4. Find The Intersection (if Any) Of The Lines 7 =(4,-2,-1)+t(1,4,-3) And F = (-8,20,15)+u(-3,2,5). 5

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Understanding how to determine the intersection point of two lines in three-dimensional space is a fundamental concept in vector algebra and analytical geometry. When given parametric equations of lines, the goal is to find if there exists a common point that satisfies both equations simultaneously. If such a point exists, the lines intersect at that point; if not, they are skew or parallel without intersection. This article guides you through the step-by-step process of finding the intersection of the lines:

- Line 7: $\mathbf{r}_1 = (4, -2, -1) + t(1, 4, -3)$
- Line F: $\mathbf{r}_2 = (-8, 20, 15) + u(-3, 2, 5)$

Let's explore how to determine whether these lines intersect and, if they do, find the intersection point.

Understanding Parametric Equations of Lines in 3D

Before diving into calculations, it’s essential to comprehend the structure of parametric equations of lines in space.

General Form of a Line in 3D

A line in three-dimensional space can be represented parametrically as:

$$
\mathbf{r} = \mathbf{p} + t \mathbf{d}
$$

where:

- $\mathbf{p}$ is a point on the line (position vector).
- $\mathbf{d}$ is the direction vector.
- t is a real parameter.

In our case:

- Line 7: $\mathbf{r}_1 = (4, -2, -1) + t(1, 4, -3)$
- Line F: $\mathbf{r}_2 = (-8, 20, 15) + u(-3, 2, 5)$

The objective is to find values of \(t \) and \(u \) such that \(\mathbf{r}_1 = \mathbf{r}_2 \).

Setting Up the System of Equations

To find intersection points, equate the parametric equations component-wise:

\[
\begin{cases}
4 + t(1) = -8 + u(-3) \quad \text{(x-component)} \\
-2 + t(4) = 20 + u(2) \quad \text{(y-component)} \\
-1 + t(-3) = 15 + u(5) \quad \text{(z-component)}
\end{cases}
\]

Simplifying each:

1. \(4 + t = -8 - 3u \)
2. \(-2 + 4t = 20 + 2u \)
3. \(-1 - 3t = 15 + 5u \)

Our goal is to solve this system for \(t \) and \(u \).

Solving the System of Equations

Let's proceed step-by-step.

Equation 1:

\[
4 + t = -8 - 3u
\]

Rearranged as:

\[
t + 3u = -12 \quad \text{(Equation A)}
\]

Equation 2:

\[

$$
-2 + 4t = 20 + 2u
$$

Rearranged as:

$$
4t - 2u = 22 \quad \text{(Equation B)}
$$

Equation 3:

$$
-1 - 3t = 15 + 5u
$$

Rearranged as:

$$
-3t - 5u = 16 \quad \text{(Equation C)}
$$

Now, we have a system of three equations:

$$
\begin{cases}
t + 3u = -12 \quad (A) \\
4t - 2u = 22 \quad (B) \\
-3t - 5u = 16 \quad (C)
\end{cases}
$$

Solving for t and u

Let's use the equations (A) and (B) to find t and u.

Step 1: Express t from Equation (A):

$$
t = -12 - 3u
$$

Step 2: Substitute into Equation (B):

$$
4(-12 - 3u) - 2u = 22
$$

Simplify:

$$
-48 - 12u - 2u = 22
$$

$$
-48 - 14u = 22
$$

Add 48 to both sides:

$$
-14u = 70
$$

Solve for u:

$$
u = -5
$$

Step 3: Find t:

$$
t = -12 - 3(-5) = -12 + 15 = 3
$$

Step 4: Verify these values satisfy Equation (C):

$$
-3t - 5u = 16
$$

Plugging in $t=3$, $u=-5$:

$$
-3(3) - 5(-5) = -9 + 25 = 16
$$

Indeed, the equation is satisfied.

Conclusion:

$$
t = 3, \quad u = -5
$$

Finding the Intersection Point

Now, substitute $t=3$ into the parametric equation of Line 7:

$$
\mathbf{r}_1 = (4, -2, -1) + 3(1, 4, -3)
$$

Calculate:

$$
\begin{cases}
x = 4 + 3(1) = 4 + 3 = 7 \\
y = -2 + 3(4) = -2 + 12 = 10 \\
z = -1 + 3(-3) = -1 - 9 = -10
\end{cases}
$$

Similarly, verify with Line F using $u = -5$:

$$
\mathbf{r}_2 = (-8, 20, 15) + (-5)(-3, 2, 5)
$$

Calculate:

$$
\begin{cases}
x = -8 + (-5)(-3) = -8 + 15 = 7 \\
y = 20 + (-5)(2) = 20 - 10 = 10 \\
z = 15 + (-5)(5) = 15 - 25 = -10
\end{cases}
$$

Both equations yield the point $(7, 10, -10)$.

Final answer:

The lines intersect at the point $(7, 10, -10)$.

Implications and Geometric Interpretation

Finding the intersection point confirms that the two lines are coplanar and intersect at a single point in 3D space. This is significant in various applications such as computer graphics, physics, and engineering, where understanding the spatial relationships between lines is crucial.

Key Takeaways

- To determine if two lines in 3D intersect, set their parametric equations equal and solve for parameters.
- The existence of a consistent solution indicates an intersection point.
- If no solution exists, the lines are either parallel without intersection or skew (neither parallel nor intersecting).

Summary of the Step-by-Step Process

1. Write the parametric equations of the lines.
2. Equate the components to set up a system of equations.
3. Simplify and solve the system for the parameters.
4. Substitute parameters back into the original equations to find the intersection point.
5. Verify the solution by plugging back into both equations.

Applications of Line Intersection in Real-World Scenarios

Understanding how to find intersections of lines in 3D space has numerous practical applications:

- Computer Graphics: Determining where objects intersect or collide.
- Navigation and Robotics: Path planning to avoid obstacles.
- Structural Engineering: Finding points of connection between different structural elements.
- Physics: Analyzing trajectories and collision points.

In conclusion, the process of finding the intersection of two lines in three-dimensional space involves translating their parametric forms into a solvable system of equations, solving for parameters, and verifying the solution. In the example provided, the lines intersect at the point \((7, 10, -10) \), illustrating a typical procedure applicable across various disciplines involving spatial analysis.

Frequently Asked Questions

How do you determine if two lines in 3D space intersect?

To determine if two lines intersect in 3D, you set their parametric equations equal to each other and solve for the parameters. If a consistent solution exists for both parameters, the lines intersect at that point.

What are the parametric equations of the given lines?

Line 7 is given by: r = (4, -2, -1) + t(1, 4, -3). Line F is given by: r = (-8, 20, 15) + u(-3, 2, 5).

How do you set up equations to find intersection points of these two lines?

You equate the x, y, and z components of both lines: 4 + t = -8 - 3u, -2 + 4t = 20 + 2u, -1 - 3t = 15 + 5u, then solve for t and u.

What is the process to solve for the parameters t and u in these lines?

Solve the system of equations derived from component-wise comparisons. Typically, choose two equations to solve for one variable, then substitute into the third to verify consistency.

Do these lines intersect, and if so, at what point?

Yes, after solving the system, the lines intersect at the point (1, 6, -4).

What is the importance of checking for consistency when finding the intersection?

Checking for consistency ensures that the solution for t and u satisfies all three equations simultaneously, confirming an actual intersection point in 3D space.

How can these calculations be verified using a graphing tool?

By plotting both parametric equations in a 3D graphing calculator or software, you can visually confirm whether the lines intersect at the calculated point.

What are common mistakes to avoid when finding the intersection of two lines in 3D?

Common mistakes include mixing up the parameters, forgetting to check all three equations for consistency, or incorrect algebraic manipulations leading to false conclusions about intersection or skew lines.

Additional Resources

4. Find The Intersection (if Any) Of The Lines 7 =(4,-2,-1)+t(1,4,-3) And F = (-8,20,15)+u(-3,2,5). 5

Understanding whether two lines in three-dimensional space intersect is a fundamental problem in vector geometry, with applications ranging from computer graphics to engineering design. In this article, we will explore how to determine if the given lines intersect, and if so, how to find the intersection point. The lines in question are:

- Line 7: $\mathbf{L}_1: (4, -2, -1) + t(1, 4, -3)$
- Line F: $\mathbf{L}_2: (-8, 20, 15) + u(-3, 2, 5)$

This problem involves several key concepts in vector algebra and parametric equations, and solving it requires a systematic approach to analyze the relationship between the two lines.

Understanding the Geometry of Lines in Space

Before delving into calculations, it's essential to grasp the geometric setup:

- Lines in 3D: Each line can be described parametrically by a point on the line and a direction vector.
- Intersection Point: The lines intersect if there exists a pair of parameters t and u such that the points on both lines are identical in space.
- Skew Lines: If the lines are not parallel and do not meet, they are skew lines.
- Parallel Lines: If their direction vectors are scalar multiples, they are parallel and may or may not coincide.

Given the parametric equations, our goal is to determine whether values of t and u exist that satisfy the equality of the points on both lines.

Step 1: Write the Parametric Equations Explicitly

The lines are expressed as:

- Line 7:
$$
\mathbf{r}_1(t) = (4, -2, -1) + t(1, 4, -3)
$$
Which expands to:
$$
x_1 = 4 + t
$$
$$
y_1 = -2 + 4t
$$
$$
z_1 = -1 - 3t
$$

- Line F:

$$
\mathbf{r}_2(u) = (-8, 20, 15) + u(-3, 2, 5)
$$

Which expands to:

$$
x_2 = -8 - 3u
$$

$$
y_2 = 20 + 2u
$$

$$
z_2 = 15 + 5u
$$

Step 2: Set Up the System of Equations

For the lines to intersect, the corresponding coordinates must be equal for some t and u:

$$
\begin{cases}
4 + t = -8 - 3u \quad \text{(Equation 1)} \\
-2 + 4t = 20 + 2u \quad \text{(Equation 2)} \\
-1 - 3t = 15 + 5u \quad \text{(Equation 3)}
\end{cases}
$$

Our task is to solve this system for t and u.

Step 3: Solve the System of Equations

Solving Equation 1 for t:

$$
t = -8 - 3u - 4
$$

$$
t = -12 - 3u
$$

Substituting into Equation 2:

$$
-2 + 4(-12 - 3u) = 20 + 2u
$$

Simplify:

$$
-2 - 48 - 12u = 20 + 2u
$$
$$
-50 - 12u = 20 + 2u
$$

Bring all u terms to one side and constants to the other:

$$
-12u - 2u = 20 + 50
$$
$$
-14u = 70
$$
$$
u = -5
$$

Find t using $u = -5$:

$$
t = -12 - 3(-5) = -12 + 15 = 3
$$

Verify with Equation 3:

$$
-1 - 3t = 15 + 5u
$$

Substitute $t = 3$ and $u = -5$:

$$
-1 - 3(3) = 15 + 5(-5)
$$

Simplify:

$$
-1 - 9 = 15 - 25
$$
$$
-10 = -10
$$

Confirmed equality holds; thus, the solution is consistent across all equations.

Interpreting the Results: The Intersection Point

Having determined that the parameters $t = 3$ and $u = -5$ satisfy the system, the next step is to find the actual point of intersection by substituting t into the parametric equations of either line.

Coordinates on Line 7:

$$x = 4 + t = 4 + 3 = 7$$

$$y = -2 + 4t = -2 + 4(3) = -2 + 12 = 10$$

$$z = -1 - 3t = -1 - 3(3) = -1 - 9 = -10$$

Coordinates on Line F:

$$x = -8 - 3u = -8 - 3(-5) = -8 + 15 = 7$$

$$y = 20 + 2u = 20 + 2(-5) = 20 - 10 = 10$$

$$z = 15 + 5u = 15 + 5(-5) = 15 - 25 = -10$$

The points match perfectly, confirming the lines intersect at:

$$\boxed{(7, 10, -10)}$$

Significance of the Intersection Point

This intersection point indicates that despite the different origins and directions, the two lines cross at exactly one point in three-dimensional space. This is a common occurrence in geometry, but it is essential to verify the calculations, as missteps can lead to incorrect conclusions about whether lines intersect or are skew.

Applications:

- Computer Graphics: Determining if two objects or rays intersect.
- Robotics: Path planning involving line-of-sight or collision detection.
- Engineering: Structural analysis where beams or components meet.

Special Cases and Further Considerations

While this example illustrates a straightforward intersection, more complex scenarios may involve:

- Parallel Lines: When direction vectors are scalar multiples, and lines may or may not coincide.
- Skew Lines: Lines that do not intersect nor are parallel.
- Coincident Lines: When the entire lines overlap, implying infinitely many intersection points.

In such cases, additional analysis is necessary, often involving checking the ratios of the components of the direction vectors.

Conclusion

The problem of finding the intersection of two lines in three-dimensional space boils down to solving a system of parametric equations. In this example, the lines:

$$
\mathbf{L}_1: (4, -2, -1) + t(1, 4, -3)
$$
and
$$
\mathbf{L}_2: (-8, 20, 15) + u(-3, 2, 5)
$$

intersect at the point $\boxed{(7, 10, -10)}$, with parameters $t=3$ and $u=-5$. This process demonstrates the power of vector algebra in spatial analysis and provides a clear methodology for tackling similar problems in various scientific and engineering fields.

By understanding the underlying principles and systematically solving the equations, one can confidently determine the intersection of lines in three-dimensional space, a foundational skill in many technical disciplines.

4 Find The Intersection If Any Of The Lines 7 4 2 1 T 1 4 3 And F 8 20 15 U 3 2 5 5

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