# 1) Consider The Following Problems: (a) Find The Sum N=1[infinity]3nn2+n (b) Solve Y+3y+2y=3+2x Using

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Introduction

Mathematics problems often challenge learners to develop a deeper understanding of concepts such as series summation and differential equations. In this article, we will explore two distinct yet fundamental problems:

1. The evaluation of an infinite series involving a complex term.
2. Solving a linear differential equation with constant coefficients.

Both problems serve as excellent examples for enhancing problem-solving skills, understanding series convergence, and mastering techniques for solving differential equations. We will analyze each problem in detail, offer step-by-step solutions, and discuss relevant mathematical principles.

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Problem (a): Find the Sum for n=1 to Infinity of $\frac{3n}{n^2 + n}$

Understanding the Series

The series in question is:

$$
\sum_{n=1}^{\infty} \frac{3n}{n^2 + n}
$$

At first glance, the series involves a rational expression, which suggests that partial fraction decomposition can be used to simplify the terms.

Step 1: Simplify the General Term

Observe that:

$$
n^2 + n = n(n+1)
$$

Thus, the general term becomes:

$$
\frac{3n}{n(n+1)} = \frac{3n}{n(n+1)}
$$

which simplifies to:

$$
\frac{3n}{n(n+1)} = \frac{3n}{n(n+1)} = \frac{3}{n+1}
$$

But wait, this simplification seems to be incorrect because the numerator is $3n$, and the denominator is $n(n+1)$. Let's carry out the partial fraction decomposition carefully.

Step 2: Apply Partial Fraction Decomposition

Express:

$$
\frac{3n}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}
$$

Multiply both sides by $n(n+1)$:

$$
3n = A(n+1) + B n
$$

which simplifies to:

$$
3n = A n + A + B n
$$

Group like terms:

$$
3n = (A + B) n + A
$$

Matching coefficients:

- Coefficient of $n$:

$$
A + B = 3
$$

- Constant term:

$$
A = 0
$$

Thus:

$$
A = 0
$$

and

$$
B = 3 - A = 3 - 0 = 3
$$

Step 3: Rewrite the Series

Therefore,

$$
\frac{3n}{n(n+1)} = \frac{0}{n} + \frac{3}{n+1} = \frac{3}{n+1}
$$

Hence, the original series simplifies to:

$$
\sum_{n=1}^{\infty} \frac{3}{n+1}
$$

which can be written as:

$$
3 \sum_{n=1}^{\infty} \frac{1}{n+1}
$$

Change the index to make it clearer:

Let $k = n + 1$

When $n = 1$, $k=2$

As $n \to \infty$, $k \to \infty$

Thus,

$$
3 \sum_{k=2}^{\infty} \frac{1}{k}
$$

This is a scaled tail of the harmonic series.

Step 4: Analyze the Series for Convergence

The harmonic series:

$$
\sum_{k=1}^{\infty} \frac{1}{k}
$$

diverges to infinity, which means:

$$
\sum_{k=2}^{\infty} \frac{1}{k}
$$

also diverges.

Therefore,

$$
\boxed{
\sum_{n=1}^{\infty} \frac{3n}{n^2 + n} \text{ diverges to infinity}
}
$$

Conclusion for Problem (a):

The series

$$
\sum_{n=1}^{\infty} \frac{3n}{n^2 + n}
$$

diverges, as it simplifies to a multiple of the harmonic series, which is divergent.

Key Takeaway:

- When dealing with infinite series involving rational functions, partial fraction decomposition can simplify the terms.
- Recognizing divergence or convergence is crucial; in this case, the harmonic series diverges.

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Problem (b): Solve the Differential Equation $y + 3y' + 2y = 3 + 2x$

Understanding the Differential Equation

The equation:

$$
y + 3y' + 2y = 3 + 2x
$$

is a linear nonhomogeneous differential equation of the first order involving derivatives $y'$.

Note that the equation can be rewritten as:

$$
(1 + 2) y + 3y' = 3 + 2x
$$

or simply:

$$
(1 + 2) y + 3y' = 3 + 2x
$$

But this is incorrect. Let's proceed carefully.

Step 1: Write in Standard Form

The standard form for a first-order linear differential equation:

$$
y' + P(x) y = Q(x)
$$

is achieved by dividing through by the coefficient of $y'$, if necessary.

In our case, the differential equation is:

$$
y + 3y' + 2y = 3 + 2x
$$

which simplifies to:

$$
3y' + (1 + 2) y = 3 + 2x
$$

or

$$
3 y' + 3 y = 3 + 2x
$$

Divide both sides by 3:

$$
y' + y = 1 + \frac{2}{3} x
$$

Now, the equation is in the standard form:

$$
y' + y = 1 + \frac{2}{3} x
$$

Step 2: Find the Integrating Factor

The integrating factor (IF):

$$
\mu(x) = e^{\int P(x) dx} = e^{\int 1 dx} = e^{x}
$$

Step 3: Multiply the Entire Equation by the Integrating Factor

Multiplying both sides by $e^{x}$:

$$
e^{x} y' + e^{x} y = e^{x} \left( 1 + \frac{2}{3} x \right)
$$

Left side simplifies to:

$$
\frac{d}{dx} \left( e^{x} y \right) = e^{x} \left( 1 + \frac{2}{3} x \right)
$$

Step 4: Integrate Both Sides

$$
e^{x} y = \int e^{x} \left( 1 + \frac{2}{3} x \right) dx + C
$$

Break the integral into two parts:

$$
\int e^{x} dx + \frac{2}{3} \int x e^{x} dx
$$

- The first integral:

$$
\int e^{x} dx = e^{x} + C_1
$$

- The second integral:

\[

$$
\int x e^{x} dx
$$

which requires integration by parts.

Step 5: Calculate $\int x e^{x} dx$

Let:

- $u = x \Rightarrow du = dx$
- $dv = e^{x} dx \Rightarrow v = e^{x}$

Applying integration by parts:

$$
\int x e^{x} dx = x e^{x} - \int e^{x} dx = x e^{x} - e^{x} + C
$$

Step 6: Complete the Integration

Putting it all together:

$$
e^{x} y = e^{x} + \frac{2}{3} \left( x e^{x} - e^{x} \right) + C
$$

Simplify:

$$
e^{x} y = e^{x} + \frac{2}{3} x e^{x} - \frac{2}{3} e^{x} + C
$$

Factor out $e^{x}$:

$$
e^{x} y = e^{x} \left( 1 + \frac{2}{3} x - \frac{2}{3} \right) + C
$$

$$
e^{x} y = e^{x} \left( \frac{1}{3} + \frac{2}{3} x \right) + C
$$

Divide both sides by $e^{x}$:

$$
y = \frac{1}{3} + \frac{2}{3} x + C e^{-x}
$$

Solution:

$$
\boxed{
y(x) = \frac{1}{3} + \frac{2}{3} x + C e^{-x}
}
$$

where $C$ is an arbitrary constant

## Frequently Asked Questions

### How do I evaluate the infinite sum N=1 to infinity of (3n n^2 + n)?

To evaluate the sum, split it into two separate sums: sum of 3n n^2 and sum of n. Use known formulas for sums of powers, such as the sum of n, n^2, and n^3, and then apply limits as n approaches infinity. Since the terms grow without bound, the sum diverges and does not have a finite value.

### Is the infinite sum � _{n=1}^� (3n n^2 + n) convergent or divergent?

The sum diverges because the terms grow proportionally to n^3, which does not tend to zero as n approaches infinity. Therefore, the infinite series diverges.

### How can I solve the differential equation y + 3y + 2y = 3 + 2x?

Note that the equation appears to have a typo; assuming it is $y'' + 3y' + 2y = 3 + 2x$, you can solve it by finding the homogeneous solution and a particular solution, then combine them. Use characteristic equations for the homogeneous part and the method of undetermined coefficients for the particular solution.

### What method can I use to solve the differential equation $y'' + 3y' + 2y = 3 + 2x$?

Use the method of solving linear nonhomogeneous differential equations: first find the general solution of the homogeneous equation $y'' + 3y' + 2y = 0$, then find a particular solution for the right-hand side using undetermined coefficients or variation of parameters.

### Can the sum � _{n=1}^� (3n n^2 + n) be simplified to a closed-form expression?

Since the sum diverges, it cannot be expressed in a finite closed-form. However, you can analyze the partial sums or consider regularization techniques if applicable, but generally, the sum is divergent.

### What are the initial steps to solve the differential equation $y'' + 3y' + 2y = 3 + 2x$?

First, solve the homogeneous equation $y'' + 3y' + 2y = 0$ to find the complementary solution. Then, propose a particular solution for the nonhomogeneous part, such as a polynomial, and determine its coefficients by plugging it into the original equation.

### Are there any special techniques needed for summing the series involving n^2 and n terms?

Yes, use known formulas for sums of powers: � n, � n^2, and � n^3, which are expressed in closed form. These formulas help evaluate or approximate the series or determine convergence.

### What is the significance of the divergence of the sum � _{n=1}^� (3n n^2 + n)?

The divergence indicates that the series does not sum to a finite value, and thus, it cannot be used to represent functions or quantities that require convergence. It highlights the importance of checking the convergence before applying series in analysis.

## How do I find the particular solution of the differential equation with a polynomial right-hand side?

Assume a polynomial form for the particular solution with undetermined coefficients, substitute it into the differential equation, and solve for these coefficients by equating coefficients of like powers of x.

# Additional Resources

Consider The Following Problems: (a) Find The Sum N=1[infinity] 3 n $n^2$ + n (b) Solve Y + 3y + 2y = 3 + 2x — these types of mathematical problems test your understanding of infinite series and differential equations, two fundamental areas in advanced mathematics. Whether you're a student preparing for exams, a professional mathematician, or an enthusiast expanding your knowledge, mastering such problems enhances your analytical skills and deepens your understanding of mathematical concepts. This comprehensive guide will walk you through the step-by-step approaches to solving these problems, offering detailed explanations, techniques, and insights to help you navigate similar challenges confidently.

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Understanding and Solving the Series Sum Problem

Introduction to Series Sums

Before diving into the specific problem, it's essential to understand the basic concepts of infinite series. An infinite series is the sum of infinitely many terms, and under certain conditions, this sum converges to a finite value. Recognizing the type of series you are dealing with—arithmetic, geometric, or more complex forms—is crucial for selecting the right approach.

The Problem: Find the sum $\sum_{n=1}^{\infty} (3 n n^{2} + n)$

Let's restate the problem for clarity:

> Calculate the value of the infinite sum:
> $$\sum_{n=1}^{\infty} (3 n n^{2} + n)$$

---

Step 1: Simplify the Series Expression

Start by simplifying the expression inside the sum:

$$
3 n n^{2} + n = 3 n^{3} + n
$$

So, the series becomes:

$$
\sum_{n=1}^{\infty} (3 n^{3} + n)
$$

This can be separated into two sums:

$$
\sum_{n=1}^{\infty} 3 n^{3} + \sum_{n=1}^{\infty} n
$$

which simplifies to:

$$
3 \sum_{n=1}^{\infty} n^{3} + \sum_{n=1}^{\infty} n
$$

---

Step 2: Analyze the Convergence

Now, analyze whether these individual series converge:

- The sum $\sum_{n=1}^{\infty} n$ diverges because the harmonic series with powers of 1 diverges.
- The sum $\sum_{n=1}^{\infty} n^{3}$ also diverges, as the sum of powers greater than 1 diverges.

Conclusion: Both series diverge, and therefore, their linear combination diverges as well.

Final insight: Since both parts diverge to infinity, the original sum does not converge to a finite value.

---

Key Takeaway: Series Divergence

This problem illustrates an important point: not all infinite series have finite sums. When dealing with such series, the first step is to check for convergence using tests such as the p-series test or comparison test. In this case, the series diverge, so the sum is infinite.

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Solving the Differential Equation

Introduction to Differential Equations

The second problem involves solving a differential equation:

> Solve the differential equation:
> $$
> y + 3 y' + 2 y = 3 + 2 x
> $$

This is a linear nonhomogeneous differential equation of the first order, which is a common type encountered in various fields like physics, engineering, and mathematics.

---

Step 1: Recognize the Standard Form

Rewrite the equation for clarity:

$$
y + 3 \frac{dy}{dx} + 2 y = 3 + 2 x
$$

Or, arranging terms:

$$
3 \frac{dy}{dx} + (1 + 2) y = 3 + 2 x
$$

which simplifies to:

$$
3 y' + 3 y = 3 + 2 x
$$

Divide through by 3:

$$
y' + y = 1 + \frac{2}{3} x
$$

Now, the differential equation is in the standard linear form:

$$
y' + p(x) y = q(x)
$$

where:

- $p(x) = 1$
- $q(x) = 1 + \frac{2}{3} x$

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Step 2: Find the Integrating Factor

The integrating factor (IF) is given by:

$$
\mu(x) = e^{\int p(x) dx} = e^{\int 1 dx} = e^{x}
$$

---

Step 3: Multiply Through by the Integrating Factor

Multiply both sides of the differential equation by $e^{x}$:

$$
e^{x} y' + e^{x} y = e^{x} \left( 1 + \frac{2}{3} x \right)
$$

Notice that the left side becomes the derivative of $e^{x} y$:

$$
\frac{d}{dx} \left( e^{x} y \right) = e^{x} \left( 1 + \frac{2}{3} x \right)
$$

---

Step 4: Integrate Both Sides

Integrate both sides with respect to $x$:

$$
e^{x} y = \int e^{x} \left( 1 + \frac{2}{3} x \right) dx + C
$$

Break the integral into two parts:

$$
\int e^{x} dx + \frac{2}{3} \int x e^{x} dx
$$

- The first integral:

$$
\int e^{x} dx = e^{x} + C_1
$$

- The second integral involves $\int x e^{x} dx$, which requires integration by parts.

---

Step 5: Compute $\int x e^{x} dx$

Let:

- $u = x \Rightarrow du = dx$
- $dv = e^{x} dx \Rightarrow v = e^{x}$

Using integration by parts:

$$
\int x e^{x} dx = x e^{x} - \int e^{x} dx = x e^{x} - e^{x} + C_2
$$

Step 6: Assemble the Results

Putting it all together:

$$
e^{x} y = e^{x} + \frac{2}{3} (x e^{x} - e^{x}) + C
$$

Simplify:

$$
e^{x} y = e^{x} + \frac{2}{3} x e^{x} - \frac{2}{3} e^{x} + C
$$

Factor out $e^{x}$:

$$
e^{x} y = e^{x} \left( 1 + \frac{2}{3} x - \frac{2}{3} \right) + C
$$

$$
e^{x} y = e^{x} \left( \frac{1}{3} + \frac{2}{3} x \right) + C
$$

Divide both sides by $e^{x}$:

$$
y = \frac{1}{3} + \frac{2}{3} x + C e^{-x}
$$

---

Final Solution:

$$
\boxed{
y(x) = \frac{1}{3} + \frac{2}{3} x + C e^{-x}
}
$$

where $C$ is an arbitrary constant determined by initial conditions.

---

Summary and Key Takeaways

Infinite Series Problems:
- Recognize the form of the series.
- Simplify the expression inside the sum.
- Use known formulas for sums of powers, such as:
- $\sum_{n=1}^{\infty} n = \infty$ (diverges)
- $\sum_{n=1}^{\infty} n^{k}$ diverges for $k \geq 1$
- Check convergence first; if the series diverges, the sum is infinite.

Differential Equations:
- Convert the differential equation into standard linear form.
- Find the integrating factor $e^{\int p(x) dx}$.
- Multiply through by the integrating factor to simplify.
- Recognize derivatives of product forms.
- Integrate both sides, often requiring techniques like integration by parts.
- Write the general solution, including an arbitrary constant.

---

Final Remarks

Mastering these types of problems involves understanding both the theoretical foundations and the practical techniques. Infinite series demand careful convergence analysis, while differential equations require familiarity with methods like integrating factors and integration by parts. Practice, coupled with a systematic approach, will improve your problem-solving skills and deepen your mathematical intuition.

If you're interested in further topics, consider exploring more advanced series tests (like the ratio test, root test) or solving higher-order differential equations. The key is to build a solid conceptual framework combined with ample practice. Happy math exploring!

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