(1 Point) A Box Contains 80 Balls Numbered From 1 To 80. If 11 Balls Are Drawn With Replacement, What

(1 Point) A Box Contains 80 Balls Numbered From 1 To 80. If 11 Balls Are Drawn With Replacement, What

Introduction to the Problem

In probability and combinatorics, understanding the outcomes of random draws from a finite set is fundamental. This particular problem involves a box containing 80 balls numbered from 1 to 80. The process involves drawing 11 balls with replacement, meaning after each draw, the ball is returned to the box, maintaining the total number of balls at 80 for every draw. The question posed typically revolves around calculating the probability of certain events, such as specific sequences, the occurrence of particular numbers, or the likelihood of certain patterns emerging after the 11 draws.

This comprehensive guide explores the problem's structure, the underlying probability principles, and detailed calculations to understand the various outcomes and their probabilities. Whether you're a student preparing for exams, a teacher designing problem sets, or a math enthusiast curious about combinatorial probability, this detailed analysis aims to clarify the concepts involved and provide practical insights.

Understanding the Basic Setup

Before diving into calculations, it’s essential to grasp the key aspects of the problem:

Key Elements of the Problem

- **Number of Balls:** 80 balls numbered from 1 to 80.
- **Number of Draws:** 11 draws are performed.
- **Replacement Policy:** After each draw, the ball is replaced back into the box, so the total number remains constant at 80.

Implications of Drawing With Replacement

Drawing with replacement ensures each draw is independent of previous draws. The probability of drawing a particular ball remains constant across all draws, which simplifies calculations significantly.

Probability Principles Involved

Understanding the foundational probability concepts is crucial to solve the problem effectively.

Sample Space and Outcomes

- Since each draw is independent and the same set of balls is available every time, the total number of possible sequences of 11 draws is:

- **Number of possible outcomes:** 80^{11}

- Each outcome is a sequence of 11 numbers, each between 1 and 80 inclusive.

Probability of Specific Events

- The probability of drawing a specific ball in a single draw: $\frac{1}{80}$.
- The probability of a particular sequence of balls: $\left(\frac{1}{80}\right)^{11}$.

Common Types of Questions and Calculations

This problem can lead to various types of questions, such as:

1. Probability of Drawing a Specific Sequence

Suppose you want the probability that the 11 draws produce a particular sequence, e.g., 5, 12, 33, ..., etc.

- Calculation: Since each draw is independent,

$$
P(\text{specific sequence}) = \left(\frac{1}{80}\right)^{11}
$$

- Interpretation: The probability is extremely small, reflecting the rarity of such an exact sequence.

2. Probability of Drawing At Least One Specific Number

For example, what is the probability that the number 7 appears at least once in the 11 draws?

- Step 1: Calculate the probability that 7 does not appear in any draw.

$$
P(\text{7 not in a single draw}) = \frac{79}{80}
$$

- Step 2: Since draws are independent,

$$
P(\text{7 not in all 11 draws}) = \left(\frac{79}{80}\right)^{11}
$$

- Step 3: Therefore, the probability that 7 appears at least once is:

$$
P(\text{7 at least once}) = 1 - \left(\frac{79}{80}\right)^{11}
$$

This approach applies to any specific number.

3. Probability of All Draws Being Distinct

Since the balls are replaced after each draw, the probability that all 11 draws are of distinct balls is interesting to consider.

- Calculation:

$$
P(\text{all distinct}) = \frac{80}{80} \times \frac{79}{80} \times \frac{78}{80} \times \cdots \times \frac{70}{80}
$$

- Explanation: The first draw can be any ball (probability 1), the second must be different from the first ($\frac{79}{80}$), the third different from the first two ($\frac{78}{80}$), and so on until the 11th draw.

- Result:

$$
P(\text{all distinct}) = \prod_{k=0}^{10} \frac{80 - k}{80}
$$

- Numerical calculation:

$$
P(\text{all distinct}) = \frac{80 \times 79 \times 78 \times \ldots \times 70}{80^{11}}
$$

4. Expected Number of Occurrences of a Particular Number

In a sequence of 11 independent draws:

- Expected value: Since the probability of drawing a specific number (say, number 10) in each draw is $\frac{1}{80}$,

$$
E(\text{number 10 appears}) = 11 \times \frac{1}{80} \approx 0.1375
$$

- Interpretation: On average, number 10 is expected to appear approximately 0.1375 times in 11 draws.

Advanced Probability Calculations

Beyond basic questions, more complex scenarios involve joint probabilities and distributions.

1. Distribution of the Number of Times a Specific Number Appears

This follows a Binomial distribution:

- Parameters:

$$
n = 11,\quad p = \frac{1}{80}
$$

- Probability that the number appears exactly k times:

$$
P(X = k) = \binom{11}{k} \left(\frac{1}{80}\right)^k \left(1 - \frac{1}{80}\right)^{11-k}
$$

- Application: This helps in understanding the likelihood of a number appearing multiple times or not at all.

2. Probability of Multiple Specific Numbers Occurring

Suppose we want the probability that both numbers 5 and 12 appear at least once:

- Step 1: Calculate the probability that each does not appear:

$$
P(\text{not 5}) = \left(\frac{79}{80}\right)^{11}
$$

$$
P(\text{not 12}) = \left(\frac{79}{80}\right)^{11}
$$

- Step 2: Calculate the probability that neither appears:

Assuming independence,

$$
P(\text{neither 5 nor 12}) = \left(\frac{78}{80}\right)^{11}
$$

- Step 3: Use inclusion-exclusion principle to find the probability that both appear at least once:

$$
P(\text{both 5 and 12 at least once}) = 1 - P(\text{not 5}) - P(\text{not 12}) + P(\text{neither 5 nor 12})
$$

$$
= 1 - 2 \left(\frac{79}{80}\right)^{11} + \left(\frac{78}{80}\right)^{11}
$$

Practical Applications of the Problem

Understanding such probability models has numerous real-world applications:

1. Lottery and Gambling

- Analyzing the odds of winning based on number draws.
- Calculating the probability of certain combinations appearing.

2. Quality Control and Random Sampling

- Ensuring randomness in sample selection.
- Estimating the likelihood of specific items being selected multiple times.

3. Data Science and Simulation

- Simulating random processes involving sampling with replacement.
- Modeling scenarios where items or data points are repeatedly sampled.

Conclusion

The problem of drawing 11 balls with replacement from a set of 80 numbered balls exemplifies core probability concepts, including sample space calculation, independent events, and binomial distributions. By understanding the fundamental principles—such as how to compute the probability of specific sequences, the likelihood of particular numbers appearing, or multiple numbers co-occurring—you can approach a wide range of similar problems with confidence.

Whether you're interested in theoretical probability, practical applications, or preparing for exams, mastering these concepts provides a solid foundation in combinatorial probability. Remember that the key to solving such problems lies in breaking down complex questions into manageable calculations, applying the appropriate probability models, and interpreting the results within the

Frequently Asked Questions

What is the probability of drawing a specific ball, say ball number 25, in one draw?

The probability is 1/80 since all balls are equally likely to be drawn.

What is the probability of drawing ball number 25 at least once in 11 draws with replacement?

The probability is 1 minus the probability of not drawing ball 25 in all 11 draws, which is 1 - (79/80)^11.

How do you calculate the expected number of times ball number 25 is drawn in 11 draws?

Since each draw is independent, the expected number is 11 multiplied by 1/80, which equals 11/80.

What is the probability of drawing any specific ball exactly 3 times in 11 draws?

It follows a binomial distribution: C(11,3) (1/80)^3 (79/80)^8.

If 11 balls are drawn with replacement, what is the probability that all balls are different?

The probability that all drawn balls are different is (80/80) (79/80) (78/80) ... (70/80), for 11 draws.

What is the probability that at least one ball is drawn more than once in 11 draws?

It's 1 minus the probability that all 11 balls are different, calculated as 1 - [(80/80) (79/80) ... (70/80)].

How does increasing the number of draws affect the probability of drawing a specific ball at least once?

The probability increases with more draws, approaching 1 as the number of draws grows large, since P(at least once) = 1 - (79/80)^n.

If you want to increase the chance of drawing ball number 25 at least once in 11 draws, what could you do?

You could increase the number of draws or reduce the total number of balls to increase the probability.

Additional Resources

Probability and Expected Outcomes in Drawing Balls With Replacement from a Box of 80

Drawing balls from a finite set, especially with replacement, is a classic problem in probability theory that has wide-ranging applications, from statistical sampling to game design. This comprehensive review explores the scenario where a box contains 80 balls numbered from 1 to 80, and 11 balls are drawn with replacement. We will analyze the problem in depth, covering fundamental concepts, calculations, and implications.

Understanding the Scenario

Basic Setup

- Total Balls: 80 balls numbered 1 through 80
- Drawing Process: 11 balls drawn sequentially
- Replacement Policy: After each draw, the ball is returned to the box, maintaining the total of 80 balls for every draw
- Goals: To analyze various probabilistic outcomes, including the likelihood of specific events, expected values, and distributions

Why Drawing With Replacement Matters

Drawing with replacement simplifies the analysis by ensuring:

- The probability of drawing any particular ball remains constant across all draws
- The independence of each draw, which allows us to apply standard probability rules without complex dependency considerations
- The ability to model the process with familiar distributions like the Binomial distribution

---

Fundamental Concepts in Probability

Probability of Drawing a Specific Number

- Since each ball is equally likely, the probability \(P \) of drawing any specific ball (say, ball number 25) in a single draw is:

$$
P(\text{drawing ball } i) = \frac{1}{80}
$$

- This probability remains constant for each draw due to replacement.

Independence of Draws

- Each draw is independent; the outcome of one does not influence the next.
- This independence allows the use of binomial and related distributions.

Key Probabilistic Metrics

- Expected Value (Mean): The average number of times a particular event occurs across many repetitions
- Variance: The measure of spread or dispersion around the expected value
- Probability Distribution: The function that assigns probabilities to all possible outcomes

---

Analyzing Specific Outcomes

Probability of Drawing a Particular Number at Least Once

Suppose you are interested in the probability that a specific ball number, say 30, appears at least once in the 11 draws.

- Complement approach:

$$
P(\text{ball 30 appears at least once}) = 1 - P(\text{ball 30 does not appear in any draw})
$$

- Calculating the probability that ball 30 does not appear in a single draw:

$$
P(\text{not drawing ball 30}) = 1 - \frac{1}{80} = \frac{79}{80}
$$

- Probability that ball 30 does not appear in all 11 draws:

$$
\left(\frac{79}{80} \right)^{11}
$$

\]

- Final probability:

$$
P(\text{ball 30 appears at least once}) = 1 - \left(\frac{79}{80} \right)^{11}
$$

- Numerical approximation:

$$
\left(\frac{79}{80} \right)^{11} \approx e^{11 \times \ln(79/80)} \approx e^{11 \times -0.01266} \approx e^{-0.1393} \approx 0.8699
$$

Thus,

$$
P(\text{ball 30 appears at least once}) \approx 1 - 0.8699 = 0.1301
$$

Interpretation: There's approximately a 13% chance that a particular ball, such as ball 30, will be drawn at least once in 11 draws with replacement.

Expected Number of Times a Specific Ball Is Drawn

Since each draw is independent and the probability of drawing a specific ball in any single draw is $\frac{1}{80}$, the expected number of times this ball appears over 11 draws is:

$$
E = 11 \times \frac{1}{80} = \frac{11}{80} \approx 0.1375
$$

Implication: On average, each particular ball will be drawn approximately 0.1375 times over 11 draws with replacement, which means that most of the time, it won't be drawn at all, but there's a small chance it might be drawn once or even twice in some cases.

Distribution of the Number of Times a Particular Ball Is Drawn

This scenario naturally follows a Binomial distribution because:

- Each draw has two outcomes: either the specific ball is drawn or not
- The number of successes (drawing the specific ball) over 11 independent trials

Parameters:

- Number of trials $n = 11$
- Success probability $p = \frac{1}{80}$

Probability mass function:

$$
P(k) = \binom{11}{k} p^{k} (1 - p)^{11 - k}
$$

where $k = 0, 1, 2, ..., 11$

Example calculations:

- Probability that the specific ball is drawn exactly once:

$$
P(1) = \binom{11}{1} \left(\frac{1}{80} \right)^{1} \left(\frac{79}{80} \right)^{10} \approx 11 \times \frac{1}{80} \times (0.987)^10
$$

which numerically approximates to a small probability, reinforcing the idea that multiple appearances are rare.

Probability of Drawing All Distinct Balls

One interesting scenario is the probability that all 11 drawn balls are distinct:

- Since each draw is independent with replacement, the probability that the second draw is different from the first:

$$
\frac{79}{80}
$$

- Similarly, for the third draw to be different from the first two:

$$
\frac{78}{80}
$$

- Continuing this logic, the probability that all 11 balls are different:

$$
P(\text{all 11 balls are distinct}) = \frac{80}{80} \times \frac{79}{80} \times \frac{78}{80} \times \dots \times \frac{70}{80}
$$

which simplifies to:

$$
P = \frac{80 \times 79 \times 78 \times \dots \times 70}{80^{11}}
$$

- Numerical evaluation:

$$
P \approx \frac{\text{Number of permutations of 11 distinct balls out of 80}}{80^{11}}
$$

- This probability is relatively low because with replacement, repeats are common, especially as the number of draws approaches the total number of balls.

Expected Number of Distinct Balls Drawn

Using the linearity of expectation, we can determine the expected number of unique balls in 11 draws:

- For each ball, the probability that it is drawn at least once:

$$
1 - \left(\frac{79}{80} \right)^{11}
$$

- The total expected number of distinct balls:

$$
E_{\text{distinct}} = 80 \times \left[1 - \left(\frac{79}{80} \right)^{11} \right]
$$

- Numerical approximation:

$$
E_{\text{distinct}} \approx 80 \times (1 - 0.8699) = 80 \times 0.1301 \approx 10.4
$$

Interpretation: Although 11 draws are made, on average, approximately 10.4 distinct balls will be observed, indicating some repeats are likely but not dominant.

Implications and Real-World Applications

1. Random Sampling and Data Collection

- Understanding the probability of drawing specific items at least once helps in designing sampling strategies, especially when the goal is to ensure coverage of rare items.
- For example, in quality control, where each ball represents a batch, the probability of missing a particular batch after multiple samples.

2. Gaming and Lottery Design

- Many lottery and gaming systems involve drawing with replacement or without replacement.
- Knowing the probabilities aids in game fairness analysis and designing odds that are engaging but not overly predictable.

3. Statistical Inference

- Drawing with replacement models is foundational for bootstrap methods in statistics, where resampling with replacement estimates properties of the population.
- Calculations like the expected number of times a particular number appears inform confidence intervals and hypothesis testing.

4. Network and Randomized Algorithms

- Algorithms that rely on random sampling within a fixed set often assume independence and uniformity, which this problem illustrates.
- Probabilistic analysis informs optimization and reliability modeling.

Advanced Considerations and Variations

1. Drawing Without Replacement

- If instead of replacement, balls are drawn without replacement:

- The distribution becomes hypergeometric
- Probabilities change significantly, especially as the number of draws approaches the total number of balls

2. Larger Number of Draws

- Increasing the number of draws increases the expected number of

1 Point A Box Contains 80 Balls Numbered From 1 To 80 If 11 Balls Are Drawn With Replacement What

1 Point A Box Contains 80 Balls Numbered From 1 To 80 If 11 Balls Are Drawn With Replacement What

Related Articles

- 1. Complete The Table To Summarize The Inputs And Outputs Of Matter And Energy That Are Shown In Your
- A Clothing Company Makes A Profit Of $2.3 Million. This Is $4.1 Million More Than Last Year. What Was
- 2. No 3 Consider The Standard Hotelling Location Model In Which Two Firms First Simultaneously Choose

Back to Home