1) Solve The IVP: Y"-9y'+18y=0; Y(0)=1; Y'(0)=-6 2) Determine The Form Of The Particular Solution For

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Introduction

Solving initial value problems (IVPs) involving second-order linear differential equations is a fundamental aspect of differential equations. These problems often arise in physics, engineering, and other scientific disciplines to model systems such as mechanical vibrations, electrical circuits, and thermal processes. In this article, we will explore how to solve the specific IVP:

$$
Y'' - 9Y' + 18Y = 0, \quad Y(0) = 1, \quad Y'(0) = -6
$$

and how to determine the form of the particular solution when dealing with nonhomogeneous equations. This comprehensive guide will walk you through the step-by-step solution process, including the characteristic equation, general solution, applying initial conditions, and understanding the structure of particular solutions.

Solving the Homogeneous Differential Equation

Step 1: Write the Differential Equation

The differential equation provided is:

$$
Y'' - 9Y' + 18Y = 0
$$

This is a second-order linear homogeneous differential equation with constant coefficients.

Step 2: Find the Characteristic Equation

To solve this, we assume a solution of the form $Y = e^{rt}$, where r is a constant to be determined. Substituting into the differential equation yields:

$$
r^2 e^{rt} - 9r e^{rt} + 18 e^{rt} = 0
$$

Dividing through by e^{rt} (which is never zero):

$$
r^2 - 9r + 18 = 0
$$

This quadratic equation is called the characteristic equation.

Step 3: Solve the Characteristic Equation

The quadratic can be factored or solved using the quadratic formula:

$$
r = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 1 \times 18}}{2}
$$

Calculate the discriminant:

$$
\Delta = 81 - 72 = 9
$$

Since $\Delta = 9$, the roots are real and distinct:

$$
r = \frac{9 \pm \sqrt{9}}{2} = \frac{9 \pm 3}{2}
$$

Thus, the roots are:

$$
r_1 = \frac{9 + 3}{2} = 6, \quad r_2 = \frac{9 - 3}{2} = 3
$$

General Solution to the Homogeneous Equation

With the roots $r_1 = 6$ and $r_2 = 3$, the general solution is:

$$
Y(t) = C_1 e^{6t} + C_2 e^{3t}
$$

where C_1 and C_2 are arbitrary constants determined by initial conditions.

Applying Initial Conditions

Step 1: Find Derivative Y'

Differentiate the general solution:

$$
Y'(t) = 6 C_1 e^{6t} + 3 C_2 e^{3t}
$$

Step 2: Use the Initial Conditions

Given:

$$
Y(0) = 1, \quad Y'(0) = -6
$$

Plug in $t=0$:

$$
Y(0) = C_1 e^{0} + C_2 e^{0} = C_1 + C_2 = 1
$$

$$
Y'(0) = 6 C_1 e^{0} + 3 C_2 e^{0} = 6 C_1 + 3 C_2 = -6
$$

Step 3: Solve for C_1 and C_2

From the first equation:

$$
C_1 + C_2 = 1 \quad \Rightarrow \quad C_2 = 1 - C_1
$$

Substitute into the second:

$$
6 C_1 + 3 (1 - C_1) = -6
$$
$$
6 C_1 + 3 - 3 C_1 = -6
$$
$$
(6 C_1 - 3 C_1) + 3 = -6
$$
$$
3 C_1 + 3 = -6

$$
3 C_1 = -9
$$

$$
C_1 = -3
$$

Now, find C_2:

$$
C_2 = 1 - (-3) = 4
$$

Final Solution to the IVP

$$
\boxed{
Y(t) = -3 e^{6t} + 4 e^{3t}
}
$$

Determining the Form of the Particular Solution

When dealing with nonhomogeneous differential equations, the method of undetermined coefficients is often employed. The form of the particular solution depends on the right-hand side of the differential equation.

Step 1: Identify the Nonhomogeneous Term

Suppose the differential equation is:

$$
Y'' + p(t) Y' + q(t) Y = g(t)
$$

where $g(t)$ is a known function (e.g., polynomial, exponential, sine, cosine, or a combination).

Step 2: Analyze the Forcing Function $g(t)$

Common types of forcing functions and their corresponding particular solutions:

Forcing Function $g(t)$	Typical Form of Particular Solution	Notes
Polynomial (e.g., $A t^n + \dots$)	Polynomial of same degree	Adjust if solution overlaps with homogeneous solution
Exponential (e.g., $A e^{k t}$)	$A e^{k t}$	Multiply by t if

$e^{k t}$ is a solution to homogeneous equation |
| Sine or Cosine (e.g., $A \sin \omega t + B \cos \omega t$) | $A \sin \omega t + B \cos \omega t$ | Multiply by t if resonance occurs |

Step 3: Formulate the Particular Solution

Based on the form of $g(t)$, choose an appropriate trial solution with undetermined coefficients. For example:

- For polynomial $g(t)$, try a polynomial.
- For exponential $g(t)$, try an exponential multiplied by a polynomial.
- For sinusoidal $g(t)$, try sinusoidal functions with undetermined coefficients.

Step 4: Adjust for Resonance

If the forcing function is similar to the homogeneous solution, multiply the trial particular solution by t (or higher powers of t) to find a suitable form.

Summary and Key Takeaways

- Homogeneous solutions are found by solving the characteristic equation associated with the differential equation.
- Initial conditions allow for solving arbitrary constants to find a particular solution that fits the problem.
- The general solution of a second-order linear homogeneous differential equation with constant coefficients is a linear combination of exponential functions based on the roots of the characteristic equation.
- When dealing with nonhomogeneous equations, the form of the particular solution depends on the nature of the forcing function $g(t)$.
- Method of undetermined coefficients is a systematic way to find particular solutions, adjusting the trial form based on resonance conditions.

Final Remarks

Understanding the process of solving second-order linear differential equations and determining particular solutions is essential for students and professionals working in applied mathematics, physics, and engineering. Properly applying initial conditions ensures the solution accurately models the specific problem scenario. Regular practice with various types of forcing functions enhances mastery of the method of undetermined coefficients and differential equation solving strategies.

If you encounter more complex or non-standard equations, other methods such as variation of parameters or Laplace transforms may be necessary. However, the fundamental principles highlighted in this article serve as a solid foundation for tackling a wide range of differential equations.

Frequently Asked Questions

What is the characteristic equation for the differential equation Y'' - 9Y' + 18Y = 0?

The characteristic equation is r^2 - 9r + 18 = 0.

How do you find the general solution to the homogeneous differential equation Y'' - 9Y' + 18Y = 0?

By solving the characteristic equation, which yields roots r = 3 and r = 6, so the general solution is Y(t) = C1 e^{3t} + C2 e^{6t}.

How are the initial conditions Y(0) = 1 and Y'(0) = -6 used to determine the specific solution?

They are substituted into the general solution and its derivative to solve for constants C1 and C2.

What are the steps to find the particular solution for a differential equation with given initial conditions?

First, find the general solution, then compute its derivative, substitute initial conditions to solve for constants, resulting in the particular solution.

In the context of the given IVP, what is the explicit form of the solution Y(t)?

The solution is Y(t) = 2 e^{3t} - 3 e^{6t}.

What does the second part of the problem, 'Determine the form of the particular solution for,' refer to?

It refers to identifying the structure of the particular solution if the differential equation includes a nonhomogeneous term, which is not specified here, but generally involves guessing based on the form of the forcing function.

If the differential equation had a nonhomogeneous term, say $Y'' - 9Y' + 18Y = f(t)$, how would you determine the particular solution?

You would choose an ansatz (trial solution) based on the form of $f(t)$, such as polynomials, exponentials, or sines and cosines, and use methods like undetermined coefficients or variation of parameters.

Why is it important to verify the initial conditions after finding the general solution?

To ensure the particular solution accurately satisfies the specific initial conditions, leading to the correct particular solution for the IVP.

What are common methods used to solve second-order linear differential equations like the one given?

Methods include solving the characteristic equation for homogeneous solutions, applying initial conditions, and using variation of parameters or undetermined coefficients for nonhomogeneous cases.

Additional Resources

Solve The IVP: $Y'' - 9Y' + 18Y = 0$; $Y(0) = 1$; $Y'(0) = -6$ – Determining the General and Particular Solutions

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Introduction

The process of solving initial value problems (IVPs) involving linear differential equations is fundamental in applied mathematics, physics, and engineering. Among these, second-order linear homogeneous differential equations with constant coefficients stand out due to their broad applicability and well-developed solution methods. This article delves deeply into solving the specific initial value problem:

$$ Y'' - 9Y' + 18Y = 0, \quad Y(0) = 1, \quad Y'(0) = -6 $$

and discusses how to determine the form of particular solutions for similar classes of equations.

Fundamental Concepts

Homogeneous Linear Differential Equations with Constant Coefficients

A second-order linear differential equation with constant coefficients has the general form:

$$ aY'' + bY' + cY = 0 $$

where a, b, c are constants. The solution involves:

- Finding the characteristic equation: $ ar^2 + br + c = 0 $.
- Solving for roots r, which determine the form of the general solution.

Step 1: Formulating the Characteristic Equation

Given the differential equation:

$$ Y'' - 9Y' + 18Y = 0 $$

we identify:

- $ a = 1 $
- $ b = -9 $
- $ c = 18 $

The characteristic equation is:

$$ r^2 - 9r + 18 = 0 $$

Step 2: Solving the Characteristic Equation

Factoring or using the quadratic formula:

$$ r = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 1 \times 18}}{2 \times 1} $$

Calculate the discriminant:

$$ \Delta = 81 - 72 = 9 $$

Thus,

$$ r = \frac{9 \pm \sqrt{9}}{2} = \frac{9 \pm 3}{2} $$

which gives two real roots:

- $ r_1 = \frac{9 + 3}{2} = 6 $
- $ r_2 = \frac{9 - 3}{2} = 3 $

Step 3: General Solution of the Homogeneous Equation

Since the roots are real and distinct, the general solution is:

$$
Y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} = C_1 e^{6t} + C_2 e^{3t}
$$

where C_1 and C_2 are constants determined by initial conditions.

Step 4: Applying Initial Conditions to Find Particular Solution

Given initial conditions:

$$
Y(0) = 1, \quad Y'(0) = -6
$$

Calculate $Y(0)$:

$$
Y(0) = C_1 e^{0} + C_2 e^{0} = C_1 + C_2 = 1
$$

Next, find $Y'(t)$:

$$
Y'(t) = 6 C_1 e^{6t} + 3 C_2 e^{3t}
$$

Evaluate $Y'(0)$:

$$
Y'(0) = 6 C_1 + 3 C_2 = -6
$$

Step 5: Solving for Constants

The system:

$$
\begin{cases}
C_1 + C_2 = 1 \\
6 C_1 + 3 C_2 = -6
\end{cases}
$$

Multiply the first equation by 3:

$$
3 C_1 + 3 C_2 = 3
$$

Subtract from the second:

$$
(6 C_1 + 3 C_2) - (3 C_1 + 3 C_2) = -6 - 3 \\
3 C_1 = -9 \\
C_1 = -3
$$

Using $C_1 + C_2 = 1$:

$$
-3 + C_2 = 1 \Rightarrow C_2 = 4
$$

Final Solution to the IVP

$$
Y(t) = -3 e^{6t} + 4 e^{3t}
$$

This explicit expression satisfies both the differential equation and initial conditions.

Determining the Form of the Particular Solution for Similar Equations

While the current problem is homogeneous, the methodology extends to nonhomogeneous equations where a particular solution must be found.

Typical Form of Particular Solutions

For nonhomogeneous equations:

$$ Y'' + p(t) Y' + q(t) Y = g(t) $$

the form of particular solutions depends on $g(t)$. Common approaches include:

- Method of Undetermined Coefficients: suitable when $g(t)$ is a polynomial, exponential, sine, cosine, or combinations.
- Variation of Parameters: used for more general $g(t)$.

Method of Undetermined Coefficients

This method involves guessing a particular solution $Y_p(t)$ based on the form of $g(t)$, then solving for unknown coefficients.

Typical Forms Based on $g(t)$:

$g(t)$ Type	Particular Solution Guess	Notes
Polynomial of degree n	Polynomial of degree n	Adjust if overlaps with homogeneous solution
$Ae^{\alpha t}$	$Y_p = Ae^{\alpha t}$	If $e^{\alpha t}$ is a root of the homogeneous solution, multiply by t
$\sin \beta t$ or $\cos \beta t$	$A \sin \beta t + B \cos \beta t$	Standard harmonic trial

---

Application to the Current Equation

Although the current problem is homogeneous, understanding the form of particular solutions is critical when dealing with inhomogeneous variants like:

$$ Y'' - 9Y' + 18Y = g(t) $$

Suppose, for example, $g(t) = e^{3t}$. Since e^{3t} is associated with the roots of the homogeneous solution, the particular solution would be guessed as:

$$ Y_p(t) = t \times A e^{3t} $$

This multiplication by t ensures linear independence from the homogeneous solution.

Conclusion

The solution of the initial value problem $Y'' - 9Y' + 18Y = 0$ with initial conditions $Y(0) = 1$ and $Y'(0) = -6$ illustrates a systematic approach to solving second-order linear differential equations with constant coefficients. The roots of the characteristic equation directly inform the form of the general solution, and applying initial conditions allows us to determine the specific constants that generate the particular solution satisfying the problem constraints.

Understanding the structure of homogeneous solutions provides essential insights when extending to nonhomogeneous equations, where the particular solution's form hinges on the nature of the nonhomogeneous term \(g(t)\). The method of undetermined coefficients, along with variation of parameters, remains integral in crafting solutions across a broad spectrum of differential equations.

References

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This detailed exploration provides a comprehensive understanding applicable for students, educators, and practitioners seeking to master the intricacies of solving second-order linear differential equations with constant coefficients.

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