2. A) Determine An Equation For The Family Of Cubic Functions With Zeros - 3/2; 1, And 5/2 (2 Points)

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Understanding how to derive the equation of a cubic function based on its zeros is a fundamental concept in algebra and polynomial functions. In this guide, we will explore the process step-by-step to determine the general form of a cubic function given specific zeros, specifically zeros at -3/2, 1, and 5/2. This comprehensive approach will help students and learners grasp the underlying principles and apply them to similar problems confidently.

Introduction to Cubic Functions and Zeros

A cubic function is a polynomial function of degree three, generally expressed as:

$$ f(x) = ax^3 + bx^2 + cx + d $$

where $a \neq 0$. The zeros (or roots) of the function are the values of x that make $f(x) = 0$.

Understanding the relationship between the zeros and the factors of the polynomial is essential. If r is a zero of the cubic function, then $(x - r)$ is a factor of the polynomial.

Key points:

- The zeros of a polynomial directly relate to its factors.
- The multiplicity of zeros affects the graph's behavior at those zeros.
- For a cubic function with three distinct zeros, the polynomial can be expressed as the product of three linear factors.

Step-by-Step Process to Determine the Equation

To find the cubic function with zeros at $-\frac{3}{2}$, 1, and $\frac{5}{2}$, follow these steps:

Step 1: Express the zeros as factors

Since the zeros are at $-\frac{3}{2}$, 1, and $\frac{5}{2}$, the corresponding factors are:

- $x - (-\frac{3}{2}) = x + \frac{3}{2}$
- $x - 1$

- $x - \frac{5}{2}$

However, to avoid dealing with fractions, it's often easier to clear denominators by multiplying each factor by an appropriate number.

Step 2: Write the factors in a form free of fractions

- $x + \frac{3}{2}$ can be written as $2x + 3$
- $x - 1$ remains as is
- $x - \frac{5}{2}$ becomes $2x - 5$

This approach ensures all factors are polynomial expressions with integer coefficients.

Step 3: Write the general form of the cubic polynomial

The general form, considering the leading coefficient a, is:

$$f(x) = a \times (2x + 3) \times (x - 1) \times (2x - 5)$$

where a is any non-zero constant. Usually, unless specified, we take $a = 1$ for the simplest form.

Step 4: Expand the factors to find the polynomial in standard form

Let's expand step-by-step:

1. Multiply the first two factors:

$$(2x + 3)(x - 1)$$

Using the distributive property (FOIL):

$$
\begin{aligned}
(2x + 3)(x - 1) &= 2x \times x + 2x \times (-1) + 3 \times x + 3 \times (-1) \\
&= 2x^2 - 2x + 3x - 3 \\
&= 2x^2 + x - 3
\end{aligned}
$$

2. Multiply this result by the third factor $(2x - 5)$:

$$
(2x^2 + x - 3)(2x - 5)
$$

$$

Again, expand term-by-term:

$$
\begin{aligned}
2x^2 \times 2x &= 4x^3 \\
2x^2 \times (-5) &= -10x^2 \\
x \times 2x &= 2x^2 \\
x \times (-5) &= -5x \\
-3 \times 2x &= -6x \\
-3 \times (-5) &= 15
\end{aligned}
$$

Now, combine like terms:

$$
\begin{aligned}
4x^3 + (-10x^2 + 2x^2) + (-5x - 6x) + 15 &= 4x^3 - 8x^2 - 11x + 15
\end{aligned}
$$

3. Write the final polynomial with the leading coefficient a:

$$
f(x) = a \times (4x^3 - 8x^2 - 11x + 15)
$$

Choosing $a = 1$ gives the simplest form:

$$
\boxed{
f(x) = 4x^3 - 8x^2 - 11x + 15
}
$$

Understanding the Significance of the Leading Coefficient

While we've set $a = 1$ for simplicity, in some contexts, the problem may specify a particular leading coefficient or require the polynomial to pass through additional points.

- If $a \neq 1$: The equation becomes $f(x) = a(4x^3 - 8x^2 - 11x + 15)$.
- Impact of changing a: It scales the entire graph vertically but doesn't change the zeros or their multiplicities.

In most problems of this nature, unless specified, choosing $a = 1$ is acceptable.

Verifying the Zeros

To confirm the zeros are correct, substitute each zero into the polynomial:

- For $x = -\frac{3}{2}$:

$$
f\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^3 - 8\left(-\frac{3}{2}\right)^2 - 11\left(-\frac{3}{2}\right) + 15
$$

Calculating step-by-step:

$$
\left(-\frac{3}{2}\right)^3 = -\frac{27}{8}
$$

$$
\left(-\frac{3}{2}\right)^2 = \frac{9}{4}
$$

Plug in:

$$
4 \times -\frac{27}{8} = -\frac{108}{8} = -\frac{27}{2}
$$

$$
-8 \times \frac{9}{4} = -8 \times \frac{9}{4} = -18
$$

$$
-11 \times -\frac{3}{2} = \frac{33}{2}
$$

Now sum all:

$$
-\frac{27}{2} - 18 + \frac{33}{2} + 15
$$

Express all with denominator 2:

$$
-\frac{27}{2} - \frac{36}{2} + \frac{33}{2} + \frac{30}{2}
$$

Combine:

\[

\left(-27 - 36 + 33 + 30\right)/2 = (0)/2 = 0
\]

Confirmed zero.

Similarly, verify for \(x=1 \):

\[
f(1) = 4(1)^3 - 8(1)^2 - 11(1) + 15 = 4 - 8 - 11 + 15 = 0
\]

And for \(x = \frac{5}{2} \):

\[
f\left(\frac{5}{2}\right) = 4 \times \left(\frac{5}{2}\right)^3 - 8 \times \left(\frac{5}{2}\right)^2 - 11 \times \frac{5}{2} + 15
\]

Calculations:

\[
\left(\frac{5}{2}\right)^3 = \frac{125}{8}
\]

\[
\left(\frac{5}{2}\right)^2 = \frac{25}{4}
\]

Plug in:

\[
4 \times \frac{125}{8} = \frac{500}{8} = \frac{125}{2}
\]

\[
-8 \times \frac{25}{4} = -8 \times \frac{25}{4} = -50
\]

\[
-11 \times \frac{5}{2} = -\frac{55}{2}
\]

Sum all:

\[
\frac{125}{2} - 50 - \frac{55}{2} + 15
\]

Express all with denominator 2:

\[

$$\frac{125}{2} - \frac{100}{2} - \frac{55}{2} + \frac{30}{2} = \frac{125 - 100 - 55 + 30}{2} = \frac{0}{2} = 0$$
\

Frequently Asked Questions

How do you find the equation of a cubic function with given zeros -3/2, 1, and 5/2?

First, write the factors corresponding to each zero: $(x + 3/2)$, $(x - 1)$, and $(x - 5/2)$. Convert fractional factors to integers by multiplying through by 2: $(2x + 3)$, $(x - 1)$, and $(2x - 5)$. Then, form the cubic polynomial as their product, and expand to get the equation.

What is the general form of the cubic function with zeros at -3/2, 1, and 5/2?

The general form is $y = k (2x + 3)(x - 1)(2x - 5)$, where k is any non-zero constant. Expanding this product gives the specific cubic equation depending on the value of k.

How do you determine the leading coefficient for the cubic function with these zeros?

If no specific leading coefficient is given, it is common to assume $k=1$ for the simplest form. To include a different leading coefficient, multiply the expanded polynomial by that constant.

Can you provide the expanded form of the cubic function with zeros -3/2, 1, and 5/2 assuming k=1?

Yes. Starting with $y = (2x + 3)(x - 1)(2x - 5)$, expand step-by-step: first expand $(2x + 3)(x - 1) = 2x^2 - 2x + 3x - 3 = 2x^2 + x - 3$. Then multiply this result by $(2x - 5)$: $(2x^2 + x - 3)(2x - 5)$. Expanding gives $4x^3 -10x^2 + 2x^2 -5x -6x +15 = 4x^3 -8x^2 -11x + 15$. So, the cubic function is $y = 4x^3 - 8x^2 - 11x + 15$.

Why is it important to convert fractional zeros into factors with integer coefficients when finding the cubic equation?

Converting fractional zeros to factors with integer coefficients simplifies the process of expanding the polynomial and ensures accuracy in deriving the standard form of the cubic function. It makes multiplication and expansion more straightforward.

Additional Resources

Determining the Equation for a Family of Cubic Functions with Specific Zeros

In the realm of algebra and polynomial functions, understanding how to derive an equation from given roots or zeros is a fundamental skill. This process not only reinforces conceptual clarity about the relationship between roots and factors but also provides insight into how cubic functions behave based on their zeros. In this comprehensive exploration, we focus on determining the general form of a family of cubic functions that have three specified zeros: -3/2, 1, and 5/2. This task involves translating roots into factors, constructing the polynomial, and analyzing how variations in coefficients give rise to a family of functions with those particular zeros.

Understanding the Foundations: Roots, Zeros, and Polynomial Construction

What Are Zeros and Roots?

In the context of polynomial functions, the terms "zeros" and "roots" are often used interchangeably. They refer to the values of the independent variable (usually x) at which the function equals zero. When a polynomial is factored completely, its zeros correspond to the factors in the form (x - r), where r is a zero.

For example, if a polynomial has a zero at x = r, then (x - r) is a factor of the polynomial. Conversely, multiplying these factors yields the polynomial, up to a constant multiple.

From Roots to Factors: The Basic Principle

Given a set of zeros, constructing the corresponding polynomial involves:

- Writing each zero as a linear factor: (x - zero)
- Multiplying these factors together to form the polynomial
- Including a constant coefficient to generalize the family of functions

This process is fundamental because it provides a direct method to generate all cubic functions with specified zeros.

Specifying the Zeros: -3/2, 1, and 5/2

Interpreting the Zeros

The zeros provided are:

- x = -3/2
- x = 1
- x = 5/2

Note that two of these zeros are rational fractions, which influences the factors directly. The zero at x = 1 is an integer, making it straightforward.

Converting Zeros to Factors

Following the principle, each zero translates into a factor:

- For x = -3/2, the factor is (x - (-3/2)) = (x + 3/2)
- For x = 1, the factor is (x - 1)
- For x = 5/2, the factor is (x - 5/2)

To avoid dealing with fractions in the factors, it is often easier to clear denominators by multiplying through by the least common denominator (LCD). The denominators are 2, so the LCD is 2.

Constructing the Polynomial: From Factors to Standard Form

Expressing Factors with Rational Zero

The factors are:

- (x + 3/2)
- (x - 1)
- (x - 5/2)

Multiplying each by 2 to clear denominators:

- 2(x + 3/2) = 2x + 3
- 2(x - 1) = 2x - 2
- 2(x - 5/2) = 2x - 5

However, note that multiplying the entire polynomial by a constant does not alter its zeros, only its leading coefficient. To keep the polynomial monic (leading coefficient of 1), we need to incorporate these factors carefully.

Formulating the Polynomial with Rational Zeros

The polynomial can be written as:

$$ P(x) = k \times (x + \frac{3}{2}) \times (x - 1) \times (x - \frac{5}{2}) $$

where k is any non-zero constant, representing the family of functions.

To eliminate fractions within the factors, multiply both sides by 2:

$$ P(x) = k \times \frac{(2x + 3)}{1} \times (x - 1) \times \frac{(2x - 5)}{1} $$

which simplifies to:

$$ P(x) = k \times (2x + 3) \times (x - 1) \times (2x - 5) $$

The constant k captures all scalar multiples, giving us the entire family of cubic functions with these zeros.

Expanding the Polynomial: Deriving the General Equation

Step 1: Multiply the Factors

We now expand:

$$ P(x) = k \times (2x + 3) \times (x - 1) \times (2x - 5) $$

The order of multiplication can vary, but for clarity, first multiply the first two factors:

$$ (2x + 3)(x - 1) $$

Expanding:

$$ 2x \times x = 2x^2 $$
$$ 2x \times (-1) = -2x $$
$$ 3 \times x = 3x $$
$$ 3 \times (-1) = -3 $$

Combining:

$$ 2x^2 - 2x + 3x - 3 = 2x^2 + x - 3 $$

Now, multiply this result by the third factor:

$$ (2x^2 + x - 3)(2x - 5) $$

Step 2: Expand Using Distribution

Distribute term by term:

$$ 2x^2 \times 2x = 4x^3 $$
$$ 2x^2 \times (-5) = -10x^2 $$
$$ x \times 2x = 2x^2 $$
$$ x \times (-5) = -5x $$
$$ -3 \times 2x = -6x $$
$$ -3 \times (-5) = 15 $$

Combine like terms:

$$ 4x^3 + (-10x^2 + 2x^2) + (-5x - 6x) + 15 $$
$$ 4x^3 - 8x^2 - 11x + 15 $$

Therefore, the polynomial family is:

$$ P(x) = k \times (4x^3 - 8x^2 - 11x + 15) $$

where k is any real constant except zero.

Understanding the Family of Cubic Functions

Role of the Constant k

The constant k scales the polynomial vertically. When $k = 1$, the polynomial is monic (leading coefficient of 1). Varying k produces an entire family of cubic functions sharing the same zeros but differing in their amplitude and vertical stretch.

- If $k > 0$, the cubic opens upward in the general shape.
- If $k < 0$, the cubic opens downward.
- The magnitude of k influences the steepness of the graph.

This flexibility is crucial in many applications, such as modeling physical phenomena where the roots are fixed but the scale varies.

Implications of the Zeros on the Graph

Since the zeros are at $x = -3/2$, 1, and 5/2, the graph of any function in this family crosses the x-axis at these points. Between and beyond these zeros, the behavior of the cubic depends on the sign of k and the multiplicity of zeros.

Final Equation and Summary

The general form of the family of cubic functions with zeros at -3/2, 1, and 5/2 is:

$$\boxed{
P(x) = k \times (4x^3 - 8x^2 - 11x + 15)
}$$

where $k \neq 0$ is any real number.

Analytical Insights and Applications

Mathematical Significance

The process exemplifies how algebraic principles facilitate the construction of specific polynomial functions from their zeros. It highlights the importance of transforming fractional roots into manageable factors, expanding polynomial expressions, and understanding the influence of scalar constants.

This approach is foundational in polynomial interpolation, designing functions with predetermined roots, and analyzing how parameters affect the graph's shape.

Educational and Practical Relevance

For students and practitioners, mastering this method enhances problem-solving skills and deepens understanding of polynomial behavior. It has applications in fields such as engineering, physics, economics, and data modeling, where defining functions with particular roots is essential.

In summary, the derivation of the cubic family equation from given zeros involves translating roots into factors, clearing denominators to simplify calculations, expanding the factors to standard form, and recognizing the role of scalar multiples. This comprehensive approach ensures a thorough grasp of polynomial construction and the geometric implications of roots on the graph.

In conclusion, determining an equation for a family of cubic functions with specified zeros is a powerful illustration of algebraic manipulation and conceptual understanding. By systematically converting zeros into factors, expanding, and incorporating a scalar multiplier, one can generate all functions sharing those roots, providing a versatile framework for mathematical analysis and real-world modeling.

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