4.Find The Equation For The Plane Through The Point Po=(2,6,5) And Normal To The Vector N=4i + 7j+2k.

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Introduction to Plane Equations in 3D Space

Understanding the equation of a plane in three-dimensional space is fundamental in geometry, vector calculus, and numerous applications in engineering and physics. A plane can be uniquely defined by a point it passes through and a normal vector perpendicular to its surface. This article provides a comprehensive guide to finding the equation of a plane given a specific point and a normal vector, with a focus on the example where the point is Po = (2, 6, 5), and the normal vector is N = 4i + 7j + 2k.

Understanding the Components of the Problem

The Given Point Po

- Po = (2, 6, 5)

- Coordinates: $x = 2$, $y = 6$, $z = 5$
- This point lies somewhere on the plane, serving as a reference point for the plane's position in space.

The Normal Vector N

- $N = 4i + 7j + 2k$
- Components: (4, 7, 2)
- The normal vector is perpendicular to the plane's surface, indicating the plane's orientation in space.

Objective

- To derive the algebraic equation of the plane that passes through Po and is perpendicular to N.

Mathematical Foundations for Plane Equations

The Vector Equation of a Plane

The general form of a plane's equation in three-dimensional space can be expressed using a point-normal form:

$$\text{(Point)} \Rightarrow \text{Po} = (x_0, y_0, z_0)$$

$$\text{Normal Vector} \Rightarrow \vec{N} = (A, B, C)$$

$$\text{Plane Equation} \Rightarrow A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

This equation states that any point (x, y, z) on the plane satisfies the linear equation derived from the dot product of the normal vector and the vector from the point Po to (x, y, z).

Why the Normal Vector Is Essential

- It defines the orientation of the plane.
- It is perpendicular to every line lying on the plane.
- It simplifies the derivation of the plane's equation once the point and normal vector are known.

Step-by-Step Solution for the Given Problem

Step 1: Write Down the Known Values

- Point Po: (2, 6, 5)
- Normal vector N: (4, 7, 2)

Step 2: Apply the Point-Normal Form of the Plane Equation

Using the formula:

$$ A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 $$

Plugging in the known values:

$$ 4(x - 2) + 7(y - 6) + 2(z - 5) = 0 $$

Step 3: Expand and Simplify

- Expand each term:

$$ 4x - 8 + 7y - 42 + 2z - 10 = 0 $$

- Combine like terms:

$$ 4x + 7y + 2z - (8 + 42 + 10) = 0 $$

$$ 4x + 7y + 2z - 60 = 0 $$

Step 4: Write the Final Equation

The standard form of the plane's equation:

$$ 4x + 7y + 2z = 60 $$

This is the explicit algebraic equation of the plane passing through the point Po and perpendicular to the vector N.

Interpreting the Equation of the Plane

Geometric Significance

- The coefficients (4, 7, 2) directly relate to the orientation of the plane.
- The constant (60) determines the plane's position relative to the origin.

Applications of the Plane Equation

- Calculating distances from points to the plane.
- Finding intersections with other geometric objects.
- Designing structures and analyzing spatial relationships in 3D space.

Additional Considerations and Variations

Normal Vector in Different Forms

- The normal vector can be scaled by any non-zero scalar; the plane's equation remains the same after dividing through by that scalar.
- For example, dividing the entire equation by 2 gives:

$$2x + \frac{7}{2} y + z = 30$$

- The normal vector in this case becomes (2, 3.5, 1), which is parallel to the original vector.

Alternate Forms of the Plane Equation

- Parametric Form: Expressed using two parameters u and v, suitable for plotting or understanding the plane's surface.
- Vector Form: Emphasizes the point-normal relationship and can incorporate vector notation directly.

Visualizing the Plane in 3D Space

While visualization can be challenging without graphical tools, understanding the orientation of the plane can be approached by:

- Considering the normal vector's direction.
- Plotting the point Po.
- Recognizing that the plane extends infinitely in directions orthogonal to N, passing through Po.

Practical Examples and Applications

Example 1: Calculating the Distance from a Point to the Plane

Suppose you have another point P1 = (x1, y1, z1). The shortest distance d from P1 to the plane is:

$$ d = \frac{|A x_1 + B y_1 + C z_1 - D|}{\sqrt{A^2 + B^2 + C^2}} $$

where D is the constant term (here, D=60).

Example 2: Finding Intersecting Lines

- The intersection of the plane with other surfaces, such as lines or other planes, can be analyzed using the derived equation.
- For example, solving for the intersection with a given line involves setting parametric equations into the plane equation.

Application in Engineering and Physics

- Designing mechanical components where surfaces must meet at specific orientations.
- Analyzing forces acting on surfaces in physics.
- Geographic Information Systems (GIS) modeling terrain surfaces.

Summary and Key Takeaways

- The equation of a plane passing through a specific point and perpendicular to a given vector can be directly obtained using the point-normal form.
- For the example Po = (2, 6, 5) and N = (4, 7, 2), the plane's equation is:

$$4x + 7y + 2z = 60$$

- Understanding the derivation process enhances spatial reasoning and mathematical problem-solving skills.

- The same methodology applies to any point and normal vector in 3D space, making it a versatile tool in geometry.

Conclusion

Finding the equation of a plane in three-dimensional space is a fundamental skill in mathematics and related fields. By leveraging the point-normal form, you can quickly derive the plane's equation given any point and normal vector. In this specific case, the plane passing through Po = (2, 6, 5) and perpendicular to N = (4, 7, 2) is given by:

$$\boxed{4x + 7y + 2z = 60}$$

This equation encapsulates the plane's position and orientation, serving as a foundation for further geometric analysis, computations, and practical applications across various disciplines.

Keywords: plane equation, point-normal form, vector calculus, 3D geometry, normal vector, algebraic plane equation, spatial reasoning, geometry applications

Frequently Asked Questions

What is the general form of the equation for a plane given a point and a normal vector?

The equation of a plane can be written as $n \cdot (r - r0) = 0$, where n is the normal vector, r is the position vector (x, y, z), and r0 is a point on the plane.

How do you find the equation of a plane passing through point Po = (2, 6, 5) with normal vector N = 4i + 7j + 2k?

Use the formula: $4(x - 2) + 7(y - 6) + 2(z - 5) = 0$, then expand and simplify to get the equation of the plane.

What is the step-by-step process to derive the plane's equation from the given point and normal vector?

First, write the point-normal form: $4(x - 2) + 7(y - 6) + 2(z - 5) = 0$. Then expand: $4x - 8 + 7y - 42 + 2z - 10 = 0$. Combine like terms: $4x + 7y + 2z - 60 = 0$.

What is the final equation of the plane passing through (2,6,5) with normal vector (4,7,2)?

The equation is $4x + 7y + 2z = 60$.

How can I verify that a point lies on the plane defined by 4x + 7y + 2z = 60?

Substitute the point's coordinates into the equation: $4(2) + 7(6) + 2(5) = 8 + 42 + 10 = 60$. Since the sum equals 60, the point lies on the plane.

What are common mistakes to avoid when deriving the plane equation from a point and normal vector?

Common mistakes include forgetting to subtract the point coordinates in the formula, miscalculating the expansion, or mixing signs. Double-check each step and verify by substituting the point back into the equation.

Can the equation of the plane be written in a different form?

Yes, the plane's equation can also be expressed in standard form $Ax + By + Cz + D = 0$, where D is calculated as $-(A x_0 + B y_0 + C z_0)$. In this case, it is $4x + 7y + 2z - 60 = 0$.

How is the normal vector related to the orientation of the plane?

The normal vector is perpendicular to the plane's surface. Its components determine the tilt or orientation of the plane in three-dimensional space.

What applications might require finding the equation of a plane given a point and a normal vector?

Applications include computer graphics, engineering design, physics simulations, navigation systems, and any field involving spatial analysis or modeling of surfaces.

Additional Resources

4. Find The Equation For The Plane Through The Point Po=(2,6,5) And Normal To The Vector N=4i + 7j + 2k

Understanding the geometric and algebraic representation of planes in three-dimensional space is fundamental in various fields, from engineering and computer graphics to physics and mathematics. One of the core tasks involves deriving the equation of a plane given specific conditions—namely, a point through which the plane passes and a vector perpendicular to it, called the normal vector. In this article, we will explore how to find the equation of a plane that passes through a point $P_0 = (2, 6, 5)$ and is perpendicular to the vector $\mathbf{N} = 4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$. We will delve into the theoretical background, step-by-step derivation, and practical applications of this process.

Understanding the Geometric Foundations

Before diving into the calculations, it’s essential to grasp the underlying concepts that define a plane in three-dimensional space.

The Equation of a Plane

A plane in 3D space can be represented mathematically by an equation of the form:

$$ ax + by + cz + d = 0 $$

where (a, b, c) are the coefficients that determine the orientation of the plane, and d is the scalar that shifts the plane relative to the origin.

The Normal Vector

The vector $\mathbf{N} = 4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ is the normal vector to the plane. The normal vector is perpendicular to every line lying within the plane, which makes it a crucial component in deriving the plane’s equation.

The Role of a Point on the Plane

Given a point $P_0 = (x_0, y_0, z_0)$ through which the plane passes, we can use it to determine the specific plane associated with a given normal vector.

Mathematical Framework for Deriving the Plane Equation

The general approach involves leveraging the dot product between the normal vector and any vector lying on the plane.

Step 1: Understanding the Point-Normal Form

The equation of a plane can be expressed using the point-normal form:

$$ \mathbf{N} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 $$

where:

- $\mathbf{r} = (x, y, z)$ is any arbitrary point on the plane,
- $\mathbf{r}_0 = (x_0, y_0, z_0)$ is a specific point on the plane (P_0),
- $\mathbf{N}$ is the normal vector.

This form states that the vector connecting P_0 to any point $P = (x, y, z)$ on the plane is orthogonal (perpendicular) to $\mathbf{N}$.

Step 2: Applying the Dot Product

Expressing the dot product explicitly:

$$ \mathbf{N} \cdot (\mathbf{r} - \mathbf{r}_0) = (4, 7, 2) \cdot (x - 2, y - 6, z - 5) = 0 $$

which simplifies to:

$$ 4(x - 2) + 7(y - 6) + 2(z - 5) = 0 $$

This is the core equation of the plane.

Step-by-Step Calculation

Now, let's methodically derive the explicit equation of the plane based on the given data.

Step 1: Write the Point and Normal Vector

- Point $P_0 = (2, 6, 5)$
- Normal vector $\mathbf{N} = (4, 7, 2)$

Step 2: Use the Point-Normal Form

Substitute into the formula:

$$4(x - 2) + 7(y - 6) + 2(z - 5) = 0$$

Step 3: Expand the Equation

Carrying out the distribution:

$$4x - 8 + 7y - 42 + 2z - 10 = 0$$

Combine like terms:

$$4x + 7y + 2z - (8 + 42 + 10) = 0$$

$$4x + 7y + 2z - 60 = 0$$

Step 4: Write in Standard Form

Thus, the equation of the plane is:

$$4x + 7y + 2z = 60$$

Alternatively, it can be written as:

$$4x + 7y + 2z - 60 = 0$$

This form clearly shows the coefficients corresponding to the normal vector and the constant term.

Practical Implications and Applications

The derived equation has numerous applications across different disciplines:

- In Engineering: Designing structures where components must be aligned or oriented in specific ways.
- In Computer Graphics: Calculating shading, reflections, and rendering scenes with accurate plane surfaces.
- In Physics: Analyzing fields and forces that are constrained to specific planes.
- In Geology: Modeling geological layers or fault planes based on specific data points and orientations.

Knowing how to derive the equation of a plane from a point and a normal vector enables professionals to model complex systems with precision.

Advanced Considerations and Variations

While the above method applies when the normal vector and a point are known, real-world scenarios may involve additional complexities:

- Multiple Points: Sometimes, a plane is defined by three non-collinear points, requiring a different approach involving cross products.
- Normal Vector Unknown: When the plane passes through a point but the normal vector is not given, other methods such as regression analysis or intersecting planes may be used.
- Finding the Normal Vector: In some cases, the normal vector is derived from other data, such as the

cross product of two vectors lying on the plane.

In such cases, a solid understanding of vector operations, especially dot and cross products, becomes invaluable.

Final Thoughts

Finding the equation of a plane through a specific point and perpendicular to a given vector is a fundamental skill in analytical geometry. The process hinges on understanding the geometric significance of the normal vector and leveraging the point-normal form of the plane equation.

By carefully applying the dot product and simplifying algebraic expressions, we derive a clear, concise equation that encapsulates the plane’s orientation and position in space. This equation not only serves as a theoretical construct but also as a practical tool across sciences and engineering disciplines, enabling precise modeling and problem-solving in three-dimensional environments.

Whether you’re designing a complex structure, analyzing physical phenomena, or creating realistic computer-generated imagery, mastering this fundamental concept opens the door to a deeper understanding of the spatial relationships that shape our world.

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