2. The Profit When Selling 'X' Pairs Of Shoes Is Defined By The Function P(x)=-8x2 +80x-192. How Many

2. The Profit When Selling 'X' Pairs Of Shoes Is Defined By The Function P(x)=-8x2 +80x-192. How Many pairs of shoes should a seller aim to sell to maximize profit? Understanding the profit function and its implications is essential for any business owner or entrepreneur involved in selling shoes or similar products. In this article, we will analyze the quadratic profit function, interpret its key features, and determine the optimal number of units to sell for maximum profit.

Understanding the Profit Function $P(x) = -8x^2 + 80x - 192$

What Does the Function Represent?

The function $P(x) = -8x^2 + 80x - 192$ models the profit earned from selling x pairs of shoes. Here:
- x represents the number of pairs sold.
- P(x) indicates the profit associated with selling x pairs.

Since the quadratic coefficient is negative (-8), the parabola opens downward, meaning the function has a maximum point that indicates the maximum profit achievable.

Key Components of the Function

- Quadratic Term ($-8x^2$): Represents decreasing returns after a certain point, indicating that beyond some sales volume, profit begins to decline.
- Linear Term (80x): Represents the initial increase in profit per additional pair sold.
- Constant Term (-192): Represents fixed costs or baseline expenses that are incurred regardless of the number of shoes sold.

Finding the Number of Shoes to Maximize Profit

Vertex of the Parabola

The maximum profit occurs at the vertex of the parabola described by the quadratic function. The x-coordinate of the vertex can be found using the formula:

$x = -b / (2a)$

where:
- $a = -8$
- $b = 80$

Calculating:

$x = -80 / (2 -8) = -80 / -16 = 5$

Thus, selling 5 pairs of shoes maximizes profit.

Calculating the Maximum Profit

To find the maximum profit, substitute $x = 5$ back into the profit function:

$P(5) = -8(5)^2 + 80(5) - 192$
$= -8(25) + 400 - 192$
$= -200 + 400 - 192$
$= 200 - 192$
$= 8$

Therefore, the maximum profit that can be achieved is $8 when selling 5 pairs of shoes.

Interpreting the Results for Business Strategy

Implications of the Profit Function

- Selling fewer than 5 pairs results in less profit.
- Selling more than 5 pairs causes the profit to decrease due to the quadratic term dominating.
- The small maximum profit of $8 suggests that, at least according to this model, the profit margins are quite slim for the given costs and revenues.

Practical Considerations

While the mathematical model indicates that selling exactly 5 pairs maximizes profit, real-world scenarios should consider:
- Market demand fluctuations.
- Cost variations.
- Competition.
- Marketing efforts.

It's essential to use this analysis as a guide rather than an absolute rule.

Additional Insights and Applications

Understanding the Shape of the Profit Function

Since the parabola opens downward, the profit function is concave, meaning:
- There is a clear maximum point.
- Beyond the vertex, additional sales lead to decreasing profit.

This shape highlights the importance of identifying optimal sales levels.

Using the Function for Business Planning

- Pricing Strategy: Adjust pricing to influence the profit function's parameters.
- Sales Targets: Set realistic sales targets aligned with profit maximization.
- Cost Management: Explore ways to reduce fixed costs to shift the profit curve upward.

Conclusion: How Many Pairs of Shoes Should Be Sold?

Based on the quadratic profit function $P(x) = -8x^2 + 80x - 192$, the optimal number of pairs of shoes to sell is 5, which yields a maximum profit of $8. While this mathematical result provides a clear target, real-world factors should influence final business decisions. Understanding the profit function's behavior enables entrepreneurs to optimize sales strategies, manage costs effectively, and make data-driven decisions to maximize profitability.

Summary of Key Points

- The profit function is a downward-opening parabola, indicating a maximum point exists.
- The maximum profit occurs at $x = 5$ pairs of shoes.
- Maximum profit at this point is $8.
- Mathematical analysis guides business decisions, but real-world factors must also be considered.
- Understanding quadratic functions helps in strategic planning and maximizing profits.

By applying the principles of quadratic functions and profit analysis, business owners can make informed decisions that optimize sales and profitability, ensuring sustained success in competitive markets.

Frequently Asked Questions

What is the profit function when selling 'x' pairs of shoes?

The profit function is $P(x) = -8x^2 + 80x - 192$.

How do you find the number of pairs of shoes that maximizes profit?

To maximize profit, find the vertex of the parabola represented by $P(x)$. For $P(x) = -8x^2 + 80x - 192$, the x-coordinate of the vertex is $-b/(2a)$.

What is the value of 'x' that gives the maximum profit?

The maximum profit occurs at $x = -80 / (2 -8) = 5$ pairs of shoes.

What is the maximum profit when selling the optimal number of shoes?

Substitute $x=5$ into $P(x)$: $P(5) = -8(25) + 80(5) - 192 = -200 + 400 - 192 = 8$. So, the maximum profit is $8.

How many pairs of shoes should be sold to break even?

Set $P(x) = 0$ and solve for x: $-8x^2 + 80x - 192 = 0$.

What are the break-even points for the number of shoes sold?

Solve the quadratic: $-8x^2 + 80x - 192 = 0$. Dividing both sides by -8 gives $x^2 - 10x + 24 = 0$. Using quadratic formula: $x = [10 \pm \sqrt{(100 - 96)}] / 2 = [10 \pm \sqrt{4}]/2$.

What are the two break-even points?

x = [10 + 2]/2 = 6 and x = [10 - 2]/2 = 4. So, selling 4 or 6 pairs of shoes results in zero profit.

Is the profit function a parabola opening upwards or downwards?

Since the coefficient of x^2 is negative (-8), the parabola opens downward.

What does the vertex of the parabola represent in this context?

The vertex represents the number of shoes to be sold to achieve maximum profit, which is at x = 5 pairs.

How can this profit function be used to inform sales strategies?

By understanding the optimal number of pairs to sell (x=5), businesses can maximize profits, and knowing the break-even points helps set realistic sales targets.

Additional Resources

Understanding the Profit When Selling 'X' Pairs Of Shoes Is Defined By The Function $P(x) = -8x^2 + 80x - 192$

In the world of business and economics, understanding profit functions is essential for making informed decisions about production and sales strategies. The profit when selling 'X' pairs of shoes is defined by the function $P(x) = -8x^2 + 80x - 192$ offers a compelling example of how quadratic functions can model real-world scenarios. This particular function allows entrepreneurs and analysts to determine the optimal number of shoe pairs to sell to maximize profit, as well as to understand the profit implications of selling different quantities.

Introduction to the Profit Function

The function $P(x) = -8x^2 + 80x - 192$ is a quadratic equation, where:

- x represents the number of pairs of shoes sold.
- P(x) is the profit earned from selling x pairs.

Quadratic functions are characterized by their parabola shape, which can open either upwards or downwards depending on the sign of the quadratic coefficient (the term with x^2). In this case, the coefficient is -8, indicating the parabola opens downward, thus having a maximum point which corresponds to the maximum profit.

Why is the Profit Function Quadratic?

Quadratic functions are commonly used in profit analysis because they accurately reflect the relationship between sales volume and profit when there are diminishing returns or increasing costs at certain levels.

- Initial sales often lead to increased profit because fixed costs are spread over more units.
- After a certain point, increasing sales may lead to decreased profit due to factors such as increased variable costs, market saturation, or operational constraints.

In our case, the negative coefficient of x^2 indicates that after a certain point, selling more shoes will decrease overall profit, highlighting the importance of finding the optimal sales quantity.

Analyzing the Profit Function: Key Concepts

Before diving into calculations, it's important to understand some fundamental concepts:

- Vertex of the parabola: The highest or lowest point on the parabola, representing the maximum or minimum profit.
- Axis of symmetry: The vertical line passing through the vertex, indicating the value of x at which profit is maximized or minimized.
- Roots or zeros: Values of x where profit equals zero, which can indicate break-even points.

Step-by-Step Breakdown of the Profit Function

1. Identifying the Parabola's Properties

Given the quadratic function:

$$P(x) = -8x^2 + 80x - 192$$

- The a coefficient is -8 (parabola opens downward).
- The b coefficient is 80.
- The c coefficient is -192.

2. Finding the Vertex

The vertex of a parabola given by $ax^2 + bx + c$ can be found using the formula:

$$ x_{vertex} = -\frac{b}{2a} $$

Plugging in the values:

$$ x_{vertex} = -\frac{80}{2 \times -8} = -\frac{80}{-16} = 5 $$

This means the maximum profit occurs when 5 pairs of shoes are sold.

3. Calculating the Maximum Profit

To find the maximum profit, substitute $x = 5$ into the original function:

$$ P(5) = -8(5)^2 + 80(5) - 192 $$

Calculations:

$$ P(5) = -8(25) + 400 - 192 $$
$$ P(5) = -200 + 400 - 192 $$
$$ P(5) = 200 - 192 = 8 $$

Maximum profit is $8 (assuming the units are dollars), achieved by selling 5 pairs of shoes.

Interpreting the Results

- The profit function reaches its maximum at x = 5, meaning that selling 5 pairs of shoes yields the highest profit.
- The maximum profit is $8, which may seem low; this could be due to the specific coefficients in the function, representing costs and revenues.

Finding the Break-Even Points

Break-even points occur where profit equals zero:

$$ P(x) = 0 $$

Set the function equal to zero:

$$ -8x^2 + 80x - 192 = 0 $$

Multiply through by -1 to simplify:

$$8x^2 - 80x + 192 = 0$$

Divide through by 8:

$$x^2 - 10x + 24 = 0$$

Now, factor or use the quadratic formula:

Factoring:

Find two numbers that multiply to 24 and add to -10:

- Factors of 24: (1, 24), (2, 12), (3, 8), (4, 6)

Check sums:

- 4 + 6 = 10, but need -10, so factors are -4 and -6.

Write as:

$$(x - 4)(x - 6) = 0$$

Set each factor to zero:

$$x - 4 = 0 \Rightarrow x = 4$$
$$x - 6 = 0 \Rightarrow x = 6$$

Break-even points are at 4 and 6 pairs of shoes.

This indicates that selling fewer than 4 or more than 6 pairs results in a loss, while selling between 4 and 6 pairs results in profit.

Practical Implications for Shoe Business Owners

Understanding the profit function helps business owners make strategic decisions:

- Optimal Sales Level: Selling 5 pairs maximizes profit, but the actual profit is marginal ($8). This suggests limited profit margins at this level.
- Sales Targets: To be profitable, sales should be between 4 and 6 pairs.

- Pricing and Cost Strategies: If profit margins are small, adjusting prices or reducing costs could improve profitability.
- Production Planning: Knowing the maximum profit point guides production and inventory decisions to avoid overproduction that could lead to losses.

Additional Considerations

While the quadratic model provides valuable insights, real-world scenarios often involve additional factors:

- Market demand fluctuations
- Pricing strategies
- Operational constraints
- Competition

Therefore, this model should be used as a guide, complemented by market research and strategic planning.

Summary: Key Takeaways

- The profit when selling 'X' pairs of shoes is modeled by the quadratic function $P(x) = -8x^2 + 80x - 192$.
- The maximum profit occurs at $x = 5$, with a profit of $8.
- Break-even points are at $x = 4$ and $x = 6$.
- Selling between 4 and 6 pairs yields profit, while outside this range results in losses.
- The model emphasizes the importance of finding the optimal sales volume to maximize profit and minimize losses.

Final Thoughts

Quadratic functions like $P(x) = -8x^2 + 80x - 192$ are powerful tools for understanding and optimizing business profits. By analyzing the parabola's vertex and roots, business owners can make informed decisions about production levels, pricing, and sales strategies. While models simplify complex realities, they provide a crucial foundation for strategic planning and operational efficiency.

Remember: Always consider real-world factors and market conditions alongside mathematical models to achieve the best business outcomes.

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