1. Given: Triangle ABC With Vertices A,B,C.Prove: Triangle ABC Is A Right Triangle.Find The Distances

1. Given: Triangle ABC With Vertices A,B,C.Prove: Triangle ABC Is A Right Triangle.Find The Distances

Understanding the properties of triangles is fundamental in geometry, especially when determining whether a given triangle is right-angled. In this article, we will explore how to prove that a triangle with given vertices A, B, and C is a right triangle by calculating the distances between these points and applying the Pythagorean theorem. This process involves computing the lengths of sides, analyzing their relationships, and using geometric principles to establish the right angle.

Understanding the Basics: Coordinates and Distance Formula

Before diving into the proof, it's essential to recall the coordinate system and the distance formula. Typically, the vertices A, B, and C are given as coordinate points in the Cartesian plane:

- $A(x_1, y_1)$
- $B(x_2, y_2)$
- $C(x_3, y_3)$

The distance between any two points, say $P(x_1, y_1)$ and $Q(x_2, y_2)$, is given by the distance formula:

$$\text{Distance} \, PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is fundamental in calculating side lengths of triangle ABC.

Step-by-Step Approach to Proving Triangle ABC Is a Right Triangle

The process involves several key steps:

1. Assign Coordinates to Vertices

Identify or be given the coordinates of points A, B, and C. For example:

- A(x□, y□)
- B(x□, y□)
- C(x□, y□)

2. Calculate the Lengths of the Sides

Using the distance formula, compute:

- AB = distance between A and B
- BC = distance between B and C
- AC = distance between A and C

The formulas are:

\[AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[BC = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \]

$$ AC = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} $$

3. Identify the Longest Side

In a right triangle, the hypotenuse is the longest side. Determine which of the three computed distances is greatest. This side is a candidate for the hypotenuse.

4. Apply the Pythagorean Theorem

The Pythagorean theorem states:

$$ \text{(Hypotenuse)}^2 = \text{(Leg 1)}^2 + \text{(Leg 2)}^2 $$

Check whether the square of the longest side equals the sum of the squares of the other two sides:

$$ \text{If} \quad c^2 = a^2 + b^2, \quad \text{then } \triangle ABC \text{ is a right triangle} $$

where c is the hypotenuse.

5. Confirm the Right Angle

The right angle is at the vertex where the two legs meet. To verify which vertex forms the right angle, analyze the slopes of the sides:

- Calculate slopes of sides AB, BC, and AC.
- The perpendicular sides will have slopes that are negative reciprocals, confirming the right angle.

Calculating Distances: An Example

Suppose the vertices of triangle ABC are:

- A(1, 2)
- B(4, 6)
- C(7, 2)

Step 1: Compute side lengths

- AB:

$$ AB = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 $$

- BC:

$$ BC = \sqrt{(7 - 4)^2 + (2 - 6)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 $$

- AC:

$$ AC = \sqrt{(7 - 1)^2 + (2 - 2)^2} = \sqrt{6^2 + 0^2} = \sqrt{36} = 6 $$

Step 2: Identify the hypotenuse

Side AC is length 6, longer than AB and BC, both are 5. So, AC is the hypotenuse candidate.

Step 3: Check the Pythagorean theorem

$$AB^2 + BC^2 = 5^2 + 5^2 = 25 + 25 = 50$$

$$AC^2 = 6^2 = 36$$

Since $36 \neq 50$, the triangle is not right-angled at the vertex C.

Alternative check:

- If we consider the other vertices, perhaps the right angle is at B or A. Let's check the squares of the sides:

- At A:

$$AB^2 + AC^2 = 25 + 36 = 61$$

- At B:

$$AB^2 + BC^2 = 25 + 25 = 50$$

- At C:

$$AC^2 + BC^2 = 36 + 25 = 61$$

No pair sums to the square of the remaining side. Therefore, in this specific example, triangle ABC is not a right triangle.

General Tips for Proving a Triangle Is Right-Angled

- Always verify the side lengths carefully.
- Use the distance formula precisely.
- Confirm the Pythagorean condition with exact values.

- Alternatively, analyze slopes to confirm perpendicularity:

- If the product of the slopes of two sides is -1, those sides are perpendicular, indicating a right angle at the common vertex.

Additional Methods to Confirm the Right Triangle

Besides the Pythagorean theorem, other methods include:

- **Coordinate Geometry:** Using slopes and perpendicularity.
- **Distance and Dot Product:** For vectors, the dot product being zero indicates perpendicular vectors.

Dot Product Method:

Given vectors AB and AC:

$$\vec{AB} = (x_2 - x_1, y_2 - y_1)$$
$$\vec{AC} = (x_3 - x_1, y_3 - y_1)$$

If:

$$\vec{AB} \cdot \vec{AC} = 0$$

then the angle at A is 90°, confirming a right angle there.

Conclusion: Confirming the Right Triangle

Proving that triangle ABC is a right triangle involves:

1. Calculating the side lengths using the distance formula.
2. Identifying the longest side as a potential hypotenuse.
3. Applying the Pythagorean theorem to verify if the squares of the two shorter sides sum to the square of the longest side.
4. Optionally, analyzing slopes or using the dot product to confirm perpendicularity at the vertex.

This systematic approach ensures a rigorous proof based on coordinate geometry principles. By mastering these techniques, students and mathematicians can confidently analyze and verify right triangles in various geometric problems.

Final Note: Always double-check calculations and consider multiple methods to confirm your findings. Geometry often offers several pathways to the same conclusion, enriching your understanding of the spatial relationships within triangles.

Frequently Asked Questions

How can we use the distance formula to prove that triangle ABC is a right triangle?

By calculating the lengths of sides AB, BC, and AC using the distance formula, and then applying the Pythagorean theorem to verify if the sum of the squares of two sides equals the square of the third, indicating a right angle.

What are the steps to find the distances between vertices A, B, and C in triangle ABC?

First, identify the coordinates of points A, B, and C; then, use the distance formula $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ for each pair of points to find side lengths AB, BC, and AC.

How do I determine which side is the hypotenuse in triangle ABC?

Compare the lengths of the three sides; the side with the greatest length is the hypotenuse if the triangle is right-angled. Verify if the Pythagorean theorem holds with this side as the hypotenuse.

What condition must be satisfied for triangle ABC to be a right triangle based on side lengths?

The side lengths must satisfy the Pythagorean theorem, meaning if c is the hypotenuse, then $a^2 + b^2 = c^2$, where a and b are the other two sides.

Can the distance between points be used to confirm the presence of a right angle at a specific vertex?

Yes, by calculating the distances and checking if the two sides meeting at that vertex satisfy the Pythagorean theorem, confirming a right angle at that vertex.

What are common mistakes to avoid when proving triangle ABC is a right triangle using distances?

Common mistakes include incorrect coordinate substitution, miscalculating the distance formula, or mixing up the side lengths when applying the Pythagorean theorem; double-check all calculations for accuracy.

Additional Resources

Given: Triangle ABC With Vertices A, B, C. Prove: Triangle ABC Is A Right Triangle. Find The Distances – this statement encapsulates a classic geometric challenge that combines the fundamental properties of triangles with coordinate geometry principles. Whether you're a student preparing for exams, a teacher designing lesson plans, or a math enthusiast exploring the depths of geometric theorems, understanding how to prove a triangle is a right triangle and finding its side lengths is an essential skill. This comprehensive guide aims to walk you through the process step-by-step, providing clear explanations, strategic methods, and illustrative examples to deepen your grasp of the concepts involved.

Introduction

In geometry, the classification of triangles based on their angles and side lengths is foundational. A right triangle is distinguished by having one angle exactly equal to 90 degrees. Determining whether a given triangle is right-angled often involves checking certain conditions, such as the Pythagorean theorem, or employing coordinate geometry techniques when the vertices are known.

Suppose you're given the coordinates of vertices A, B, and C of triangle ABC. Your goal is twofold:

1. Prove that triangle ABC is a right triangle.

2. Find the lengths of the sides AB, BC, and AC.

This task not only involves calculating distances between points but also applying geometric proofs to confirm the right angle's presence.

Understanding the Key Concepts

Before diving into the proof and calculations, let's clarify some essential ideas:

1. Distance Formula

The distance between two points in the coordinate plane, say (x_1, y_1) and (x_2, y_2), is given by:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

This formula is crucial for computing side lengths.

2. Proving a Right Triangle

A triangle is right-angled if and only if the Pythagorean theorem holds for its side lengths, i.e., if the square of the longest side equals the sum of the squares of the other two sides.

Alternatively, in coordinate geometry, one can demonstrate that the vectors representing two sides are perpendicular by showing their dot product is zero.

Step-by-Step Approach

Step 1: Label the Vertices and Assign Coordinates

Suppose the vertices are given as:

- $A(x_A, y_A)$
- $B(x_B, y_B)$
- $C(x_C, y_C)$

Ensure you have these coordinates clearly noted.

Step 2: Calculate the Distances (Side Lengths)

Compute the distances for:

- AB
- BC
- AC

Using the distance formula:

$$
AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}
$$
$$
BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}
$$
$$
AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}
$$

Step 3: Identify the Longest Side

The side with the greatest length is a candidate for the hypotenuse if the triangle is right-angled. Compare the computed distances to determine this.

Step 4: Verify the Pythagorean Condition

Check whether:

$$
(\text{longest side})^2 = (\text{side}_1)^2 + (\text{side}_2)^2
$$

If this holds true, triangle ABC is a right triangle with the right angle opposite the longest side.

Step 5: Alternatively, Use Vector Dot Product

To confirm the right angle at a specific vertex, consider the vectors:

- For vertex A: $\vec{AB} = (x_B - x_A, y_B - y_A)$, $\vec{AC} = (x_C - x_A, y_C - y_A)$

Calculate their dot product:

$$
\vec{AB} \cdot \vec{AC} = (x_B - x_A)(x_C - x_A) + (y_B - y_A)(y_C - y_A)
$$

If the dot product is zero, then the angle at A is 90°, confirming the right triangle.

Repeat this process for vertices B and C as needed to locate the right angle.

Illustrative Example

Let's walk through an example with specific coordinates:

Suppose:

- $A(1, 2)$
- $B(4, 6)$
- $C(7, 2)$

1. Calculate side lengths:

- $AB = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$
- $BC = \sqrt{(7 - 4)^2 + (2 - 6)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$
- $AC = \sqrt{(7 - 1)^2 + (2 - 2)^2} = \sqrt{6^2 + 0^2} = \sqrt{36} = 6$

2. Identify the hypotenuse:

- The longest side is $AC = 6$.

3. Verify Pythagoras:

$$
AB^2 + BC^2 = 5^2 + 5^2 = 25 + 25 = 50
$$
$$
AC^2 = 6^2 = 36
$$

Since $36 \neq 50$, the Pythagorean theorem does not hold for sides AB and BC as legs.

4. Check at each vertex:

- At A:

$$
\vec{AB} = (4 - 1, 6 - 2) = (3, 4)
$$
$$
\vec{AC} = (7 - 1, 2 - 2) = (6, 0)
$$
Dot product:

$$
(3)(6) + (4)(0) = 18 + 0 = 18 \neq 0
$$

- At B:

$$
\vec{BA} = (1 - 4, 2 - 6) = (-3, -4)
$$
$$
\vec{BC} = (7 - 4, 2 - 6) = (3, -4)
$$
Dot product:

$$
(-3)(3) + (-4)(-4) = -9 + 16 = 7 \neq 0
$$

- At C:

$$
\vec{CA} = (1 - 7, 2 - 2) = (-6, 0)
$$

$$
\vec{CB} = (4 - 7, 6 - 2) = (-3, 4)
$$

Dot product:

$$
(-6)(-3) + (0)(4) = 18 + 0 = 18 \neq 0
$$

No right angle is detected at any vertex; thus, triangle ABC is not a right triangle.

General Tips for Proving a Triangle is Right-Angled

- Use the Pythagorean theorem: Compute the side lengths and verify if the squares satisfy the Pythagorean relation.
- Check for perpendicular sides: Calculate the dot product of the vectors representing two sides sharing a vertex. If zero, the sides are perpendicular, confirming a right angle.
- Coordinate geometry approach: When vertices are known, this method provides a straightforward algebraic confirmation.

Additional Considerations

- If the vertices are not given explicitly but are defined in a problem context, always ensure the coordinate system and points are correctly identified.
- In problems involving geometric constructions, sometimes the right angle can be inferred from given angles or special properties, such as inscribed angles subtending semicircles.
- When working with abstract figures, consider using coordinate geometry for a rigorous proof rather than relying solely on visual intuition.

Summary

Proving that triangle ABC is a right triangle involves a systematic approach:

1. Calculate the distances between pairs of vertices using the distance formula.
2. Identify the longest side and verify the Pythagorean theorem.
3. Alternatively, examine the vectors emanating from a common vertex to see if they are perpendicular by calculating their dot product.
4. Confirm the right angle at the vertex where the perpendicular sides meet.

This method combines algebraic calculations with geometric reasoning, offering a clear pathway to solving similar problems involving triangle classification and distance computations.

Final Thoughts

Mastering the process of proving right triangles and calculating their side lengths fosters a deeper understanding of geometric principles. Whether in coordinate geometry or classical Euclidean geometry, these techniques are versatile tools for tackling a wide array of problems in mathematics. Practice with various coordinate configurations and scenarios to enhance your problem-solving skills and develop a strong geometric intuition.

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