WILL GIVE BRAINLIEST How Long Would It Take 308 G Of The Sample To Decay To 4. 8125 Grams? Show Your

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Understanding radioactive decay is fundamental in fields such as nuclear physics, radiometric dating, medical imaging, and nuclear energy. When working with radioactive samples, one common question is: How long will it take for a certain amount of a radioactive substance to decay to a specific lower quantity? In this article, we will explore the detailed process of calculating the decay time for a sample starting at 308 grams and reducing to 4.8125 grams. We will explain the principles behind decay, introduce the relevant formulas, walk through a step-by-step calculation, and discuss important concepts to deepen your understanding.

Understanding Radioactive Decay

Radioactive decay is a spontaneous process where unstable atomic nuclei emit particles or radiation to become more stable. The decay process follows a probabilistic pattern, characterized by the half-life — the time it takes for half of the radioactive nuclei to decay.

Key Concepts in Radioactive Decay

- Radioactive isotope: The unstable element undergoing decay.
- Decay constant (λ): A parameter that indicates the probability per unit time that a nucleus will decay.
- Half-life ($T_{1/2}$): The time required for half of the sample to decay.
- Exponential decay law: The mathematical model describing the decay process.

Mathematical Foundations of Radioactive Decay

The fundamental formula governing radioactive decay is:

$$N(t) = N_0 \times e^{-\lambda t}$$

Where:
- $N(t)$ = amount of substance remaining after time t,
- N_0 = initial amount of substance,
- λ = decay constant,
- t = elapsed time.

The decay constant λ relates to the half-life as:

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Step-by-step Calculation Process

To determine how long it takes for a sample to decay from an initial amount N_0 to a final amount $N(t)$, we rearrange the decay law:

$$ t = \frac{1}{\lambda} \times \ln \left(\frac{N_0}{N(t)} \right) $$

Application to Our Problem

Given:
- Initial sample $N_0 = 308 \text{ g}$,
- Final sample $N(t) = 4.8125 \text{ g}$,
- The decay process is assumed to be from a specific isotope with a known half-life.

Important: To proceed, we must know the isotope's half-life $T_{1/2}$. For illustration, let's assume the isotope is Radon-222, which has a half-life of approximately 3.8 days. However, the process is similar for any isotope; you just need its specific half-life.

Calculating Decay Time Step-by-Step

Step 1: Find the decay constant λ.

$$ \lambda = \frac{\ln 2}{T_{1/2}} $$

For Radon-222:

$$ T_{1/2} = 3.8 \text{ days} $$

$$
\lambda = \frac{\ln 2}{3.8} \approx \frac{0.6931}{3.8} \approx 0.1824 \text{ days}^{-1}
$$

Step 2: Plug values into the decay formula.

$$
t = \frac{1}{\lambda} \times \ln \left(\frac{N_0}{N(t)} \right)
$$

$$
t = \frac{1}{0.1824} \times \ln \left(\frac{308}{4.8125} \right)
$$

Calculate the ratio:

$$
\frac{308}{4.8125} \approx 64
$$

Calculate the natural logarithm:

$$
\ln 64 \approx 4.1589
$$

Step 3: Compute the decay time t.

$$
t \approx \frac{1}{0.1824} \times 4.1589 \approx 5.481 \times 4.1589 \approx 22.8 \text{ days}
$$

Summary of the Calculation

Parameter	Value
Initial mass N_0	308 g
Final mass $N(t)$	4.8125 g
Half-life $T_{1/2}$	3.8 days (for Radon-222)
Decay constant λ	0.1824 day^{-1}
Calculated time t	approximately 22.8 days

Therefore, it would take approximately 22.8 days for 308 grams of Radon-222 to decay to 4.8125 grams.

Factors Affecting Decay Calculations

While the above calculations assume a perfect exponential decay process and a known half-life, several factors can influence real-world measurements:

- Isotope purity: Contaminants can alter effective decay rates.
- Environmental conditions: Temperature, pressure, and chemical environment may influence the decay process minimally but are generally negligible.
- Measurement accuracy: Precise measurements of initial and final amounts are essential.
- Decay chain complexities: Some isotopes decay through a series of steps, which may complicate calculations.

Additional Considerations and Variations

What if the isotope's half-life is different?

If you know the half-life $T_{1/2}$ of the isotope you are dealing with, simply substitute into the decay constant formula:

$$
\lambda = \frac{\ln 2}{T_{1/2}}
$$

and repeat the calculation.

How to handle different starting and ending quantities?

The formula:

$$
t = \frac{1}{\lambda} \times \ln \left(\frac{N_0}{N(t)} \right)
$$

\]

works universally for any initial and final amounts, provided the decay follows an exponential law.

Example: Decay of Carbon-14

Carbon-14 has a half-life of approximately 5730 years. If a sample starts at 100 grams and reduces to 25 grams, how long does this take?

$$
\lambda = \frac{\ln 2}{5730} \approx 1.21 \times 10^{-4} \text{ year}^{-1}
$$

$$
t = \frac{1}{1.21 \times 10^{-4}} \times \ln \left(\frac{100}{25} \right) = 8264 \text{ years}
$$

Practical Applications of Decay Time Calculations

Understanding decay times is crucial in various fields:

- Radiometric dating: Determining the age of archaeological finds.
- Medical treatments: Planning for radioactive isotope decay in imaging or therapy.
- Nuclear power: Managing fuel decay and waste.
- Environmental monitoring: Tracking radioactive contamination over time.

Conclusion

Calculating the decay time of a radioactive sample from an initial amount to a specific lower amount involves understanding the decay law, knowing the isotope's half-life, and applying the exponential decay formula. In our example, assuming Radon-222 with a half-life of 3.8 days, we estimated that approximately 22.8 days are needed for 308 grams to decay to 4.8125 grams. This approach can be adapted to any isotope by substituting the respective half-life and initial quantities.

By mastering these calculations, scientists and engineers can effectively predict decay timelines, plan experiments, and manage radioactive materials safely and efficiently.

Remember: Always verify the isotope and its half-life before performing such calculations to ensure accuracy.

Frequently Asked Questions

How do I calculate the decay time for 308 g of a sample to reach 4.8125 g?

You can use the exponential decay formula: $N = N_0 e^{(-\lambda t)}$. First, find λ using the half-life or decay constant, then solve for t by rearranging the formula: $t = (1/\lambda) \ln(N_0 / N)$.

What is the significance of the decay constant (λ) in this calculation?

The decay constant λ determines the rate at which the sample decays. It is related to the half-life and is essential for calculating the time required for the sample to decay from 308 g to 4.8125 g.

How can I find the decay constant if I know the half-life of the substance?

The decay constant λ can be calculated using the relation $\lambda = \ln(2)$ / half-life. Once you have the half-life, plug it into this formula to find λ.

Is it necessary to know the half-life of the substance to solve this problem?

Yes, knowing the half-life or decay constant is necessary to determine the time it takes for the sample to decay from 308 g to 4.8125 g.

What is the formula to calculate the time for radioactive decay from an initial to a final amount?

The formula is $t = (1/\lambda) \ln(N_0 / N)$, where N_0 is the initial amount, N is the final amount, and λ is the decay constant.

If the decay constant is unknown, how can I find it?

If the half-life is known, use $\lambda = \ln(2)$ / half-life. If neither is known, you need additional data about the substance's decay properties.

Can I use half-lives directly to determine the total decay time in this problem?

Yes, by calculating how many half-lives it takes for the initial amount to decay to the target amount, then multiplying by the half-life duration.

What is the step-by-step process to solve this problem?

1. Find the decay constant λ using known half-life or data. 2. Use the formula $t = (1/\lambda) \ln(N_0 / N)$ with N_0=308 g and N=4.8125 g. 3. Calculate t to find the decay time.

Are there any assumptions made in this type of decay calculation?

Yes, the calculation assumes the decay follows a perfect exponential model and that the decay constant remains constant over time.

What additional information is needed to accurately solve this problem?

You need the decay constant (λ) or the half-life of the substance to perform an accurate calculation of the decay time.

Additional Resources

WILL GIVE BRAINLIEST How Long Would It Take 308 G Of The Sample To Decay To 4.8125 Grams? Show Your Work

Understanding radioactive decay is fundamental in nuclear physics, radiometric dating, and various scientific applications. When posed with a question like "How long would it take 308 g of the sample to decay to 4.8125 grams?", it requires a clear grasp of exponential decay formulas, half-life concepts, and proper calculation techniques. This article aims to provide a comprehensive, step-by-step guide to solving this kind of problem, illustrating the process with detailed explanations, formulas, and example calculations.

The Basics of Radioactive Decay

Radioactive decay is a random process at the level of single atoms, but it follows a predictable statistical pattern for large numbers of atoms. The decay of a radioactive isotope over time is modeled mathematically by exponential decay.

Key Concepts

- Radioactive Isotope: An unstable atom that decays over time, emitting radiation.
- Half-life ($t_{1/2}$): The time required for half of the radioactive nuclei in a sample to decay.
- Decay Constant (λ): The probability per unit time that a given nucleus will decay.
- Exponential Decay Law: Describes how the quantity of a radioactive substance decreases over time.

The Decay Formula

The fundamental equation governing radioactive decay is:

$$N(t) = N_0 \times e^{-\lambda t}$$

Where:

- $N(t)$ = amount of substance remaining at time t
- N_0 = initial amount of substance
- λ = decay constant
- t = elapsed time

Alternatively, using half-life:

$$N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{t_{1/2}}}$$

Step-by-Step Guide to the Problem

1. Identify Known Values

- Initial amount, $N_0 = 308\, \text{g}$
- Final amount, $N(t) = 4.8125\, \text{g}$

Note: The problem as posed does not specify the isotope or its half-life. To proceed, we'll assume a specific isotope, such as Uranium-238, which has a well-known half-life of approximately 4.468 billion years. If the problem is hypothetical, we can use any isotope's half-life for demonstration purposes.

2. Find the Half-life (if not given)

Assuming Uranium-238:

$$ t_{1/2} = 4.468 \times 10^9\, \text{years} $$

3. Calculate the Decay Constant (λ)

Using the relationship between half-life and decay constant:

$$ \lambda = \frac{\ln 2}{t_{1/2}} $$

Calculating:

$$ \lambda = \frac{0.6931}{4.468 \times 10^9} \approx 1.551 \times 10^{-10}\, \text{year}^{-1} $$

Applying the Decay Formula

4. Set Up the Equation

Using the exponential decay formula:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Plugging in the values:

$$ 4.8125 = 308 \times e^{- (1.551 \times 10^{-10}) t} $$

5. Isolate the Exponential Term

Divide both sides by 308:

$$ \frac{4.8125}{308} = e^{-\lambda t} $$

Calculate:

$$ 0.01563 \approx e^{-\lambda t} $$

6. Solve for t

Take the natural logarithm of both sides:

$$ \ln(0.01563) = -\lambda t $$

$$ -4.159 = - (1.551 \times 10^{-10}) t $$

Solve for t:

$$ t = \frac{4.159}{1.551 \times 10^{-10}} $$

$$ t \approx 2.681 \times 10^{10}\, \text{years} $$

Interpretation of the Result

The calculated time:

$$ t \approx 26.81\, \text{billion years} $$

This indicates that, assuming Uranium-238, it would take approximately 26.81 billion years for a 308 g sample to decay to 4.8125 g, which exceeds the age of the universe (~13.8 billion years). This suggests that, for isotopes with such a long half-life, the decay process is extremely slow.

Generalization and Additional Considerations

1. Isotope Choice Matters

Different isotopes have vastly different half-lives, from fractions of a second to billions of years. When solving similar problems:

- Always identify the specific isotope.
- Use the official half-life value for that isotope.
- Convert units consistently.

2. Alternative Approach: Using Half-life

If you prefer, you can directly use the half-life formula:

$$N(t) = N_0 \times \left(\frac{1}{2} \right)^{t / t_{1/2}}$$

Rearranged to solve for t:

$$t = t_{1/2} \times \frac{\ln (N(t)/N_0)}{\ln (1/2)}$$

Applying this to the same numbers:

$$t = 4.468 \times 10^9 \times \frac{\ln (4.8125/308)}{\ln(1/2)}$$

Calculate numerator:

$$\ln (0.01563) = -4.159$$

Calculate denominator:

$$\ln(1/2) = -0.6931$$

Calculate:

$$t = 4.468 \times 10^9 \times \frac{-4.159}{-0.6931}$$

$$t \approx 4.468 \times 10^9 \times 6.000$$

$$t \approx 26.81 \times 10^9\, \text{years}$$

Same as previous result, confirming consistency.

Summary of Key Steps

- Identify initial and final amounts.
- Determine the isotope's half-life.
- Calculate the decay constant λ.
- Use the exponential decay formula or half-life relation to solve for time.
- Interpret the result in context, considering isotope properties and physical meaning.

Practical Implications

Understanding how long it takes for a radioactive sample to decay to a certain level is critical in fields like:

- Radiometric dating (e.g., determining geological ages).
- Nuclear medicine (e.g., decay of isotopes within the body).
- Nuclear power (e.g., managing radioactive waste).

Knowing how to perform these calculations helps scientists estimate decay timelines, safety protocols, and age estimates accurately.

Final Thoughts

The key to solving decay time problems lies in understanding the exponential decay law, having accurate isotope data, and carefully performing the calculations. While the example used Uranium-238, the same principles apply to any isotope, with adjustments based on its specific half-life. Always double-check your units and assumptions, and remember that in real-world scenarios, factors such as environmental influences or measurement uncertainties can affect results.

In conclusion, calculating how long it takes for a sample to decay from one mass to another involves a combination of understanding exponential decay formulas, knowing the isotope's half-life, and performing careful algebraic manipulations. With practice, these calculations become intuitive and invaluable tools in scientific analysis and research.

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