1. Find Sn For The Arithmetic Series Where A1= 3 , An= 42 , N= 142. Find Sn For The Arithmetic Series

1. Find Sn For The Arithmetic Series Where A1= 3 , An= 42 , N= 142. Find Sn For The Arithmetic Series

Understanding how to find the sum of an arithmetic series is a fundamental concept in mathematics, especially in algebra and calculus. When dealing with an arithmetic sequence, knowing certain parameters allows you to calculate the sum of the series efficiently. In this guide, we will explore the process of finding the sum S_n of an arithmetic series where the first term $A_1 = 3$, the nth term $A_n = 42$, and the number of terms $N = 142$. We will go through each step comprehensively, providing clear explanations, formulas, and example calculations to deepen your understanding.

Understanding the Components of an Arithmetic Series

Before diving into calculations, it's crucial to understand the basic components involved:

1. First term (A_1)

- The initial term of the series.
- In our problem, $A_1 = 3$.

2. nth term (A_n)

- The term in the sequence at position n.
- For our problem, $A_n = 42$.

3. Number of terms (N or n)

- The total terms in the series.
- Here, $N = 142$.

4. Common difference (d)

- The constant difference between successive terms.
- To find d, we use the known first and nth terms.

Formulas for Arithmetic Series

To analyze the series and compute its sum, several fundamental formulas are used:

1. nth term of an arithmetic sequence:

$$
A_n = A_1 + (n - 1) \times d
$$

where:
- A_n is the nth term,
- A_1 is the first term,
- d is the common difference,
- n is the term number.

2. Sum of the first n terms of an arithmetic series:

$$
S_n = \frac{n}{2} \times (A_1 + A_n)
$$

or equivalently,

$$
S_n = \frac{n}{2} \times [2A_1 + (n - 1) \times d]
$$

These formulas are essential for calculating the sum once the relevant parameters are known.

Step 1: Find the Common Difference d

Since the sequence is arithmetic, the difference between successive terms remains constant. We can compute d using the known first and nth terms.

$$
A_n = A_1 + (n - 1) \times d
$$

Given:
- $A_1 = 3$,
- $A_n = 42$,
- $n = 142$.

Plugging these into the formula:

$$
42 = 3 + (142 - 1) \times d
$$

$$
42 = 3 + 141 \times d

\]

Now, isolate \(d \):

\[
42 - 3 = 141 \times d
\]
\[
39 = 141 \times d
\]
\[
d = \frac{39}{141}
\]

Simplify the fraction:

\[
d = \frac{13}{47}
\]

Thus, the common difference:

\[
\boxed{
d = \frac{13}{47}
}
\]

Step 2: Confirm the nth term with the calculated \(d \)

It's prudent to verify that our computed \(d \) correctly leads to the nth term:

\[
A_n = A_1 + (n - 1) \times d
\]
\[
A_{142} = 3 + (142 - 1) \times \frac{13}{47}
\]
\[
A_{142} = 3 + 141 \times \frac{13}{47}
\]

Calculate \(141 \times \frac{13}{47} \):

- Note that \(141 = 3 \times 47 \),
- So,

\[
141 \times \frac{13}{47} = (3 \times 47) \times \frac{13}{47} = 3 \times 13 = 39
\]

Adding to the first term:

$$
A_{142} = 3 + 39 = 42
$$

This confirms our value of d is correct.

Step 3: Calculate the Sum S_{142}

Now, with A_1, A_{142}, and n, we can compute the sum:

$$
S_{142} = \frac{n}{2} \times (A_1 + A_n)
$$

Substituting the known values:

$$
S_{142} = \frac{142}{2} \times (3 + 42)
$$
$$
S_{142} = 71 \times 45
$$

Calculate:

$$
S_{142} = 71 \times 45
$$

Break down the multiplication:

$$
71 \times 45 = 71 \times (40 + 5) = 71 \times 40 + 71 \times 5
$$
$$
= 2840 + 355 = 3195
$$

Therefore, the sum of the first 142 terms of the arithmetic series is:

$$
\boxed{
S_{142} = 3195
}
$$

Understanding the Significance of the Result

The sum $S_{142} = 3195$ represents the total of all the terms from the first to the 142nd in the sequence starting at 3 and increasing by $\frac{13}{47}$ each time. This calculation is essential in various real-world applications such as finance (calculating total payments over time), physics (summation of discrete measurements), and computer science (algorithm complexity analysis).

Additional Insights and Tips for Calculations

To ensure accuracy and efficiency when working with arithmetic series, consider the following:

1. **Always verify the common difference:** Use the known first and nth terms to compute d, especially when the sequence parameters are given separately.

2. **Use the sum formula directly:** Once A_1, A_n, and n are known, the sum calculation becomes straightforward.

3. **Simplify fractions:** When dealing with fractional common differences or terms, simplify to reduce computational errors.

4. **Check your work:** Confirm the nth term calculation to avoid mistakes in the sum formula.

5. **Practice with different values:** Changing the parameters helps build confidence and understanding of the method.

Practical Applications of Arithmetic Series Calculations

Understanding how to find the sum of an arithmetic series extends beyond pure mathematics. Here are some real-world scenarios where these calculations are vital:

1. Financial Planning and Loan Payments

- Calculating total payments over time with fixed installment increases.
- Estimating total interest paid when payments increase uniformly.

2. Engineering and Physics

- Summing evenly spaced measurements.
- Calculating total displacement over uniform acceleration.

3. Computer Science

- Analyzing algorithms with linear time complexity.
- Summing iterative processes with consistent increments.

4. Business and Economics

- Summing revenue or costs that grow linearly over periods.
- Planning investments with predictable incremental returns.

Summary of the Process

To summarize the steps involved in finding S_n for an arithmetic series like the one given:

1. Identify the first term A_1, the nth term A_n, and the total number of terms n.

2. Use the nth term formula to calculate the common difference d:

   $$
   A_n = A_1 + (n - 1) \times d
   $$

3. Verify your d by plugging it back into the nth term formula to ensure correctness.

4. Calculate the sum using the formula:

   $$
   S_n = \frac{n}{2} \times (A_1 + A_n)
   $$

5. Perform the calculations carefully, simplifying where possible.

Final Remarks

Mastering the process of finding the sum of an arithmetic series enhances problem-solving skills and mathematical intuition. The specific example with $A_1 = 3$, $A_n = 42$, and $N = 142$ demonstrates how to systematically approach such problems. Remember, the key steps involve determining the common difference accurately, confirming the nth term, and applying the sum formula. With consistent practice, these calculations become straightforward, enabling you to tackle more complex series and their applications confidently.

Frequently Asked Questions

How do I find the sum of the first 142 terms in an arithmetic series with A1=3 and An=42?

Use the formula for the sum of an arithmetic series: Sn = n/2 (A1 + An). Plug in the values: Sn = 142/2 (3 + 42) = 71 45 = 3195.

What is the formula to calculate the sum of an arithmetic series when given the first and nth term?

The sum is given by Sn = n/2 (A1 + An), where n is the number of terms, A1 is the first term, and An is the nth term.

Given A1=3, An=42, and N=142, how do I determine the value of the sum Sn?

Calculate Sn by substituting the known values into the formula Sn = n/2 (A1 + An): Sn = 142/2 (3 + 42) = 71 45 = 3195.

Is it necessary to find the common difference to calculate Sn for this series?

No, since you already know the first and nth terms, you can directly find the sum using the formula Sn = n/2 (A1 + An) without calculating the common difference.

Can I verify the sum of the arithmetic series with these values using an alternative method?

Yes, you can verify by first finding the common difference d = (An - A1) / (N - 1), then using the sum formula. However, directly applying Sn = n/2 (A1 + An) is simpler and sufficient in this case.

Additional Resources

Arithmetic Series

Understanding the Arithmetic Series: A Deep Dive into the Calculation of Sum (Sn)

In the realm of mathematics, sequences and series serve as foundational concepts that underpin much of algebra, calculus, and applied mathematics. Among these, the arithmetic series stands out due to its simplicity and wide-ranging applications—from financial calculations to engineering design. Today, we explore how to determine the sum of an arithmetic series when specific terms are known, focusing on the example where the first term \(A_1

$= 3$), the nth term $A_n = 42$, and the total number of terms $N = 142$. This journey not only showcases the mechanics of the calculation but also illuminates the underlying principles that make the process straightforward and reliable.

Fundamentals of Arithmetic Series

What Is an Arithmetic Series?

An arithmetic series is the sum of the terms of an arithmetic sequence. An arithmetic sequence is a list of numbers where each term after the first is obtained by adding a fixed number, called the common difference (d), to the previous term. Formally, an arithmetic sequence can be expressed as:

$$
A_n = A_1 + (n - 1)d
$$

where:
- A_1 = first term
- d = common difference
- n = position of the term in the sequence
- A_n = nth term

The series is the sum of all these terms:

$$
S_N = A_1 + A_2 + A_3 + \dots + A_N
$$

which can be written as:

$$
S_N = \sum_{k=1}^N A_k
$$

Why Is the Sum Important?

Calculating the sum of an arithmetic series is essential in various contexts, including:
- Financial planning (computing total payments over installments)
- Engineering (total load over repeated structures)
- Statistics (cumulative totals)
- Everyday problem-solving (budgeting, resource allocation)

Having a clear formula simplifies these calculations, especially for large N, where manual summation would be impractical.

Given Data and Objective

In our specific example, the data provided is:

- First term (A_1): 3
- Nth term (A_n): 42
- Number of terms (N): 142

Our goal: Find the sum of the series (S_N).

Step-by-Step Approach to Find S_N

To determine S_N, we need:
1. The common difference d.
2. The sum S_N.

Let's analyze how to find each.

Step 1: Find the Common Difference d

Given the first term A_1 and the nth term A_n, the general formula for the nth term of an arithmetic sequence is:

$$
A_n = A_1 + (n - 1)d
$$

Rearranged to solve for d:

$$
d = \frac{A_n - A_1}{n - 1}
$$

Plugging in the known values:

$$
d = \frac{42 - 3}{142 - 1} = \frac{39}{141} = \frac{13}{47}
$$

Thus, the common difference:

$$
\boxed{
d = \frac{13}{47} \approx 0.2766
}
$$

This value indicates each successive term increases by approximately 0.2766.

Step 2: Verify the nth term with the calculated d

To ensure our calculations are accurate, let's verify A_n:

$$
A_n = A_1 + (n - 1)d = 3 + (142 - 1) \times \frac{13}{47}
$$

Calculating:

$$
A_n = 3 + 141 \times \frac{13}{47}
$$

Notice that:

$$
141 \div 47 = 3
$$

since $47 \times 3 = 141$. Therefore:

$$
A_n = 3 + 3 \times 13 = 3 + 39 = 42
$$

which confirms our value for A_n matches perfectly.

Step 3: Find the Sum S_N

The sum of the first N terms of an arithmetic series can be calculated using the formula:

$$
S_N = \frac{N}{2} (A_1 + A_N)
$$

where:
- A_1 = first term
- A_N = nth term
- N = total number of terms

Substituting the known values:

$$
S_{142} = \frac{142}{2} (3 + 42) = 71 \times 45
$$

Calculating:

$$
S_{142} = 71 \times 45
$$

Breaking down:

$$
71 \times 45 = 71 \times (40 + 5) = 71 \times 40 + 71 \times 5 = 2840 + 355 = 3195
$$

Therefore, the sum of the series is:

$$
\boxed{
S_{142} = 3195
}
$$

Summary of the Process

Let's summarize the key steps involved:

1. Identify known values: $A_1 = 3$, $A_N = 42$, $N = 142$.
2. Calculate the common difference d:

$$
d = \frac{A_N - A_1}{N - 1} = \frac{42 - 3}{142 - 1} = \frac{39}{141} = \frac{13}{47}
$$

3. Verify A_N:

$$
A_N = A_1 + (N - 1)d = 3 + 141 \times \frac{13}{47} = 42
$$

4. Compute the sum S_N:

$$
S_N = \frac{N}{2} (A_1 + A_N) = 71 \times 45 = 3195
$$

Additional Insights and Practical Applications

Implications in Real-World Contexts

The methodology demonstrated here is essential beyond academic exercises. For instance:
- Financial Planning: Calculating total payments over a fixed period with consistent increments.
- Construction: Estimating total material usage when quantities increase uniformly.
- Data Analysis: Summing linearly increasing datasets.

Tips for Efficient Calculation

- Always verify the common difference before proceeding.
- Use simplified fractions to avoid calculation errors.
- When the first and nth terms are known, the sum formula is the most efficient route.
- For large \(N\), the formula saves significant time compared to manual addition.

Conclusion

The process of finding the sum of an arithmetic series, given specific parameters, is both elegant and practical. By understanding the fundamental formula and verifying each step, one can confidently perform calculations that are critical in numerous fields. In our example, the sum of 142 terms starting from 3 and ending at 42, with consistent increments, totals 3195. This not only exemplifies the power of algebraic formulas but also reinforces the importance of precision and verification in mathematical problem-solving.

In essence, mastering the calculation of the sum of an arithmetic series empowers practitioners across disciplines to make informed decisions, optimize resources, and understand patterns within data—making it an indispensable tool in the analytical toolkit.

1 Find Sn For The Arithmetic Series Where A1 3 An 42 N 142 Find Sn For The Arithmetic Series

1 Find Sn For The Arithmetic Series Where A1 3 An 42 N 142 Find Sn For The Arithmetic Series

Related Articles

- 10. Let (X,F) Be A Topological Space And A And B Subsets Of X. A) If A And B Are Separated, Show That
- 1) A Man Flies A Kite At A Height Of 120 M. The Wind Carries The Kite Horizontally Away From Him At A
- A Client Arrives At The Outpatient Clinic With A Painful Leg Ulcer, And The Nurse Performs A Physical

Back to Home