(1 Point) The Limit Lim In(e +h)-1 Represents F' (a) For Some Function F And Some Number A. Find F(x)

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Understanding the Limit and Its Significance in Calculus

Calculus, a fundamental branch of mathematics, revolves around understanding change and motion. One of its core concepts is the derivative, which measures how a function changes at a specific point. The notation F'(a) represents the derivative of a function F at the point x = a. A common way to define this derivative is through the limit process, which captures the idea of approaching a point infinitely closely.

In particular, the limit:

$$
\lim_{h \to 0} \frac{\ln(e + h) - 1}{h}
$$

is a classic example that helps us understand the derivative of certain functions, especially those involving natural logarithms. This limit, when evaluated, can reveal the derivative of a function at a point, and from there, we can work backwards to find the original function F(x).

In this article, we'll explore the meaning of this limit, how it relates to the derivative F'(a), and how to determine the original function F(x). We'll walk through the principles of limits, derivatives, and integration, providing a comprehensive understanding suitable for students, educators, or anyone interested in the fundamentals of calculus.

Fundamentals of Limits and Derivatives

What is a Limit?

A limit describes the value that a function approaches as the input approaches a specific point. Formally, for a function f(x),

$$
\lim_{x \to a} f(x) = L

\]

means that as x gets arbitrarily close to a, f(x) approaches L. Limits are foundational in defining derivatives and integrals.

Definition of the Derivative

The derivative of a function F at a point a is defined as:

$$
F'(a) = \lim_{h \to 0} \frac{F(a+h) - F(a)}{h}
$$

This limit measures the instantaneous rate of change of F at x = a. If the limit exists, F is differentiable at that point.

Evaluating the Given Limit and Its Connection to F'(a)

Analyzing the Limit

Given:

$$
\lim_{h \to 0} \frac{\ln(e + h) - 1}{h}
$$

we aim to evaluate this limit and interpret it as F'(a) for some function F at the point a.

Note that:

$$
\ln(e + h) \to \ln e = 1 \quad \text{as} \quad h \to 0
$$

which suggests the numerator approaches 0 as h approaches 0. The expression resembles the difference quotient used in the definition of derivatives.

Recognizing the Derivative of a Logarithmic Function

Recall that the derivative of $\ln x$ is:

$$
\frac{d}{dx} \ln x = \frac{1}{x}
$$

\]

Applying this knowledge, if we consider the function:

$$
F(x) = \ln x
$$

then:

$$
F'(a) = \frac{1}{a}
$$

To connect this to our limit, observe that:

$$
\lim_{h \to 0} \frac{\ln(e + h) - \ln e}{h}
$$

which is precisely the definition of the derivative of $\ln x$ at $x = e$:

$$
F'(e) = \frac{1}{e}
$$

In our original limit, the numerator is $\ln(e + h) - 1$, and since $\ln e = 1$, this matches the form:

$$
\frac{\ln(e + h) - \ln e}{h}
$$

Thus, the limit:

$$
\lim_{h \to 0} \frac{\ln(e + h) - 1}{h}
$$

is equal to the derivative of $\ln x$ at $x = e$:

$$
F'(e) = \frac{1}{e}
$$

Therefore, the limit equals $1/e$.

Finding the Original Function F(x)

Having identified that the limit represents the derivative of $\ln x$ at $x = e$, and that:

$$
F'(a) = \frac{1}{a}
$$

we can infer that:

$$
F(x) = \ln x + C
$$

where C is an integration constant. This is because the derivative of $\ln x$ is $1/x$, and adding a constant doesn't affect the derivative.

General Form of F(x)

The original function F(x) can be expressed as:

$$
F(x) = \ln x + C
$$

where:

- $x > 0$, since $\ln x$ is defined for positive x,
- C is an arbitrary constant, representing the family of all antiderivatives of $1/x$.

Using the Limit to Confirm F(x)

To verify, consider the derivative of $F(x) = \ln x + C$:

$$
F'(x) = \frac{1}{x}
$$

At $x = e$:

$$
F'(e) = \frac{1}{e}
$$

which matches the value of the limit we computed earlier. This confirms our identification of the function.

Applications and Significance of the Derivative in Real-World Problems

Understanding how to derive functions from limits is not just an academic exercise—it has practical implications:

- **Rate of Change Analysis:** In physics, derivatives model velocity, acceleration, and other rates.
- **Optimization Problems:** Derivatives identify maxima and minima in economics, engineering, and science.
- **Modeling Natural Phenomena:** Logarithmic functions model phenomena like radioactive decay, pH levels, and sound intensity.

The ability to reverse-engineer a function from its derivative or limit is vital for solving complex real-world problems.

Summary and Key Takeaways

- The limit $\lim_{h \to 0} \frac{\ln(e + h) - 1}{h}$ evaluates to $1/e$, representing the derivative of $\ln x$ at $x = e$.
- The function $F(x)$ with derivative $F'(x) = 1/x$ is $F(x) = \ln x + C$, where C is a constant.
- Understanding derivatives through limits enables us to find original functions and analyze their behavior.
- This process exemplifies the fundamental theorem of calculus, linking derivatives and integrals.

Conclusion

In essence, the limit $\lim_{h \to 0} \frac{\ln(e + h) - 1}{h}$ is a gateway to understanding the derivative of the natural logarithmic function. Recognizing this limit as $1/e$ helps us identify the original function $F(x) = \ln x + C$. Mastering these concepts is essential for anyone studying calculus, as they form the foundation for analyzing and modeling change across numerous scientific and engineering disciplines.

Whether you're solving theoretical problems or applying calculus to real-world scenarios, understanding how to interpret limits and derivatives is crucial. Remember, limits serve as the building blocks for derivatives, and derivatives reveal how functions behave and change—fundamental insights that drive progress in mathematics and beyond.

Frequently Asked Questions

What does the limit $\lim (h \to 0)$ [F(a + h) - F(a)] / h represent in calculus?

It represents the derivative of the function F at the point a, denoted as F'(a).

Given that $\lim (h \to 0)$ [F(a + h) - F(a)] / h = L, how can we find the function F(x)?

If we know the derivative F'(a) = L, then F(x) can be found by integrating F'(x), provided initial conditions are known.

How is the limit $\lim (h \to 0)$ [F(a + h) - 1] / h related to F'(a)?

If the limit equals F'(a), then it indicates that F'(a) = $\lim (h \to 0)$ [F(a + h) - 1] / h, which suggests F(a) = 1 + C for some constant C.

In the expression $\lim (h \to 0)$ [F(a + h) - 1] / h, why is the value 1 significant?

Because the limit resembles the derivative of F at a, and if the numerator involves subtracting 1, it suggests F(a) approaches 1, indicating F(a) = 1.

If $\lim (h \to 0)$ [F(a + h) - 1] / h = F'(a), what does that tell us about the function F at x = a?

It indicates that the instantaneous rate of change of F at x = a is F'(a), and at the point, F(a) is likely close to 1.

How can we derive F(x) from the given limit expression?

By recognizing that the limit represents F'(a), we can find F(x) by integrating F'(x), which requires knowing F'(x) explicitly or additional conditions.

What is the connection between the limit $\lim (h \to 0)$ [F(a + h) - 1] / h and the derivative of F at x = a?

They are directly related; the limit equals F'(a), the derivative of F at a, especially if F(a) = 1.

Can we determine the explicit form of F(x) solely from the limit $\lim_{h \to 0} [F(a + h) - 1] / h$?

No, without additional information about F or its values at specific points, we cannot determine the exact form of F(x); the limit gives F'(a), the derivative at a.

What assumptions are needed to find F(x) from the limit $\lim_{h \to 0} [F(a + h) - 1] / h$?

We need to assume F is differentiable at a, and typically know F(a) or initial conditions to integrate the derivative and find F(x).

Additional Resources

Understanding the Limit $\lim_{h\to 0} [\ln(e + h) - 1]$ and Its Connection to Derivatives

In the realm of calculus, the concept of a limit plays a pivotal role in understanding how functions behave as variables approach specific points. One of the most fundamental applications of limits is in defining derivatives — the instantaneous rate of change of a function at a given point. A particular limit that often appears in introductory calculus problems is:

$$\lim_{h\to 0} [\ln(e + h) - 1]$$

This limit is not just a standalone expression; it serves as a gateway to understanding the derivative of a function at a specific point, especially when framed as:

> For some function F and some number A, find F(x).

In this guide, we will explore this limit in detail, interpret its significance, and demonstrate how it reveals the derivative of a particular function, ultimately guiding us to identify the function F(x).

The Significance of Limits in Calculus

Before delving into the specific limit, it’s essential to grasp why limits are fundamental. They provide a rigorous foundation for defining derivatives, integrals, and continuity. The derivative of a function F at a point A is defined as:

$$F'(A) = \lim_{h\to 0} [F(A + h) - F(A)] / h$$

This expression measures the instantaneous rate of change of F at A. Notice that the numerator involves the difference in function values, and the limit as h approaches zero captures the behavior infinitesimally close to A.

Analyzing the Limit: $\lim_{h\to 0} [\ln(e + h) - 1]$

Let's examine the given limit:

$\lim_{h\to0} [\ln(e + h) – 1]$

At first glance, this resembles the numerator of a difference quotient but lacks the division by h. To interpret this as a derivative, we need to connect it with the definition of F'(A).

Step 1: Recognize the Expression

Observe that:

- The function inside the natural logarithm is (e + h).
- As h approaches 0, (e + h) approaches e.
- The expression ln(e + h) – 1 resembles the difference in the natural logs.

Step 2: Think of the Limit as a Derivative

Recall the derivative of the natural logarithm function:

$d/dx [\ln x] = 1/x$

Evaluated at x = e:

$(d/dx) \ln x |_{x=e} = 1/e$

But our limit involves ln(e + h) – 1, which suggests examining the behavior of ln(e + h) near h = 0.

Connecting the Limit to Derivatives

Recognizing the Derivative of F(x) = ln x

Suppose we define a function:

$F(x) = \ln x$

Then, the derivative at a point A is:

$F'(A) = \lim_{h\to0} [\ln(A + h) – \ln A] / h$

Notice that this is similar in structure to the classic difference quotient.

Step 1: Express the limit in terms of F(x)

If we want to relate the original limit to F'(A), we can compare:

$\lim_{h\to0} [\ln(e + h) – 1]$

to

$\lim_{h\to 0}$ [ln(e + h) – ln e]

since ln e = 1.

This leads us to:

$\lim_{h\to 0}$ [ln(e + h) – ln e]

which is exactly the difference quotient numerator for the derivative of F(x) = ln x at x = e:

$F'(e) = \lim_{h\to 0} [\ln(e + h) - \ln e] / h$

Step 2: Recognize the derivative

From this, we see that:

$\lim_{h\to 0} [\ln(e + h) - 1] = \lim_{h\to 0} [\ln(e + h) - \ln e] = h F'(e)$ as $h\to 0$

But note that this limit as $h\to 0$ of [ln(e + h) – 1] is just zero because:

- ln(e + h) approaches ln e = 1,
- So, ln(e + h) – 1 approaches 0.

Thus, the original limit evaluates to 0.

Interpreting the Limit as a Derivative and Finding F(x)

The key insight here is that the limit:

$\lim_{h\to 0}$ [ln(e + h) – 1]

represents the change in ln x around x = e, but without dividing by h, it's not directly a derivative—it's just the difference in function values at x = e and x = e + h as h approaches zero.

Step 1: Recognize the derivative at x = e

Given:

F(x) = ln x

then,

F'(x) = 1 / x

and specifically,

F'(e) = 1 / e

Step 2: Connect the limit to F'(e)

Recall the derivative definition:

$F'(e) = \lim_{h\to 0} [\ln(e + h) - \ln e] / h = \lim_{h\to 0} [\ln(e + h) - 1] / h$

Our original limit is similar but lacks division by h. Therefore, the limit as $h\to 0$ of $[\ln(e + h) - 1]$ alone is 0, which is consistent with the fact that the numerator approaches zero as h approaches zero.

Step 3: Find F(x)

Given the context, the problem asks to find F(x) such that the limit:

$\lim_{h\to 0} [\ln(e + h) - 1]$

represents F' at some point A, with the derivative involved.

From the structure, we can see that:

- At $x = e$, $F(x) = \ln x$,
- The derivative $F'(x) = 1 / x$,
- The limit in question relates to the derivative at $x = e$.

Thus, $F(x) = \ln x$, defined on $(0, \infty)$.

Summary: Connecting the Limit to the Function F(x)

Main Takeaways:

- The limit $\lim_{h\to 0} [\ln(e + h) - 1]$ evaluates to 0 because as h approaches 0, $\ln(e + h)$ approaches $\ln e = 1$, so the difference tends to zero.
- This limit resembles the numerator in the derivative of the logarithm function at $x = e$.
- The derivative of $F(x) = \ln x$ at $x = e$ is $F'(e) = 1 / e$.
- The function $F(x) = \ln x$ satisfies the properties implied by the limit, connecting the derivative and the function.

Final Thoughts and Broader Context

Understanding the limit:

$\lim_{h\to 0} [\ln(e + h) - 1]$

provides insight into the behavior of the natural logarithm function near $x = e$. It exemplifies how limits are used to define derivatives, which in turn reveal the nature of functions. Recognizing these connections is essential for mastering calculus concepts, enabling you to analyze and derive properties of functions systematically.

In essence, the limit demonstrates that:

- The natural logarithm function, ln x, has a derivative of 1/x.
- At x = e, the derivative is 1/e.
- The limit involving ln(e + h) – 1 confirms the smoothness and differentiability of ln x at x = e.

Quick Recap: Key Points

- The limit limh→0 [ln(e + h) – 1] evaluates to zero.
- This expression relates to the derivative of ln x at x = e.
- The derivative of F(x) = ln x is F'(x) = 1/x.
- The limit is a stepping stone to understanding how derivatives are defined via limits.
- Recognizing the structure of limits helps identify the underlying functions they describe.

By mastering these limit concepts, you lay a strong foundation for exploring more advanced topics in calculus and mathematical analysis, sharpening your ability to analyze functions and their behaviors at specific points.

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