(1 Point) Find All The Unit Vectors That Are Parallel To The Tangent Line To The Curve Y = 9 Sin X At

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Understanding the relationship between tangent lines and vectors is fundamental in calculus and vector analysis. When dealing with curves defined by functions such as $y = 9 \sin x$, the tangent line at a specific point provides valuable geometric and analytical insights. One common problem involves finding all the unit vectors that are parallel to the tangent line at a particular point on the curve. This task not only deepens comprehension of derivatives and tangent vectors but also enhances skills in vector calculus, particularly in applications involving directionality and orientation of curves.

In this article, we will explore the process of finding all unit vectors parallel to the tangent line to the curve $y = 9 \sin x$ at a given point. We will systematically analyze the steps involved, including calculating derivatives, identifying tangent vectors, normalizing these vectors, and understanding their geometric significance. By the end, readers will have a comprehensive understanding of how to approach similar problems involving tangent vectors and unit vectors in calculus.

Understanding the Problem: Tangent Lines and Vectors

Before diving into calculations, it is essential to understand what it means for a vector to be parallel to a tangent line at a point on a curve.

What is a Tangent Line?

- The tangent line to a curve at a point is the straight line that just "touches" the curve at that point.
- It represents the instantaneous direction of the curve at that point.
- Mathematically, the tangent line can be described using the derivative of the function at that point.

What Are Vectors Parallel to the Tangent Line?

- A vector is parallel to the tangent line if it points in the same or exactly the opposite direction as the tangent line.
- Since the tangent line has an infinite number of vectors parallel to it, the question focuses on identifying the unit vectors that align with this direction.

Mathematical Foundations: Derivatives and Tangent Vectors

To find vectors parallel to the tangent line of the curve $y = 9 \sin x$, we need to understand how derivatives relate to tangent directions.

Calculating the Derivative

- The derivative $\frac{dy}{dx}$ gives the slope of the tangent line at any point x.
- For $y = 9 \sin x$, the derivative is:

$$
\frac{dy}{dx} = 9 \cos x
$$

Interpreting the Derivative as a Vector

- The tangent vector at a point x can be represented as:

$$
\vec{T} = \langle 1, \frac{dy}{dx} \rangle = \langle 1, 9 \cos x \rangle
$$

- This vector points in the direction of the tangent line at the point (x, y).

Finding the Direction of the Tangent Vector

The tangent vector's direction is critical in determining the parallel vectors.

Tangent Vector at a Point x

- At a specific point $x = x_0$, the tangent vector is:

$$
\vec{T}(x_0) = \langle 1, 9 \cos x_0 \rangle
$$

- This vector indicates the slope and direction of the tangent at x_0.

Expressing the Direction as a Vector

- The tangent vector points in the same direction as the line tangent to the curve at x_0.
- To find all unit vectors parallel to it, we need to normalize $\vec{T}$.

Normalizing the Tangent Vector: Finding Unit Vectors

A unit vector has a magnitude (length) of 1 and points in a specific direction.

Calculating the Magnitude of the Tangent Vector

- The magnitude $|\vec{T}|$ at x_0 is:

$$
|\vec{T}| = \sqrt{(1)^2 + (9 \cos x_0)^2} = \sqrt{1 + 81 \cos^2 x_0}
$$

Unit Vectors Parallel to the Tangent Line

- To find the unit vectors in the same direction, divide the tangent vector by its magnitude:

$$
\vec{u} = \frac{\vec{T}}{|\vec{T}|} = \left\langle \frac{1}{\sqrt{1 + 81 \cos^2 x_0}}, \frac{9 \cos x_0}{\sqrt{1 + 81 \cos^2 x_0}} \right\rangle
$$

- The vector $\vec{u}$ is a unit vector parallel to the tangent line at x_0.

All Unit Vectors Parallel to the Tangent Line

Since vectors pointing in the same direction as $\vec{T}$ are parallel, and those pointing in the opposite direction are also parallel but in the opposite sense, the complete set of unit vectors parallel to the tangent line includes:

1. The vector $\vec{u}$ as given above.
2. The vector $-\vec{u}$, pointing in the opposite direction.

Expressed explicitly:

$$
\boxed{
\begin{aligned}
\text{Unit vector in the same direction:} \quad & \vec{u} = \left\langle \frac{1}{\sqrt{1 + 81 \cos^2 x_0}}, \frac{9 \cos x_0}{\sqrt{1 + 81 \cos^2 x_0}} \right\rangle \\
\text{Unit vector in the opposite direction:} \quad & -\vec{u} = -\left\langle \frac{1}{\sqrt{1 + 81 \cos^2 x_0}}, \frac{9 \cos x_0}{\sqrt{1 + 81 \cos^2 x_0}} \right\rangle
\end{aligned}
}
$$

Application: Finding Specific Unit Vectors at a Given Point

To make this process concrete, let's consider a specific point on the curve, such as $x = \frac{\pi}{4}$.

Step 1: Calculate y at $x = \frac{\pi}{4}$

$$
y = 9 \sin \left(\frac{\pi}{4} \right) = 9 \times \frac{\sqrt{2}}{2} = \frac{9 \sqrt{2}}{2}
$$

Step 2: Compute the derivative $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

$$
\frac{dy}{dx} = 9 \cos \left(\frac{\pi}{4} \right) = 9 \times

$$
\frac{\sqrt{2}}{2} = \frac{9 \sqrt{2}}{2}
$$

Step 3: Find the tangent vector at this point

$$
\vec{T} = \langle 1, \frac{9 \sqrt{2}}{2} \rangle
$$

Step 4: Calculate the magnitude of $\vec{T}$

$$
|\vec{T}| = \sqrt{1^2 + \left(\frac{9 \sqrt{2}}{2} \right)^2} = \sqrt{1 + \frac{81 \times 2}{4}} = \sqrt{1 + \frac{162}{4}} = \sqrt{1 + 40.5} = \sqrt{41.5}
$$

Step 5: Find the unit vectors

$$
\vec{u} = \left\langle \frac{1}{\sqrt{41.5}}, \frac{\frac{9 \sqrt{2}}{2}}{\sqrt{41.5}} \right\rangle
$$

and the opposite

$$
-\vec{u} = -\left\langle \frac{1}{\sqrt{41.5}}, \frac{\frac{9 \sqrt{2}}{2}}{\sqrt{41.5}} \right\rangle
$$

This explicit calculation demonstrates how to derive the unit vectors at any point along the curve.

Visualizing the Unit Vectors and the Curve

Visualization enhances understanding, especially in vector calculus. Plotting the curve $y = 9 \sin x$ along with the tangent lines at various points can provide intuitive insight into the directions of the tangent vectors and their corresponding unit vectors.

Steps for visualization:

- Plot the curve $y = 9 \sin x$ over a suitable interval, such as $x \in [0, 2\pi]$.
- At selected points x_0, draw the tangent line based on the

derivative.
- Represent the tangent vector \(

Frequently Asked Questions

How do I find the tangent line to the curve y = 9 sin x at a specific point?

First, find the derivative y' = 9 cos x, which gives the slope at any point. Then, evaluate y' at the specific x-value to get the slope, and use the point-slope form to write the tangent line equation.

What is a unit vector parallel to the tangent line of y = 9 sin x?

A unit vector parallel to the tangent line can be obtained by taking the slope of the tangent line at a point and converting it into a vector with magnitude 1, typically using (1, slope) normalized to unit length.

How do I find all unit vectors parallel to the tangent line of y = 9 sin x?

Calculate the derivative y' = 9 cos x to find the slope at each x, then form vectors (1, y') and normalize them to magnitude 1. These form all possible unit vectors parallel to the tangent line.

Are the unit vectors parallel to the tangent line the same at all points on y = 9 sin x?

No, the unit vectors depend on the slope at each point, which varies with x. They change as x varies because the slope 9 cos x varies.

What is the significance of finding unit vectors parallel to the tangent line?

Finding these vectors helps understand the direction of the tangent line at any point on the curve, which is useful in analyzing the curve's behavior and directionality.

How do I compute the unit vector for a specific point x = a on the curve y = 9 sin x?

Evaluate y' = 9 cos a at x = a to get the slope m. Then, form the vector (1, m) and normalize it: divide by its magnitude $\sqrt{(1 + m^2)}$.

Can the unit vectors parallel to the tangent line point in opposite directions?

Yes, since the tangent line has a direction, both the vector and its negative are parallel but point in opposite directions.

What is the general formula for all unit vectors parallel to the tangent line to y = 9 sin x?

For each x, the unit vectors are ± (1, 9 cos x) divided by their magnitude $\sqrt{1 + (9 \cos x)^2}$.

Why do we normalize the vectors when finding unit vectors parallel to the tangent line?

Normalization ensures the vectors have a magnitude of 1, making them true unit vectors that only indicate direction.

How does the derivative y' = 9 cos x relate to the direction of the tangent line?

The derivative y' at a point x gives the slope of the tangent line at that point, which determines its direction and helps in forming the tangent vector.

Additional Resources

Unit Vectors Parallel to the Tangent Line of the Curve \(y = 9 \sin x \)

Introduction

In the realm of calculus and vector analysis, understanding the relationship between a curve and its tangent lines is fundamental. Whether you're a student delving into calculus principles or a seasoned mathematician exploring the depths of vector calculus, the task of finding unit vectors parallel to a tangent line offers both challenge and insight. This article provides an exhaustive examination of the problem: finding all the unit vectors that are parallel to the tangent line to the curve \(y = 9 \sin x \) at a given point. We'll approach this with the precision and clarity of a technical review, dissecting the problem step-by-step, and illuminating each concept with thorough explanations.

Understanding the Problem Framework

Before diving into calculations, it’s crucial to clarify what the problem entails:

- Curve Definition: $y = 9 \sin x$ – a classic sinusoidal function scaled vertically by a factor of 9.
- Tangent Line: The line tangent to the curve at a specific point $x = a$.
- Unit Vectors: Vectors of length 1, pointing in the direction of the tangent line.
- Parallelism: The unit vectors sought are those parallel to the tangent line, implying they share the same or opposite direction.

In essence, we're looking for all unit vectors that align with the tangent vector at a particular point $x=a$. Since tangent vectors are directional, the problem reduces to finding the unit vector (or vectors) pointing along the tangent direction at a specific point on the curve.

Mathematical Foundations and Approach

1. Deriving the Slope of the Tangent Line

The tangent line at a point $x = a$ on the curve $y = 9 \sin x$ is characterized by its slope $\frac{dy}{dx}$ at that point.

- Derivative of y:

$$
\frac{dy}{dx} = 9 \cos x
$$

- At $x = a$:

$$
\left. \frac{dy}{dx} \right|_{x=a} = 9 \cos a
$$

This derivative provides the slope of the tangent line at $x = a$.

2. Constructing the Tangent Vector

The tangent vector at $x = a$ can be represented as:

$$
\vec{T} = \langle 1, \frac{dy}{dx} \rangle = \langle 1, 9 \cos a \rangle
$$

This vector points in the direction of the tangent line but is not necessarily of unit length.

Finding the Unit Vectors Parallel to the Tangent Line

1. The General Direction of the Tangent Vector

The tangent vector \(\vec{T} \) is:

\[
\vec{T} = \langle 1, 9 \cos a \rangle
\]

Any vector parallel to \(\vec{T} \) must be a scalar multiple of it:

\[
\vec{v} = k \langle 1, 9 \cos a \rangle
\]

where \(k \) is a scalar.

To find unit vectors parallel to \(\vec{T} \), we normalize \(\vec{v} \):

\[
\hat{\mathbf{u}} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{k \langle 1, 9 \cos a \rangle}{|k| \|\langle 1, 9 \cos a \rangle\|} = \pm \frac{\langle 1, 9 \cos a \rangle}{\|\langle 1, 9 \cos a \rangle\|}
\]

The sign depends on the direction (forward or backward along the tangent line).

2. Calculating the Magnitude of \(\vec{T} \)

The magnitude \(\|\vec{T}\| \) is:

\[
\|\vec{T}\| = \sqrt{(1)^2 + (9 \cos a)^2} = \sqrt{1 + 81 \cos^2 a}
\]

Thus, the unit vectors parallel to the tangent at \(x = a \) are:

\[
\boxed{
\hat{\mathbf{u}} = \pm \frac{1}{\sqrt{1 + 81 \cos^2 a}} \langle 1, 9 \cos a \rangle
}
\]

Expressed component-wise, the unit vectors are:

\[

\boxed{
\hat{\mathbf{u}}_{+} = \frac{1}{\sqrt{1 + 81 \cos^2 a}} \langle 1, 9 \cos a \rangle
}
\]

and

\[
\boxed{
\hat{\mathbf{u}}_{-} = -\frac{1}{\sqrt{1 + 81 \cos^2 a}} \langle 1, 9 \cos a \rangle
}
\]

These vectors point in the direction of the tangent and opposite to it, respectively.

Practical Implications and Visualization

1. Geometrical Interpretation

- The unit vector \(\hat{\mathbf{u}}_{+} \) points in the same direction as the tangent at \(x = a \), indicating the forward direction along the curve.
- The unit vector \(\hat{\mathbf{u}}_{-} \) points in the opposite direction, representing the tangent in the reverse direction.

2. Special Cases

- When \(\cos a = 0 \), i.e., at \(a = \frac{\pi}{2} + n\pi \), the tangent vector simplifies:

\[
\hat{\mathbf{u}} = \pm \frac{\langle 1, 0 \rangle}{\sqrt{1}} = \pm \langle 1, 0 \rangle
\]

indicating purely horizontal tangent lines at these points.

- When \(\cos a = \pm 1 \), i.e., at \(a = n\pi \), the tangent vector becomes:

\[
\langle 1, \pm 9 \rangle
\]

with magnitude:

\[

$$\sqrt{1 + 81} = \sqrt{82}$$

giving the steepest slopes in the tangent direction.

Comprehensive Summary and Final Remarks

The process of determining all unit vectors parallel to the tangent line to the curve $y = 9 \sin x$ at any point a involves:

1. Computing the derivative $\frac{dy}{dx} = 9 \cos a$.
2. Forming the tangent vector $\langle 1, 9 \cos a \rangle$.
3. Calculating its magnitude $\sqrt{1 + 81 \cos^2 a}$.
4. Normalizing to obtain the unit vectors:

$$\boxed{
\hat{\mathbf{u}} = \pm \frac{\langle 1, 9 \cos a \rangle}{\sqrt{1 + 81 \cos^2 a}}
}$$

This elegant mathematical construct provides both the forward and backward unit vectors aligned with the tangent line, encapsulating the directional essence of the curve's behavior at any point.

Final Thoughts

Understanding the relationship between a curve and its tangent vectors is not just an academic exercise; it forms the backbone of many applications in physics, engineering, and computer graphics. Whether analyzing the direction of motion, designing control systems, or rendering curves in digital environments, the ability to find and interpret such unit vectors is invaluable. This comprehensive exploration underscores the beauty of calculus, illustrating how a simple sinusoidal function reveals complex, elegant geometric relationships when examined through the lens of vectors.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Anton, H., Bivens, I., & Davis, S. (2013). Calculus: Early Transcendental. Wiley.
- Larson, R., & Edwards, B. H. (2016). Calculus. Cengage Learning.

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Note: This detailed breakdown is designed to serve as an expert resource,
ensuring clarity, precision, and depth in understanding the problem of
finding all unit vectors parallel to the tangent line of the curve \( y = 9
\sin x \).
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