2. What Is The Equation Of A Circle With Center (0, 4) And Point (-1, 8) On The Circle?A X2 + (- 4)2

Understanding the Equation of a Circle: Center at (0, 4) and a Point at (-1, 8)

2. What Is The Equation Of A Circle With Center (0, 4) And Point (-1, 8) On The Circle?A X2 + (- 4)2 might seem like a confusing statement at first glance, but it actually introduces a fundamental concept in coordinate geometry—the equation of a circle given its center and a point on its circumference. In this article, we'll explore how to derive the equation of a circle with a specific center at (0, 4) and a point at (-1, 8) lying on the circle. We'll break down the process step-by-step, clarify the formula involved, and provide practical examples to deepen your understanding of circle equations.

What Is the Standard Equation of a Circle?

Before delving into the specific problem, it's essential to understand the general form of a circle's equation. The standard (or equation) of a circle with center at point (h, k) and radius r is expressed as:

- **$(x - h)^2 + (y - k)^2 = r^2$**

In this formula:
- (h, k) represents the center of the circle.
- r is the radius, the distance from the center to any point on the circle.

Understanding this form is crucial because, given the center and a point on the circle, calculating the radius allows us to write the complete equation.

Determining the Radius Using a Given Point

Given the center at (0, 4) and a point (-1, 8) on the circle, our goal is to find the radius r. This is achieved using the distance formula, which calculates the distance between two points in the coordinate plane:

- **Distance formula:**
  $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Applying this to our points:
- Center: (0, 4)
- Point on circle: (-1, 8)

The steps are:

1. Subtract the x-coordinates: (-1) - 0 = -1
2. Subtract the y-coordinates: 8 - 4 = 4
3. Square both differences: $(-1)^2 = 1$, $4^2 = 16$
4. Sum the squares: 1 + 16 = 17
5. Take the square root: $\sqrt{17} \approx 4.1231$

Thus, the radius r is $\sqrt{17}$, approximately 4.1231 units.

Formulating the Equation of the Circle

Now that we know the center (h, k) = (0, 4) and the radius $r = \sqrt{17}$, we can write the equation:

$$(x - 0)^2 + (y - 4)^2 = (\sqrt{17})^2$$

Simplify:

$$x^2 + (y - 4)^2 = 17$$

This is the standard form of the circle's equation with the given center and point.

Expanding and Simplifying the Equation

For clarity or particular applications, you might want to expand the equation:

1. Distribute the squared term: $(y - 4)^2 = y^2 - 8y + 16$
2. Combine everything: $x^2 + y^2 - 8y + 16 = 17$
3. Bring all to one side: $x^2 + y^2 - 8y + 16 - 17 = 0$
4. Simplify: $x^2 + y^2 - 8y - 1 = 0$

This form is useful for graphing or further algebraic analysis.

Common Mistakes and Clarifications

When working with circle equations, beginners often make errors such as:

- Confusing the sign in the standard form: Remember, it's $(x - h)^2$, not $(x + h)^2$, unless h is negative.
- Forgetting to square the radius: The radius squared appears on the right side of the equation.
- Miscalculating the distance: Always double-check the subtraction and squaring steps.

Additionally, the initial statement referencing "A X2 + (- 4)2" appears to be an incomplete or miswritten part of the original problem. The correct approach involves calculating the radius as shown above.

Practical Applications of Circle Equations

Understanding how to derive the equation of a circle has numerous real-world applications, including:

1. **Engineering and Design:** Designing circular components or structures.

2. **Navigation and GPS:** Calculating distances between points to determine circular zones.
3. **Computer Graphics:** Rendering circles and arcs accurately.
4. **Physics:** Modeling circular motion or fields.

Knowing how to determine the circle's equation facilitates these applications and enhances problem-solving skills in mathematics and related fields.

Additional Tips for Solving Circle Equations

To master the process of deriving circle equations, consider these tips:

- Always identify the center and a point on the circle first.
- Use the distance formula to find the radius accurately.
- Write the standard form directly after calculating the radius.
- If needed, expand or simplify the equation for specific purposes.

Practice with different centers and points to strengthen your understanding.

Summary: Deriving the Equation of a Circle with Given Center and Point

In summary:

- The standard form of a circle's equation is $(x - h)^2 + (y - k)^2 = r^2$.
- Given the center (0, 4) and a point (-1, 8), calculate the radius using the distance formula:
- Distance $r = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]} = \sqrt{17}$.

- Plug the radius and center into the standard form to get the complete equation: $x^2 + (y - 4)^2 = 17$.

Mastering this process is fundamental for solving many geometric problems involving circles.

Conclusion

Understanding the derivation of a circle's equation from its center and a point on the circle is a vital skill in coordinate geometry. By applying the distance formula and the standard form, you can efficiently determine the equation of any circle given its center and a point on its circumference. Remember to double-check your calculations and practice with different scenarios to strengthen your grasp of this essential mathematical concept. Whether for academic purposes or real-world applications, knowing how to find and manipulate circle equations opens doors to numerous analytical and design opportunities.

Frequently Asked Questions

What is the general form of the equation of a circle with a given center and a point on the circle?

The general form is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

How do I find the radius of a circle given its center and a point on the circle?

Calculate the distance between the center and the point using the distance formula: $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

What are the steps to determine the equation of a circle with center (0, 4) and passing through (-1, 8)?

First, find the radius using the distance formula between (0,4) and (-1,8), then substitute the center and radius into the standard circle equation.

How do I compute the radius for the circle with center (0, 4) and point (-1, 8)?

Calculate $r = \sqrt{(-1 - 0)^2 + (8 - 4)^2} = \sqrt{1^2 + 4^2} = \sqrt{1 + 16} = \sqrt{17}$.

What is the equation of the circle with center (0, 4) passing through (-1, 8)?

The equation is $x^2 + (y - 4)^2 = (\sqrt{17})^2$, which simplifies to $x^2 + (y - 4)^2 = 17$.

Why is the equation of the circle written as $x^2 + (y - 4)^2 = 17$?

Because the center is at (0, 4) and the radius squared is 17, so the standard form becomes $x^2 + (y - 4)^2 = 17$.

Can the equation $x^2 + (-4)^2$ be used directly to find the circle's equation?

No, because that form is incomplete; you need the full standard form and the radius squared, which is found from the distance between the center and the point.

What is the significance of the point (−1, 8) in determining the circle's equation?

It confirms the circle passes through that point, allowing you to calculate the radius and complete the equation.

Is the equation of the circle symmetric about the center (0, 4)?

Yes, the standard form equation $x^2 + (y - 4)^2 = 17$ is symmetric with respect to the center, reflecting the circle's symmetry.

How can I verify that the point (−1, 8) lies on the circle with equation $x^2 + (y - 4)^2 = 17$?

Substitute $x = -1$ and $y = 8$ into the equation: $(-1)^2 + (8 - 4)^2 = 1 + 16 = 17$, which matches the radius squared, confirming the point lies on the circle.

Additional Resources

Equation of a Circle with Center (0, 4) and Point (-1, 8) on the Circle is a fundamental topic in coordinate geometry that connects the concepts of the circle's center, a point on the circle, and the algebraic representation of the circle's equation. Understanding how to derive the equation of a circle given its center and a point on the circle is essential for students and professionals working in mathematics, engineering, and related fields. This article offers a comprehensive overview of this process, breaking down each step systematically, and exploring the underlying concepts, formulas, and applications involved.

Introduction to the Equation of a Circle

A circle in the Cartesian coordinate plane is defined as the set of all points that are equidistant from a fixed point called the center. The standard form of the equation of a circle with center (h, k) and radius r is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Here, (h, k) represents the circle's center, and r is the radius. To find the specific equation of a circle given certain conditions, one must determine the radius and substitute the center coordinates into the standard form.

Understanding the Given Data

In the problem at hand, the circle's center is at $(0, 4)$, and a point $(-1, 8)$ lies on the circle. The goal is to find the explicit equation of this circle.

Key points:

- Center: $(h, k) = (0, 4)$
- Point on circle: $(x_1, y_1) = (-1, 8)$

The critical step is to determine the radius r, which can be calculated using the distance formula between the center and the given point.

Calculating the Radius

The radius r is the distance between the center $(0, 4)$ and the point $(-1, 8)$. The distance formula in coordinate geometry is:

$$
r = \sqrt{(x_1 - h)^2 + (y_1 - k)^2}
$$

Substituting the known values:

$$
r = \sqrt{(-1 - 0)^2 + (8 - 4)^2} = \sqrt{(-1)^2 + (4)^2} = \sqrt{1 + 16} = \sqrt{17}
$$

Thus, the radius of the circle is $\sqrt{17}$.

Note: The radius is always positive, so we consider only the positive square root.

Formulating the Equation of the Circle

Now that we have the center $(0, 4)$ and radius $r = \sqrt{17}$, we can write the equation in standard form:

$$
(x - 0)^2 + (y - 4)^2 = (\sqrt{17})^2
$$

Simplifying:

$$
x^2 + (y - 4)^2 = 17
$$

This is the explicit equation of the circle passing through $(-1, 8)$ with the given center.

Expanding the Equation (Optional)

For many applications, it might be useful to expand the equation into the general quadratic form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

Starting from:

$$
x^2 + (y - 4)^2 = 17

\]

Expand \((y - 4)^2\):

\[
x^2 + y^2 - 8y + 16 = 17
\]

Bring all terms to one side:

\[
x^2 + y^2 - 8y + 16 - 17 = 0
\]

Simplify:

\[
x^2 + y^2 - 8y - 1 = 0
\]

This is the expanded, general quadratic form of the circle's equation.

Features, Pros, and Cons of the Equation of a Circle

Features:

- The standard form explicitly shows the center and radius, making it easy to identify these elements.
- The equation can be easily manipulated to find intersections, tangent points, and other geometric properties.
- The geometric interpretation is straightforward: the set of all points \((x, y)\) satisfying the equation.

Pros:

- Clear and concise representation of the circle's geometry.
- Facilitates calculation of the radius and center directly.
- Useful for graphing and analyzing the circle analytically.

Cons:

- The standard form isn't always the most convenient for certain types of algebraic manipulations.
- When expanded, the equation may become cumbersome, especially for complex circles.

- Requires familiarity with algebraic expansion and completing the square if converting from general to standard form.

Additional Considerations and Applications

- Graphing: The standard form makes graphing straightforward; plotting the center and drawing a circle with radius $\sqrt{17}$ is simple.
- Intersection problems: Equations in this form are useful when solving systems involving circles and other curves.
- Real-world applications: From designing circular components in engineering to calculating coverage areas in telecommunications, understanding circle equations is vital.

Example: Verifying the Point on the Circle

To confirm that $(-1, 8)$ lies on the circle, substitute into the equation:

$$
x^2 + (y - 4)^2 = 17
$$

$$
(-1)^2 + (8 - 4)^2 = 1 + 16 = 17
$$

Since the equality holds, the point indeed lies on the circle.

Conclusion

The process of deriving the equation of a circle given its center and a point on the circle is a fundamental skill in coordinate geometry. By calculating the radius via the distance formula and then substituting into the standard form, one obtains a precise algebraic representation of the circle. The equation:

$$
x^2 + (y - 4)^2 = 17
$$

encapsulates all points (x, y) that are equidistant from the center $(0, 4)$ at a distance $\sqrt{17}$. Understanding this derivation not only reinforces geometric intuition but also provides tools for solving a wide range of mathematical problems involving circles.

Summary:

- The key step is calculating the radius from the center to the known point.
- The standard form is the most straightforward way to express the circle's equation.
- Converting to expanded form offers insights into the circle's algebraic properties.
- The approach is applicable in theoretical and practical contexts, from academic exercises to engineering designs.

This comprehensive understanding of the equation of a circle with given parameters enhances one's ability to analyze, graph, and utilize circles effectively across various mathematical and scientific applications.

2 What Is The Equation Of A Circle With Center 0 4 And Point 1 8 On The Circle A X2 4 2

2 What Is The Equation Of A Circle With Center 0 4 And Point 1 8 On The Circle A X2 4 2

Related Articles

- A 1.80-m Tall Person Lifts A 2.50-kg Book From The Ground So It Is 2.50 M Above The Ground.Part AWhat
- A Bag Contains Chips Of Which 27. 5 Percent Are Blue. A Random Sample Of 5 Chips Will Be Selected One
- Why Did President Carter Create The Department Of Energy? To Overturn The 1973 Oil Embargo To Reduce

Back to Home