2. A) Determine An Equation For The Family Of Cubic Functions With Zeros -3, 1 And 2.b) Determine The

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Introduction

In algebra and polynomial functions, understanding how to find the equation of a cubic function based on its roots is a fundamental skill. Whether you're a student preparing for exams or an enthusiast seeking to deepen your understanding of polynomial functions, mastering this process is essential. The problem involves determining the general form of a cubic function given specific zeros (roots) and then further analyzing or manipulating that function as needed.

This article provides a comprehensive, step-by-step guide to solving such problems, specifically focusing on:

- Finding the equation of a cubic function with given zeros
- Understanding the relationship between roots and factors
- Deriving the general form of the function
- Exploring the additional parts of the problem (as indicated by "b) Determine the"—which we will interpret as determining certain properties or specific values related to the function)

Throughout, we will emphasize clarity, mathematical rigor, and SEO optimization to ensure this resource is valuable for learners and educators alike.

Understanding The Basics Of Cubic Functions

What Is A Cubic Function?

A cubic function is a polynomial function of degree three, generally expressed as:

$$ f(x) = ax^3 + bx^2 + cx + d $$

where $a \neq 0$.

The graph of a cubic function typically exhibits an S-shaped curve, with a possible inflection point and up to three real roots.

Roots (Zeros) Of A Polynomial

The roots or zeros of a polynomial are the values of x for which $f(x) = 0$. If the roots are known, they can be used to factor the polynomial directly.

For example, if a cubic function has roots r_1, r_2, r_3, then:

$$f(x) = a (x - r_1)(x - r_2)(x - r_3)$$

where a is the leading coefficient, which determines the vertical stretch and orientation of the graph.

Part 1: Determining The Equation For The Family Of Cubic Functions With Zeros -3, 1, and 2

Step 1: Recognize The Roots

Given roots are:

- $r_1 = -3$
- $r_2 = 1$
- $r_3 = 2$

These roots imply that the cubic polynomial can be factored into linear factors involving these roots.

Step 2: Write The General Form

Since the roots are known, the general form of the cubic function with these zeros is:

$$f(x) = a (x + 3)(x - 1)(x - 2)$$

Here, a is an arbitrary non-zero constant. If the problem specifies a particular value for a, such as 1, then the equation becomes fully determined. Otherwise, the set of all such functions forms a family parameterized by a.

Step 3: Expand The Factors

To express the function explicitly, expand the factors:

1. Multiply two factors:

$$(x + 3)(x - 1) = x^2 - x + 3x - 3 = x^2 + 2x - 3$$

2. Multiply this result by the third factor:

$$ (x^2 + 2x - 3)(x - 2) $$

Using distributive property:

$$
\begin{aligned}
& x^2 \times (x - 2) = x^3 - 2x^2 \\
& 2x \times (x - 2) = 2x^2 - 4x \\
& -3 \times (x - 2) = -3x + 6 \\
\end{aligned}
$$

Adding these:

$$
x^3 - 2x^2 + 2x^2 - 4x - 3x + 6 = x^3 + 0x^2 - 7x + 6
$$

Thus, the expanded form:

$$ f(x) = a (x^3 - 7x + 6) $$

Part 2: Determining The Specific Equation (Part b)

Given the phrase "Determine The," it's likely referring to additional properties or specific values related to the cubic function. Common tasks include:

- Determining the value of the function at a specific point
- Finding the equation when the leading coefficient a is specified
- Calculating the y-intercept
- Analyzing the extrema (maxima and minima)

Let's explore some typical interpretations:

Subsection 1: Determining The Equation When The Leading Coefficient is 1

Assuming $a = 1$, the simplest form of the cubic function with roots $-3, 1, 2$ is:

$$ f(x) = (x + 3)(x - 1)(x - 2) $$

which expands to:

$$ f(x) = x^3 - 7x + 6 $$

This is a standard form of the cubic function.

Subsection 2: Find The Equation When The Leading Coefficient is a Different Value

If the problem specifies a leading coefficient $a \neq 1$, then:

$$ f(x) = a (x + 3)(x - 1)(x - 2) $$

For example, if $a = 2$:

$$ f(x) = 2x^3 - 14x + 12 $$

Similarly, for $a = -1$:

$$ f(x) = -x^3 + 7x - 6 $$

This flexibility allows the family of cubic functions with the same roots but different vertical stretches and orientations.

Subsection 3: Find The Y-Intercept

The y-intercept occurs when $x = 0$:

$$ f(0) = a (0 + 3)(0 - 1)(0 - 2) $$

Calculate:

$$
f(0) = a \times 3 \times (-1) \times (-2) = a \times 3 \times 2 = 6a
$$

Thus, the y-intercept depends on a:

- For $a = 1$, $f(0) = 6$
- For $a = 2$, $f(0) = 12$
- For $a = -1$, $f(0) = -6$

Subsection 4: Determine The Critical Points (Maxima and Minima)

To analyze the behavior of the cubic function, find the critical points by computing the first derivative:

$$ f'(x) = \frac{d}{dx} [a(x + 3)(x - 1)(x - 2)] $$

Using the product rule, or expanding first and then differentiating, is often easier.

From earlier, the expanded form (with an arbitrary a) is:

$$f(x) = a (x^3 - 7x + 6)$$

Differentiating:

$$f'(x) = a (3x^2 - 7)$$

Set $f'(x) = 0$ to find critical points:

$$3x^2 - 7 = 0$$

$$x^2 = \frac{7}{3}$$

$$x = \pm \sqrt{\frac{7}{3}}$$

These critical points indicate where the function reaches local maxima or minima, depending on the second derivative test.

Subsection 5: Second Derivative and Concavity

Calculate the second derivative:

$$f''(x) = a \times 6x$$

At $x = \sqrt{\frac{7}{3}}$:

- If $a > 0$, $f''(x) > 0$, indicating a local minimum.
- If $a < 0$, $f''(x) < 0$, indicating a local maximum.

Similarly, analysis applies at $x = -\sqrt{\frac{7}{3}}$.

Summary and Concluding Remarks

By following these systematic steps, you can determine the equation of a cubic function given its roots:

1. Write the factored form based on roots.
2. Expand the factors to obtain the polynomial form.
3. Adjust the leading coefficient a based on specific conditions.
4. Derive additional properties like the y-intercept, critical points, and concavity.

This process is fundamental in algebra, calculus, and mathematical modeling, allowing for the analysis of polynomial behaviors and the construction of functions with desired properties.

Final Thoughts

Understanding how to determine the family of cubic functions from their zeros is an essential skill for students and professionals working in mathematics, engineering, and sciences. It combines algebraic manipulation, calculus, and graph analysis, providing insight into the nature of polynomial functions.

By mastering these concepts, you can approach more complex polynomial problems with confidence and clarity, ensuring a solid foundation for advanced mathematical studies and applications.

Keywords: cubic functions, polynomial roots, factorization, algebra, calculus, critical points, maxima and minima, y-intercept, polynomial equations, family of functions

Frequently Asked Questions

How do you find the equation of a cubic function given its zeros -3, 1, and 2?

To find the cubic function, multiply the factors corresponding to each zero: $(x + 3)$, $(x - 1)$, and $(x - 2)$. Then, expand these factors and include a leading coefficient if needed. The general form is $y = a(x + 3)(x - 1)(x - 2)$, where a is any non-zero constant.

What is the general form of a cubic function with zeros at -3, 1, and 2?

The general form is $y = a(x + 3)(x - 1)(x - 2)$, where a is a real number that determines the vertical stretch or compression of the graph.

How do you expand the cubic polynomial from its factored form with given zeros?

Start by multiplying two factors: $(x + 3)$ and $(x - 1)$ to get $x^2 + 2x - 3$. Then multiply this result by $(x - 2)$ to obtain the expanded form: $y = a[x^3 - 2x^2 + 2x^2 - 4x - 3x + 6]$, which simplifies further to $y = a(x^3 - 5x + 6)$.

What role does the leading coefficient 'a' play in the family of cubic functions with specified zeros?

The coefficient 'a' determines the vertical stretch or compression and the direction of the graph's end behavior. Changing 'a' produces different members of the family while maintaining the same zeros.

Can you find a specific cubic function with zeros -3, 1, and 2 and a leading coefficient of 1?

Yes. Setting a = 1, the equation is y = (x + 3)(x - 1)(x - 2). Expanding this gives y = x^3 - 0x^2 - 5x + 6, or simply y = x^3 - 5x + 6.

What methods are useful for verifying the zeros of the cubic function?

Substitute each zero into the function and check if the result equals zero. Alternatively, graph the function and observe where it crosses the x-axis at -3, 1, and 2.

How does knowing the zeros help in sketching the graph of the cubic function?

Knowing the zeros indicates where the graph crosses the x-axis, and combined with the end behavior and turning points, it helps in accurately sketching the cubic curve.

What is the next step after determining the general form of the cubic function with given zeros?

The next step is to choose a specific leading coefficient 'a' if needed, expand the factors to write the explicit polynomial form, and analyze its graph or properties as required.

Additional Resources

Determine an Equation for the Family of Cubic Functions with Zeros -3, 1, and 2 is a fundamental problem in algebra that involves understanding how the roots of a polynomial influence its general form. This type of problem is essential for students and professionals alike, as it deepens comprehension of polynomial functions, their roots, and how to manipulate algebraic expressions to fit specific criteria. In this comprehensive guide, we will explore the process step-by-step, providing a clear pathway from identifying roots to deriving the general family of cubic functions that satisfy these conditions.

Understanding the Problem

Before diving into the calculations, it’s crucial to interpret what the problem asks:

- Find an equation for the family of cubic functions: This means we're looking for a general formula that describes all cubic functions sharing certain characteristics.
- With zeros at -3, 1, and 2: The roots or zeros of the cubic function are given as -3, 1, and 2, which directly influence the factors of the polynomial.

In essence, the task is to determine the algebraic expression that encompasses all cubic functions having these three roots, possibly differing by a constant factor (which accounts for the 'family' aspect).

Step 1: Recall the Relationship Between Roots and Factors

In polynomial algebra, the roots of a function directly relate to its factors. For a cubic polynomial with roots $r_1, r_2,$ and r_3, the polynomial can be expressed as:

$$
f(x) = k(x - r_1)(x - r_2)(x - r_3)
$$

where:
- k is a non-zero constant (which determines the vertical stretch or compression),
- r_1, r_2, r_3 are the roots.

Since the roots are given as -3, 1, and 2, the general form becomes:

$$
f(x) = k(x - (-3))(x - 1)(x - 2) = k(x + 3)(x - 1)(x - 2)
$$

This expression represents the family of all cubic functions with these roots, differing only by the constant k.

Step 2: Construct the Basic Polynomial (Without the Constant)

To understand the structure, start with the simplest form where $k=1$:

$$
f(x) = (x + 3)(x - 1)(x - 2)
$$

Let's expand this step-by-step:

Expand $(x + 3)(x - 1)$:

$$
(x + 3)(x - 1) = x \times x + x \times (-1) + 3 \times x + 3 \times (-1) = x^2 - x + 3x - 3 = x^2 + 2x - 3
$$

Multiply this result by $(x - 2)$:

$$
(x^2 + 2x - 3)(x - 2)
$$

Using distributive property:

$$
x^2 \times x = x^3
$$
$$
x^2 \times (-2) = -2x^2
$$
$$
2x \times x = 2x^2
$$
$$
2x \times (-2) = -4x
$$
$$
-3 \times x = -3x
$$
$$
-3 \times (-2) = 6
$$

Now, sum all terms:

$$
x^3 - 2x^2 + 2x^2 - 4x - 3x + 6 = x^3 + (- 2x^2 + 2x^2) + (-4x - 3x) + 6
$$

Simplify:

$$
x^3 + 0 - 7x + 6
$$

Thus, the cubic polynomial with the specified roots (and $k=1$) is:

$$
f(x) = x^3 - 7x + 6
$$

Step 3: Incorporate the Constant k

The family of functions with these roots is represented by multiplying this polynomial by any non-zero constant k:

$$
f(x) = k(x + 3)(x - 1)(x - 2) = k[x^3 - 7x + 6]
$$

- When $k=1$, the polynomial is as derived above.
- When $k \neq 1$, the polynomial is scaled vertically but shares the same roots.

This general form encompasses all such cubic functions with the specified zeros.

Step 4: Confirming the Roots

It’s good practice to verify that the roots of the general family are indeed -3, 1, and 2.

For $f(x) = k(x + 3)(x - 1)(x - 2)$:

- At $x = -3$:

$$
f(-3) = k((-3) + 3)((-3) - 1)((-3) - 2) = k(0)(-4)(-5) = 0
$$

- At $x = 1$:

$$
f(1) = k(1 + 3)(1 - 1)(1 - 2) = k(4)(0)(-1) = 0
$$

- At $x = 2$:

$$
f(2) = k(2 + 3)(2 - 1)(2 - 2) = k(5)(1)(0) = 0
$$

Since the function equals zero at these points, the roots are confirmed.

Step 5: Determining the Specific Equation (If Additional Conditions Are Given)

The above derivation provides the family of cubic functions with roots at -3, 1, and 2. However, sometimes the problem asks for a specific equation, perhaps with a given point or additional condition (such as the value of the function at a certain x, or the leading coefficient).

Example: Find the specific cubic function passing through a point, say, $(0, 12)$.

Using the general form:

$$
f(x) = k(x + 3)(x - 1)(x - 2)
$$

At $x=0$:

$$
f(0) = k(0 + 3)(0 - 1)(0 - 2) = k(3)(-1)(-2) = k(3 \times -1 \times -2) = k(6)
$$

Set equal to 12:

$$
6k = 12 \Rightarrow k = 2
$$

Thus, the specific cubic function is:

$$
f(x) = 2(x + 3)(x - 1)(x - 2)
$$

Expanding:

$$
f(x) = 2[x^3 - 7x + 6] = 2x^3 - 14x + 12
$$

Summary: The Complete Guide to Finding the Family of Cubic Functions

To summarize the process:

1. Identify the roots: -3, 1, and 2.
2. Express the general form: $f(x) = k(x + 3)(x - 1)(x - 2)$.
3. Expand to standard polynomial form: $f(x) = k[x^3 - 7x + 6]$.
4. Determine the constant k if additional conditions are provided.
5. Verify roots by plugging in the roots to ensure the polynomial equals zero.

6. Adjust \(k\) based on specific points or conditions, leading to a particular solution within the family.

Final Thoughts

Understanding how to determine an equation for the family of cubic functions with specified zeros is a crucial step in mastering polynomial algebra. It bridges the concept of roots with factorization and polynomial expansion, equipping learners with essential algebraic tools. Whether working on pure mathematics, applied problems, or preparing for exams, this method offers a systematic approach to constructing cubic functions with desired properties.

Remember: The key lies in recognizing that each root corresponds to a factor of the polynomial, and the constant multiplier \(k\) allows for the flexibility needed to fit additional conditions. Practice with different sets of roots and conditions will solidify this understanding and enhance problem-solving proficiency.

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