area and radius in certain geometric contexts.
- Dynamics where the rate of change depends inversely on the current state y, such as certain cooling or population models.

Understanding the form is crucial because it guides the method of solution. Since the derivative involves y and x multiplicatively, it hints at potential separation of variables.

Solving the Differential Equation: The General Approach

The primary goal is to find y as a function of x, denoted $y(x)$, that satisfies the differential equation.

Step 1: Recognize the Type of Equation

This is a separable differential equation, meaning it can be rewritten so that all y-terms are on one side and all x-terms on the other.

Step 2: Rearrange for Separation of Variables

Starting with:

$$\frac{dy}{dx} = \frac{2x}{y}$$

multiply both sides by y:

$$ y \frac{dy}{dx} = 2x $$

or equivalently,

$$ y \, dy/dx = 2x $$

which leads to:

$$ y \, dy = 2x \, dx $$

This step separates the variables, making integration straightforward.

Integration: Finding the General Solution

Having separated variables, the next step is integrating both sides:

$$ \int y \, dy = \int 2x \, dx $$

Computing the Integrals

- Left side:

$$ \int y \, dy = \frac{1}{2} y^2 + C_1 $$

- Right side:

$$ \int 2x \, dx = x^2 + C_2 $$

Since the constants of integration on both sides can be combined into a single constant, say C, the general solution becomes:

$$ \frac{1}{2} y^2 = x^2 + C $$

or, multiplying through by 2:

$$ y^2 = 2x^2 + 2C $$

Let’s denote $2C$ as a new constant K:

$$ y^2 = 2x^2 + K $$

where K is an arbitrary constant determined by initial conditions.

Finding the Particular Solution: Applying Initial Conditions

The general solution contains an arbitrary constant K. To find the particular solution, we need an initial condition, such as:

$$y(x_0) = y_0$$

which specifies the value of y at some point x_0.

Step 1: Substitute the Initial Condition

Plugging in $x = x_0$ and $y = y_0$:

$$y_0^2 = 2 x_0^2 + K$$

Solve for K:

$$K = y_0^2 - 2 x_0^2$$

Step 2: Write the Particular Solution

Substitute K back into the general solution:

$$y^2 = 2 x^2 + y_0^2 - 2 x_0^2$$

which simplifies to:

$$y^2 = 2 (x^2 - x_0^2) + y_0^2$$

Thus, the particular solution is:

$$y(x) = \pm \sqrt{2(x^2 - x_0^2) + y_0^2}$$

The sign depends on initial conditions and the context of the problem; typically, the positive root is chosen if y is known to be positive.

Final Form of the Particular Solution

In summary, the particular solution to the differential equation:

$$\frac{dy}{dx} = \frac{2x}{y}$$

given the initial condition $y(x_0) = y_0$, is:

$$y(x) = \pm \sqrt{2 \left(x^2 - x_0^2 \right) + y_0^2}$$

This expression describes how y varies with x, adjusted to fit the starting point.

Visualizing the Solution

Graphing the particular solution reveals a family of curves dependent on the initial condition y_0 and x_0. These curves are symmetric with respect to the y-axis due to the x^2 term, often forming hyperbolic shapes or similar conic sections.

The choice of the positive or negative root affects the part of the curve the solution describes. In practical applications, physical constraints or initial conditions guide this choice.

Additional Considerations

- Existence and Uniqueness: Given a continuous function on a specific interval, the initial condition defines a unique solution per the Picard-Lindelöf theorem.
- Domain of the Solution: The expression under the square root must be non-negative:

$$ 2(x^2 - x_0^2) + y_0^2 \geq 0 $$

which imposes restrictions on the interval where the solution is valid.

- Special Cases: When $y_0 = 0$ and $x_0 = 0$, the solution simplifies to $y(x) = \pm \sqrt{2 x^2} = \pm \sqrt{2} |x|$.

Practical Applications and Significance

Finding particular solutions to differential equations like this one isn't just a mathematical exercise; it’s crucial in modeling real-world phenomena where initial states are known, and predictions are needed.

For example:

- Physics: Modeling the motion of particles or systems where initial velocity or position is specified.
- Biology: Describing population growth or decay with specific initial populations.
- Engineering: Analyzing systems with initial energy, charge, or other parameters.

Understanding how to derive these solutions equips practitioners to interpret data accurately and make informed decisions based on mathematical models.

Conclusion

The differential equation $\frac{dy}{dx} = \frac{2x}{y}$ exemplifies the power of separation of variables and integration in solving nonlinear first-order differential equations. By carefully applying these techniques and integrating initial conditions, we obtain a particular solution that precisely models the situation at hand.

The journey from recognizing the equation's form, separating variables, integrating, and applying initial conditions underscores the elegance and utility of mathematical methods in understanding dynamic systems. Whether in academic research or practical applications, mastering these techniques enhances our ability to interpret, predict, and control complex phenomena.

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