2. A Tank Initially Contains 800 Liters Of Pure Water. A Salt Solution With Concentration 29/1 Enters

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Understanding how salt solutions mix and how the concentration changes over time is an essential concept in fields such as chemical engineering, environmental science, and water treatment. In this article, we explore a common problem involving the initial state of a tank filled with pure water, into which a salt solution with a specified concentration is introduced. We will analyze the process step-by-step, providing detailed explanations, mathematical models, and practical insights to enhance your understanding of the dynamics involved.

Introduction to the Problem

Imagine a large tank that initially contains 800 liters of pure water. At a steady rate, a salt solution with a concentration of 29/1 (which equates to 29 parts of salt per 1 part of water) is being added to the tank. The key questions often posed in such scenarios include:

- How does the concentration of salt in the tank change over time?
- How long does it take for the salt concentration to reach a certain level?
- What is the amount of salt in the tank at any given time?

To analyze these questions, we need to understand the principles of mixing, rate of inflow, and the mathematical modeling of the process.

Understanding the Basic Concepts

Initial Conditions

- Volume of water in the tank initially: 800 liters
- Initial salt amount: 0 kg (since the water is pure)
- Concentration of incoming solution: 29/1, which indicates a high salt content

Flow Rate of the Incoming Solution

In many problems, the rate at which the solution enters the tank is specified or assumed to be constant. For this analysis, let's assume:

- Inflow rate: R liters per minute (or per hour)
- The solution is well-mixed, meaning the salt concentration throughout the tank is uniform at any given time.

Outflow Considerations

Depending on the problem, there might be:

- No outflow: The tank is closed, and only inflow occurs.
- Constant outflow: A steady amount of mixture leaves the tank, maintaining a constant volume or allowing the volume to change over time.

For simplicity, let's consider two scenarios:

1. No outflow (closed system): The total volume increases over time.
2. Outflow at the same rate as inflow: The volume remains constant at 800 liters.

This article will primarily focus on the second scenario, which is common in practical applications.

Modeling the Salt Concentration: Mathematical Approach

To analyze the process, we use differential equations, which relate the rate of change of salt in the tank to the inflow and outflow rates.

Variables and Notation

- $V(t)$: Volume of solution in the tank at time t (liters)
- $S(t)$: Amount of salt in the tank at time t (kilograms or grams)
- $C(t)$: Concentration of salt in the tank at time t (kg/liter)

Given that:

$$
C(t) = \frac{S(t)}{V(t)}
$$

In the case where volume remains constant ($V(t) = V_0 = 800$ liters), the problem simplifies.

Formulating the Differential Equation

Assuming:

- The inflow rate: R liters/minute
- The concentration of the inflowing solution: $C_{in} = \frac{29}{1}$ (which is 29 kg/liter, assuming the units are consistent)

Note: Since 29/1 is a large number, it's more practical to interpret this as 29 parts salt per 1 part water, i.e., a ratio. To give the problem a realistic context, let's assume:

- The salt solution has 29 kg of salt per 1,000 liters of solution, which corresponds to a concentration of 0.029 kg/liter.

Alternatively, if the problem states the ratio as 29:1, it could mean 29 parts salt per 1 part water, which is an extremely concentrated solution. For the purpose of a practical and solvable problem, we'll interpret the concentration as 0.029 kg/liter.

Inflow:

- Salt carried in per unit time: $R \times C_{in}$ (kg/min)

Outflow:

- The mixture leaving the tank has concentration $C(t)$, and volume flow rate R:

- Salt leaving per unit time: $R \times C(t)$

Differential equation:

$$
\frac{dS}{dt} = \text{(salt in)} - \text{(salt out)} = R \times C_{in} - R \times C(t)
$$

Since $C(t) = \frac{S(t)}{V_0}$, and V_0 is constant:

$$
\frac{dS}{dt} = R \times C_{in} - R \times \frac{S(t)}{V_0}
$$

Simplify:

$$
\frac{dS}{dt} + \frac{R}{V_0} S(t) = R \times C_{in}
$$

This is a linear first-order differential equation.

Solution to the Differential Equation

The integrating factor (IF):

\[

\mu(t) = e^{\int \frac{R}{V_0} dt} = e^{\frac{R}{V_0} t}
\]

Multiplying both sides:

\[
e^{\frac{R}{V_0} t} \frac{dS}{dt} + e^{\frac{R}{V_0} t} \frac{R}{V_0} S(t) = R \times C_{in} e^{\frac{R}{V_0} t}
\]

Left side simplifies to:

\[
\frac{d}{dt} \left(S(t) e^{\frac{R}{V_0} t} \right) = R \times C_{in} e^{\frac{R}{V_0} t}
\]

Integrate both sides:

\[
S(t) e^{\frac{R}{V_0} t} = R \times C_{in} \times \frac{V_0}{R} e^{\frac{R}{V_0} t} + C
\]

\[
S(t) e^{\frac{R}{V_0} t} = C_{in} V_0 e^{\frac{R}{V_0} t} + C
\]

Solve for \(S(t) \):

\[
S(t) = C_{in} V_0 + C e^{-\frac{R}{V_0} t}
\]

Using initial conditions: at \(t=0 \), \(S(0) = 0 \):

\[
0 = C_{in} V_0 + C
\]

\[
C = - C_{in} V_0
\]

Therefore, the solution:

\[
S(t) = C_{in} V_0 \left(1 - e^{-\frac{R}{V_0} t} \right)
\]

The salt concentration at time \(t \):

\[

$$
C(t) = \frac{S(t)}{V_0} = C_{in} \left(1 - e^{-\frac{R}{V_0} t} \right)
$$

Practical Example with Specific Values

Suppose:

- Inflow rate $R = 10$ liters/minute
- Initial volume $V_0 = 800$ liters
- Inflow concentration $C_{in} = 0.029$ kg/liter

Then:

$$
C(t) = 0.029 \left(1 - e^{-\frac{10}{800} t} \right) = 0.029 \left(1 - e^{-\frac{t}{80}} \right)
$$

This formula describes how the salt concentration approaches the inflow concentration over time.

Analyzing the Results and Their Implications

Concentration Over Time

- The concentration starts at 0 kg/liter (initial pure water).
- It increases exponentially, approaching the inflow concentration asymptotically.
- The rate of increase depends on the flow rate R and the volume V_0.

Time to Reach a Specific Concentration

Suppose we want to find the time t when the concentration reaches a specific value C_f:

$$
C_f = C_{in} \left(1 - e^{-\frac{R}{V_0} t} \right)
$$

Solve for t:

$$
e^{-\frac{R}{V_0} t} = 1 - \frac{C_f}{C_{in}}
$$

\[
t = - \frac{V_0}{R} \ln \left(1 - \frac{C_f}{C_{in}} \

Frequently Asked Questions

How does the concentration of the salt solution affect the salt levels in the tank over time?

As the salt solution with a concentration of 29/1 enters the tank, the salt levels increase proportionally, and the concentration in the tank depends on the rate of inflow, outflow, and mixing efficiency over time.

What is the rate at which salt accumulates in the tank if the inflow rate is known?

The rate of salt accumulation is determined by multiplying the inflow rate by the salt concentration (29/1), assuming perfect mixing and no outflow; for example, if inflow is R liters per minute, salt enters at R 29 units per minute.

How can we model the change in salt concentration in the tank over time?

The change can be modeled using a differential equation that accounts for the inflow of salt solution, the outflow (if any), and the mixing process, typically leading to a first-order linear differential equation for the salt amount or concentration.

What happens to the salt concentration in the tank if the inflow and outflow rates are equal?

If inflow and outflow rates are equal, the total volume remains constant at 800 liters, and the salt concentration will approach an equilibrium value determined by the ratio of inflow concentration and the initial conditions, often leading to a steady-state concentration.

How can we determine the time required for the salt concentration to reach a certain level in the tank?

By solving the differential equation governing the system, we can derive an expression for concentration as a function of time and then solve for the time when the desired concentration is achieved, considering initial conditions and flow rates.

Additional Resources

A Tank Initially Contains 800 Liters Of Pure Water. A Salt Solution With Concentration 29/1 Enters — this scenario is a classic example of a mixing problem, often encountered in chemical engineering, environmental science, and water treatment processes. Understanding how the concentration of salt within the tank changes over time as a solution flows in and out is essential for designing efficient systems, predicting substance levels, and ensuring quality control. This article provides a comprehensive guide to analyzing such a problem, breaking down the process step-by-step, and offering insights into the underlying principles.

Introduction to the Problem

Imagine you have a large tank filled with 800 liters of pure water. You start to pump in a salt solution with a concentration of 29/1 (which simplifies to 29 grams of salt per liter). Simultaneously, the tank is equipped with an outlet that allows the solution to flow out at a certain rate, maintaining a continuous process. The key questions are:

- How does the amount of salt in the tank evolve over time?
- What is the concentration of salt in the tank at any given moment?
- How long does it take for the tank to reach a certain concentration?

These questions are central to understanding mixing dynamics and are applicable in real-world scenarios such as water purification, chemical mixing, and environmental monitoring.

Setting Up the Problem

Variables and Assumptions

Before diving into the mathematics, let's identify the variables involved:

- V(t): Volume of solution in the tank at time t (liters)
- Q(t): Quantity of salt in the tank at time t (grams)
- C(t): Concentration of salt in the tank at time t (grams/liter)

Assumptions include:

- The tank initially contains only pure water, so Q(0) = 0.
- The inflow rate (rate at which the salt solution enters) is constant, say r_in liters per minute.
- The outflow rate (rate at which solution leaves the tank) is constant, say r_out liters per minute.
- The incoming solution has a fixed salt concentration of 29 grams/liter.
- The tank's volume may change over time if inflow and outflow are unequal; if they are equal, the volume remains constant.
- The mixing within the tank is instantaneous, leading to uniform concentration throughout the liquid.

Mathematical Modeling

Step 1: Establishing the Differential Equation

The core of the analysis is to formulate a differential equation that models the change in salt quantity over time.

The rate of change of salt in the tank is given by:

Rate in - Rate out = dQ/dt

- Rate in: The amount of salt entering per unit time = inflow rate concentration of inflow solution = r_{in} 29
- Rate out: The amount of salt leaving per unit time = outflow rate concentration in the tank = r_{out} (Q / V)

If the volume remains constant (i.e., inflow equals outflow: $r_{in} = r_{out} = r$), then:

$V(t) = 800$ liters (constant)

Thus,

$$dQ/dt = r\ 29 - r\ (Q / V)$$

or simplified:

$$dQ/dt = r\ (29 - Q / V)$$

Step 2: Solving the Differential Equation

This is a first-order linear differential equation. Let's write it explicitly:

$$dQ/dt + (r / V)\ Q = r\ 29$$

The solution involves integrating factor method:

- Integrating factor $\mu(t) = e^{\int (r / V)\, dt} = e^{(r / V) t}$

Multiplying through by $\mu(t)$:

$$e^{(r / V) t}\ dQ/dt + (r / V)\ e^{(r / V) t}\ Q = r\ 29\ e^{(r / V) t}$$

Recognizing the left side as the derivative of $Q e^{(r / V) t}$:

$$d/dt\ [Q e^{(r / V) t}] = r\ 29\ e^{(r / V) t}$$

Integrate both sides:

$$Q e^{(r / V) t} = \int r\ 29\ e^{(r / V) t}\, dt + C$$

Calculating the integral:

$$Q e^{(r / V) t} = 29\ V e^{(r / V) t} + C$$

Solve for $Q(t)$:

$Q(t) = 29 V + C e^{-(r / V) t}$

Applying the initial condition $Q(0) = 0$ (initially pure water):

$Q(0) = 0 = 29 V + C$

So,

$C = -29 V$

Thus, the solution:

$Q(t) = 29 V [1 - e^{-(r / V) t}]$

Interpreting the Results

Salt Quantity Over Time

The quantity of salt in the tank at any time t is:

$Q(t) = 29 V [1 - e^{-(r / V) t}]$

Given V = 800 liters:

$Q(t) = 29 800 [1 - e^{-(r / 800) t}] = 23,200 [1 - e^{-(r / 800) t}]$

Salt Concentration Over Time

The concentration in the tank at time t:

$C(t) = Q(t) / V = 29 [1 - e^{-(r / V) t}]$

This expression shows that the concentration approaches the inflow concentration (29 g/liter) exponentially as time progresses.

Practical Scenarios and Calculations

Example 1: Equal Inflow and Outflow Rate of 10 L/min

Suppose:

- $r_{in} = r_{out} = 10$ liters per minute
- V = 800 liters

Then,

$r / V = 10 / 800 = 0.0125 \text{ min}^{-1}$

The salt quantity over time:

$Q(t) = 23,200 [1 - e^{-0.0125 t}]$

The concentration:

$C(t) = 29 [1 - e^{-0.0125 t}]$

Key observations:

- After a long time ($t \to \infty$), $e^{-(r / V) t} \to 0$, so:

$Q(\infty) = 23,200$ grams

$C(\infty) = 29$ grams/liter

- At $t = 0$:

$Q(0) = 0$

$C(0) = 0$ (initially pure water)

- Time to reach 95% of the steady-state concentration:

Set $C(t) = 0.95\ 29 = 27.55$ grams/liter

Solve for t:

$0.95\ 29 = 29 [1 - e^{-(0.0125) t}]$

$\Rightarrow 0.95 = 1 - e^{-(0.0125) t}$

$\Rightarrow e^{-(0.0125) t} = 0.05$

$\Rightarrow -(0.0125) t = \ln(0.05)$

$\Rightarrow t = -\ln(0.05) / 0.0125 \approx 3.00 / 0.0125 \approx 240$ minutes

Interpretation: It takes approximately 4 hours to reach 95% of the steady-state concentration.

Example 2: Varying Inflow/Outflow Rates

If the flow rates differ, the model adjusts accordingly:

$Q(t) = (r_{in}\ 29\ V) / (r_{in} - r_{out}) [1 - e^{-(r_{in} - r_{out}) / V t}]$

This more complex formula applies when $r_{in} \neq r_{out}$. The analysis involves similar steps but requires careful attention to the signs and conditions to ensure the solution makes physical sense.

Additional Considerations

Volume Changes

If inflow and outflow rates are not equal, the volume $V(t)$ varies with time:

$$V(t) = V(0) + (r_{in} - r_{out}) t$$

This introduces additional complexity, as the differential equation becomes non-linear. In such cases, solving involves integrating with variable volume and concentration, often requiring numerical methods.

Steady-State Conditions

The system reaches a steady state when:

$$dQ/dt = 0$$

which occurs when:

$$Q_{ss} = 29 V$$

and the concentration at steady state:

$$C_{ss} = 29 \text{ grams/liter}$$

This is intuitive: the salt concentration in the tank matches the inflow concentration when the system stabilizes.

Practical Applications and Implications

Understanding the dynamics of salt concentration in a tank has broad applications:

- Water Treatment: Ensuring the final water meets quality standards by controlling inflow concentrations and flow rates.
- Chemical Manufacturing: Managing reactant concentrations during mixing processes.
- Environmental Monitoring: Predicting pollutant levels in bodies of water subject to inflow and outflow streams.
- Industrial Processes: Designing tanks and flow systems for optimal mixing and minimal lag times.

Summary and Key Takeaways

- The problem of a tank initially containing pure water being filled with a salt solution and having outflow can be modeled using first-order differential equations.
- The salt quantity and concentration evolve exponentially towards their steady-state values,

determined by inflow concentration and flow rates.
- The time to reach near steady

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