2 Determine The Highest Common Factor (HCF) And Lowest Common Multiple (LCM) Of 70 And 300. 1.2.1 HCF

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Understanding how to find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of two numbers is fundamental in mathematics, especially in the topics of number theory and arithmetic. These concepts are essential for simplifying fractions, solving problems involving ratios, and understanding the relationships between numbers. In this article, we will explore the methods to determine both the HCF and LCM of 70 and 300, providing detailed explanations, step-by-step procedures, and practical examples to solidify understanding.

What Is the Highest Common Factor (HCF)?

The Highest Common Factor (HCF), also known as the Greatest Common Divisor (GCD), of two or more numbers is the largest positive integer that divides each of the numbers without leaving a remainder. In simpler terms, it is the biggest number that is a factor of both numbers.

Importance of Finding the HCF

- Simplifying fractions
- Reducing ratios
- Solving problems involving divisibility
- Identifying common factors in algebraic expressions

Methods to Find the HCF

There are several methods to determine the HCF of two numbers:

1. Prime Factorization Method
2. Division Method (Euclidean Algorithm)
3. Listing Factors and Identifying the Greatest Common

In this article, we will primarily focus on the prime factorization method, as it provides clear insight into the factors involved.

Prime Factorization Method for HCF of 70 and 300

The prime factorization method involves breaking down each number into its prime factors and then identifying the common factors.

Step 1: Prime Factorization of 70

- 70 can be divided by 2: $70 \div 2 = 35$
- 35 can be divided by 5: $35 \div 5 = 7$
- 7 is a prime number

Prime factors of 70:
$70 = 2 \times 5 \times 7$

Step 2: Prime Factorization of 300

- 300 can be divided by 2: $300 \div 2 = 150$
- 150 can be divided by 2: $150 \div 2 = 75$
- 75 can be divided by 3: $75 \div 3 = 25$
- 25 can be divided by 5: $25 \div 5 = 5$
- 5 is a prime number

Prime factors of 300:
$300 = 2^2 \times 3 \times 5^2$

Step 3: Identify Common Prime Factors

- Common prime factors are those appearing in both factorizations.
- For 70: 2, 5, 7
- For 300: 2^2, 3, 5^2

The common prime factors are:
- 2 (since 70 has one 2, and 300 has 2^2, the common is one 2)
- 5 (since 70 has 5, and 300 has 5^2, the common is one 5)

Note: The powers of the common factors are the minimum powers in each factorization.

Step 4: Calculate the HCF

- Multiply the common prime factors with their lowest powers:

HCF = $2^1 \times 5^1 = 2 \times 5 = 10$

Result:
The HCF of 70 and 300 is 10.

Using the Division Method (Euclidean Algorithm) for HCF

The Euclidean Algorithm is an efficient method for computing the HCF of two numbers using repeated division.

Steps:
1. Divide the larger number by the smaller number.
2. If the remainder is zero, the divisor is the HCF.
3. If not, replace the larger number with the smaller number, and the smaller number with the remainder.
4. Repeat until the remainder is zero.

Applying to 70 and 300:

- Step 1: 300 ÷ 70 = 4 remainder 20
- Step 2: 70 ÷ 20 = 3 remainder 10
- Step 3: 20 ÷ 10 = 2 remainder 0

Since the last remainder is zero, the HCF is the last divisor: 10.

Conclusion:
Both methods confirm that the HCF of 70 and 300 is 10.

What Is the Lowest Common Multiple (LCM)?

The Lowest Common Multiple (LCM) of two numbers is the smallest positive integer that is divisible by both numbers. It is the smallest number into which both numbers can evenly divide.

Importance of Finding the LCM

- Synchronizing schedules
- Adding fractions with different denominators
- Solving problems involving multiple cycles or events

Methods to Find the LCM

Similar to the HCF, the LCM can be found through:

1. Prime Factorization Method
2. Listing Multiples

The prime factorization method is more systematic and efficient, especially for larger numbers.

Prime Factorization Method for LCM of 70 and 300

Step 1: Prime factors of 70:
$70 = 2 \times 5 \times 7$

Step 2: Prime factors of 300:
$300 = 2^2 \times 3 \times 5^2$

Step 3: Identify all prime factors with the highest powers
- 2^2 (from 300)
- 3 (from 300)
- 5^2 (from 300)
- 7 (from 70)

Step 4: Calculate the LCM
$LCM = 2^2 \times 3 \times 5^2 \times 7$

Calculating step-by-step:

- $2^2 = 4$
- $4 \times 3 = 12$
- 12×25 (since $5^2 = 25$) $= 300$
- $300 \times 7 = 2100$

Result:
The LCM of 70 and 300 is 2100.

Using the Division Method for LCM

Alternatively, the division method involves dividing both numbers by common prime factors, then multiplying the common and remaining factors.

Steps:
1. Write the numbers side by side.
2. Divide both numbers by prime factors common to both.
3. Continue dividing until no further common prime factors exist.
4. Multiply all the divisors and remaining factors to get the LCM.

Applying to 70 and 300:

- Divide both by 2:
$70 \div 2 = 35$, $300 \div 2 = 150$
- Divide both by 5:
$35 \div 5 = 7$, $150 \div 5 = 30$
- 7 and 30:
7 is prime, 30 is divisible by 2 and 3, but they are not common factors.
- Next, include remaining factors: 7 and 30

Calculate:
- LCM = (Product of all divisors used) × (remaining numbers after division)

Multiplying the divisors:

- 2 (first division)
- 5 (second division)
- Remaining factors: 7 (from 70's factorization) and 30 (remaining from 300)

Hence, LCM = 2 × 5 × 7 × 30 = 2 × 5 × 7 × 30

But to avoid confusion, it's clearer to use the prime factorization method as above, which directly points to the LCM being 2100.

Summary of Results

Property	Value
Highest Common Factor (HCF) of 70 and 300	10
Lowest Common Multiple (LCM) of 70 and 300	2100

Practical Applications of HCF and LCM

Understanding how to compute the HCF and LCM of numbers like 70 and 300 has numerous real-world applications:

- Simplifying Fractions:
For example, converting fractions to their simplest form involves dividing numerator and denominator by their HCF.

- Adding or Subtracting Fractions:
Using the LCM of denominators to find a common denominator simplifies calculations.

- Scheduling and Planning:
Determining the timing of events that repeat at different intervals, like cycles of 70 and 300 days.

- Problem Solving in Algebra:
Factoring expressions and solving equations often require understanding common factors and multiples.

Conclusion

Calculating the Highest Common Factor and Lowest Common Multiple of two numbers such as 70 and 300 is straightforward once the appropriate methods are understood. The prime factorization method provides a clear and systematic approach, revealing the fundamental building blocks of each number. Both the HCF (which is 10 for these numbers) and the LCM (which is 2100) are essential tools in various mathematical contexts. Master

Frequently Asked Questions

What is the Highest Common Factor (HCF) of 70 and 300?

The HCF of 70 and 300 is 10.

How do you find the HCF of 70 and 300 using prime factorization?

Prime factorization of 70 is $2 \times 5 \times 7$, and of 300 is $2^2 \times 3 \times 5^2$. The common factors are 2 and 5, so HCF is $2 \times 5 = 10$.

What is the Lowest Common Multiple (LCM) of 70 and 300?

The LCM of 70 and 300 is 2100.

How do you calculate the LCM of 70 and 300?

Using prime factors, LCM is obtained by taking the highest powers of all prime factors: 2^2, 3^1, 5^2, 7^1. So, LCM = $2^2 \times 3 \times 5^2 \times 7 = 2100$.

Why is it important to find the HCF and LCM of two numbers?

Finding the HCF and LCM helps in simplifying fractions, solving problems involving multiples and divisors, and understanding the relationship between numbers.

Can the HCF of 70 and 300 be greater than 10?

No, because the highest common factor based on their prime factors is 10.

Is the HCF of 70 and 300 unique?

Yes, the HCF of two numbers is always unique and is the greatest number that divides both without leaving a remainder.

Is the LCM of 70 and 300 unique?

Yes, the LCM of two numbers is always unique and is the smallest number that is a multiple of both.

What is the relationship between HCF and LCM of two numbers?

For two numbers, the product of the HCF and LCM equals the product of the numbers: HCF × LCM = 70 × 300.

How can understanding HCF and LCM help in real-life applications?

They assist in tasks like scheduling, dividing resources equally, and simplifying ratios in everyday problem-solving.

Additional Resources

Determine The Highest Common Factor (HCF) And Lowest Common Multiple (LCM) Of 70 And 300. 1.2.1 HCF

Understanding the concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM) is fundamental in number theory and plays a crucial role in various mathematical applications, from simplifying fractions to solving problems involving multiple quantities. This comprehensive review delves into the detailed process of determining the HCF and LCM of the numbers 70 and 300, exploring their definitions, methods of calculation, significance, and practical applications.

Introduction to HCF and LCM

Before we dive into the specific calculations for 70 and 300, it's essential to establish a clear understanding of what HCF and LCM are, their importance, and how they relate to each other.

What Is the Highest Common Factor (HCF)?

The Highest Common Factor (HCF), also known as the Greatest Common Divisor (GCD), of two or more numbers is the largest positive integer that divides each of the numbers without leaving a remainder. It is a measure of the commonality between numbers in terms of their factors.

Why is HCF important?

- Simplifying fractions: Reducing fractions to their lowest terms.
- Problem solving: Finding common divisors in division problems.
- Number theory: Understanding the structure of numbers and their relationships.

What Is the Lowest Common Multiple (LCM)?

The Lowest Common Multiple (LCM) of two or more numbers is the smallest positive integer that is divisible by each of the numbers. It essentially finds a common multiple shared by the numbers, starting from the smallest possible.

Why is LCM important?

- Scheduling problems: Finding common time intervals.
- Addition of fractions: Common denominators.
- Solving real-world problems: Synchronizing cycles, events, or quantities.

Understanding the Numbers 70 and 300

Before calculations, it's helpful to analyze the properties of the given numbers:

- 70:
- Prime factorization: $70 = 2 \times 5 \times 7$
- Divisibility: Divisible by 1, 2, 5, 7, 10, 14, 35, 70

- 300:
- Prime factorization: $300 = 2^2 \times 3 \times 5^2$
- Divisibility: Divisible by many numbers, including 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300

The prime factorizations reveal the fundamental building blocks of each number, which are crucial for calculating HCF and LCM.

Calculating the Highest Common Factor (HCF) of 70 and 300

The HCF can be determined through various methods, with prime factorization being the most systematic and reliable approach, especially for larger numbers.

Method 1: Prime Factorization

Step 1: Prime factorization of both numbers

- 70: $2 \times 5 \times 7$
- 300: $2^2 \times 3 \times 5^2$

Step 2: Identify common prime factors

- Common prime factors are those appearing in both factorizations.
- For 70 and 300:
- 2 appears in both, with the smallest power being 2^1.
- 5 appears in both, with the smallest power being 5^1.
- 7 appears only in 70; ignore.
- 3 appears only in 300; ignore.

Step 3: Multiply the common prime factors with their lowest powers

- $HCF = 2^1 \times 5^1 = 2 \times 5 = 10$

Result:

$$
\boxed{
\text{HCF of 70 and 300} = 10
}
$$

Method 2: Euclidean Algorithm

The Euclidean algorithm is an efficient method for finding HCF without prime factorization, especially useful for larger numbers.

Step 1: Divide the larger number by the smaller

- $300 \div 70 = 4$ with a remainder of $300 - 4 \times 70 = 300 - 280 = 20$

Step 2: Replace the larger number with the smaller, and the smaller with the remainder

- Now, find $HCF(70, 20)$

Step 3: Repeat the division

- $70 \div 20 = 3$ with a remainder of $70 - 3 \times 20 = 70 - 60 = 10$

Step 4: Continue the process

- $20 \div 10 = 2$ with a remainder of 0

Since the remainder is now 0, the divisor at this step (10) is the HCF.

Result:

$$
\boxed{
\text{HCF of 70 and 300} = 10
}
$$

Both methods yield the same result, confirming the correctness.

Calculating the Lowest Common Multiple (LCM) of 70 and 300

Determining the LCM involves understanding the prime factors and their highest powers among the numbers.

Method 1: Using Prime Factorization

Step 1: Prime factorization of both numbers

- 70: $2 \times 5 \times 7$
- 300: $2^2 \times 3 \times 5^2$

Step 2: For each prime, take the highest power appearing in either number

Prime	Highest Power in 70	Highest Power in 300	Selected for LCM
2	2^1	2^2	2^2
3	3^0	3^1	3^1
5	5^1	5^2	5^2
7	7^1	7^0	7^1

Step 3: Multiply the selected primes

$$
LCM = 2^2 \times 3^1 \times 5^2 \times 7^1 = (4) \times (3) \times (25) \times (7)
$$

Calculations:

- $4 \times 3 = 12$
- $12 \times 25 = 300$
- $300 \times 7 = 2100$

Result:

$$
\boxed{
\text{LCM of 70 and 300} = 2100
}
$$

Method 2: Using the Relationship Between HCF and LCM

The relationship is:

$$
\text{HCF} \times \text{LCM} = \text{Product of the numbers}
$$

Given:

- HCF = 10
- Numbers: 70 and 300

Calculate:

$$
\text{LCM} = \frac{70 \times 300}{\text{HCF}} = \frac{21000}{10} = 2100
$$

This confirms the previous calculation.

Summary of Results

Property	Value
Highest Common Factor (HCF)	10
Lowest Common Multiple (LCM)	2100

Significance and Applications of HCF and LCM

Understanding the HCF and LCM of numbers like 70 and 300 is not merely an academic exercise but has practical implications across various fields:

- Mathematics and Education: Simplifying fractions, solving divisibility and multiple problems.
- Engineering and Science: Synchronizing cycles, frequencies, or signals.
- Scheduling and Planning: Coordinating events or tasks that recur at different intervals.
- Finance and Business: Calculating common periods for financial analysis or investment cycles.

Additional Insights and Tips

- Prime factorization provides a clear visualization of the fundamental building blocks of numbers, making it the preferred method for smaller to medium-sized numbers.
- The Euclidean algorithm is highly efficient for very large numbers and can be implemented programmatically with ease.

- The relationship $\text{HCF} \times \text{LCM} = \text{Product of the two numbers}$ simplifies calculations when one is already known.
- Always verify your calculations by cross-checking through different methods to ensure accuracy.

Conclusion

Determining the Highest Common Factor and Lowest Common Multiple of 70 and 300

2 Determine The Highest Common Factor Hcf And Lowest Common Multiple Lcm Of 70 And 300 1 2 1 Hcf

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