4. Given That Points A(-3,-2,1), B(-1,2,-5) And C(2,4,1) Are Three Vertices Of Triangle ABC, Find: (3

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Understanding how to analyze the properties of a triangle in three-dimensional space is fundamental in both pure mathematics and applied sciences. When points are provided in 3D coordinates, the process involves vector operations, distance calculations, and geometric reasoning to determine various attributes such as side lengths, area, and type of triangle. In this article, we will explore step-by-step how to analyze the triangle formed by the points A(-3, -2, 1), B(-1, 2, -5), and C(2, 4, 1). We will cover the calculation of side lengths, the area of the triangle, the classification of the triangle (scalene, isosceles, equilateral, right-angled), and discuss relevant concepts to deepen your understanding.

Understanding the Coordinates and Basic Concepts

Before diving into calculations, let's review some fundamental concepts essential to analyzing points in 3D space.

Coordinate Points in 3D Space

- Each point is represented as (x, y, z), indicating its position along the x-axis, y-axis, and z-axis respectively.
- In our case:
- Point A: (-3, -2, 1)
- Point B: (-1, 2, -5)
- Point C: (2, 4, 1)

Distance Between Two Points

The distance d between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in 3D space is given by the formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
$$

This fundamental formula allows us to compute the lengths of the sides of triangle ABC.

Vector Operations

- The vector from point P to Q is $\vec{PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$.
- Dot product of vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$:

$$
\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z
$$

- The magnitude (length) of a vector $\vec{u}$:

$$
|\vec{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2}
$$

- The dot product helps determine whether the angle between two vectors is acute, right, or obtuse.

Calculating the Side Lengths of Triangle ABC

To analyze triangle ABC, the first step is calculating the lengths of sides AB, BC, and CA.

Length of Side AB

Using the distance formula:

$$
AB = \sqrt{(-1 - (-3))^2 + (2 - (-2))^2 + (-5 - 1)^2}
$$

Breaking down:

- x-difference: $-1 - (-3) = 2$
- y-difference: $2 - (-2) = 4$
- z-difference: $-5 - 1 = -6$

Calculating:

$$
AB = \sqrt{(2)^2 + (4)^2 + (-6)^2} = \sqrt{4 + 16 + 36} = \sqrt{56} \approx 7.48
$$

Length of Side BC

Similarly,

$$
BC = \sqrt{(2 - (-1))^2 + (4 - 2)^2 + (1 - (-5))^2}
$$

Calculations:

- x-difference: $2 - (-1) = 3$
- y-difference: $4 - 2 = 2$
- z-difference: $1 - (-5) = 6$

Thus,

$$
BC = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7
$$

Length of Side CA

Finally,

$$
CA = \sqrt{(2 - (-3))^2 + (4 - (-2))^2 + (1 - 1)^2}
$$

Calculations:

- x-difference: $2 - (-3) = 5$
- y-difference: $4 - (-2) = 6$
- z-difference: $1 - 1 = 0$

Therefore,

$$
CA = \sqrt{5^2 + 6^2 + 0^2} = \sqrt{25 + 36 + 0} = \sqrt{61} \approx 7.81
$$

Analyzing the Triangle: Side Lengths and Classification

Having computed the side lengths:

- AB ≈ 7.48
- BC = 7
- CA ≈ 7.81

We can now analyze the type of triangle ABC based on side lengths.

Side Lengths Summary:

Side	Length (approximate)
AB	7.48
BC	7
CA	7.81

Classifying the Triangle

- Scalene Triangle: All three sides are of different lengths. Here, since 7, 7.48, and 7.81 are all distinct, triangle ABC is scalene.

- Isosceles Triangle: At least two sides are equal. Not the case here.

- Equilateral Triangle: All three sides equal. Not the case here.

Thus, triangle ABC is scalene.

Determining if Triangle ABC is Right-Angled

A key property to analyze is whether the triangle is right-angled. To check this, we can use the Pythagorean theorem on the side lengths or, more precisely, verify if the square of the longest side equals the sum of squares of the other two sides.

- Longest side: CA ≈ 7.81

Check:

$$
(CA)^2 \stackrel{?}{=} (AB)^2 + (BC)^2
$$

Calculate:

$$
(7.81)^2 \approx 61
$$
$$
(7.48)^2 \approx 56
$$
$$
7^2 = 49
$$

Sum:

$$
56 + 49 = 105
$$

Compare with:

$$
61 \neq 105
$$

Since the sum of squares of the two smaller sides does not equal the square of the largest side, the triangle is not right-angled.

Calculating the Area of Triangle ABC in 3D Space

The area of a triangle in three dimensions can be computed using the cross product of two vectors originating from a common vertex.

Method: Vector Cross Product

- Choose point A as a reference.
- Compute vectors:

$$
\vec{AB} = \text{from A to B} = (x_B - x_A, y_B - y_A, z_B - z_A)
$$
$$
\vec{AC} = \text{from A to C} = (x_C - x_A, y_C - y_A, z_C - z_A)
$$

Calculations:

\[
\vec{AB} = (-1 - (-3), 2 - (-2), -5 - 1) = (2, 4, -6)
\]
\[
\vec{AC} = (2 - (-3), 4 - (-2), 1 - 1) = (5, 6, 0)
\]

- The area \(\Delta \) of triangle ABC is:

\[
\Delta = \frac{1}{2} |\vec{AB} \times \vec{AC}|
\]

where \(|\cdot| \) denotes the magnitude of the cross product.

Calculating the Cross Product \(\vec{AB} \times \vec{AC} \)

The cross product:

\[
\vec{AB} \times \vec{AC} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 4 & -6 \\
5 & 6 & 0 \\
\end{vmatrix}
\]

Calculating determinants:

\[
\mathbf{i} (4 \cdot 0 - (-6) \cdot 6) - \mathbf{j}

Frequently Asked Questions

How do we find the area of triangle ABC with vertices A(-3,-2,1), B(-1,2,-5), and C(2,4,1)?

To find the area, first determine two vectors from one vertex, such as AB and AC. Then compute their cross product, find its magnitude, and divide by 2. Specifically: AB = B - A = (2,4,-6), AC = C - A = (5,6,0). Cross product AB x AC = (36,30,2). Magnitude = $\sqrt{(36^2 + 30^2 + 2^2)} = \sqrt{(1296 + 900 + 4)} = \sqrt{2200} \approx 46.90$. Area = 0.5 46.90 ≈ 23.45 units^2.

What is the formula to compute the area of a triangle in 3D space given three vertices?

The area is given by (1/2) times the magnitude of the cross product of two vectors originating from the same vertex: Area = (1/2) |(AB) x (AC)|, where AB and AC are vectors from the same vertex.

How do you find the vectors AB and AC for points A(-3,-2,1), B(-1,2,-5), and C(2,4,1)?

Subtract the coordinates of A from B and C: AB = B - A = (-1 - (-3), 2 - (-2), -5 - 1) = (2,4,-6); AC = C - A = (2 - (-3), 4 - (-2), 1 - 1) = (5,6,0).

Why is the cross product used to find the area of a triangle in 3D space?

Because the magnitude of the cross product of two vectors gives the area of the parallelogram they span. Half of this magnitude gives the area of the triangle formed by those vectors.

Can the area of triangle ABC be zero? What does that indicate?

Yes, if the vectors AB and AC are collinear, the cross product will be zero, indicating the points are collinear and the triangle has zero area.

What is the significance of the cross product magnitude in triangle area calculation?

It directly relates to the area of the parallelogram formed by the two vectors; halving it gives the triangle's area.

How do you verify if points A, B, and C are collinear?

Calculate vectors AB and AC; if their cross product is zero (or near zero considering precision), the points are collinear, and the triangle area is zero.

What are common mistakes when calculating the area of a triangle in 3D space?

Common mistakes include incorrect vector subtraction, forgetting to take the absolute value of the cross product magnitude, or miscalculating the cross product components.

How can the concept of vectors and cross product be applied in real-world problems involving triangles in 3D space?

They are used in computer graphics, physics simulations, engineering designs, and navigation to determine areas, orientations, and surface calculations involving 3D objects.

Additional Resources

Comprehensive Analysis and Solution for Triangle ABC with Given Vertices

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Introduction

In the realm of coordinate geometry, understanding the properties of triangles positioned in three-dimensional space is fundamental. When points are specified in the 3D coordinate system, several key aspects such as side lengths, area, perimeter, centroid, and classification (acute, right, or obtuse) depend on precise calculations derived from their coordinates. This detailed review will focus on the problem where points $A(-3, -2, 1)$, $B(-1, 2, -5)$, and $C(2, 4, 1)$ are vertices of triangle ABC. The goal is to explore the process of finding specific attributes of this triangle, especially the lengths of its sides, area, and other significant properties, providing comprehensive insights into each step.

Understanding the Problem

The problem states that points A, B, and C are vertices of triangle ABC, and the task involves calculating particular quantities associated with this triangle. Although the problem mentions "Find: (3," which appears incomplete or possibly a formatting error, the typical context suggests that the problem might be asking to:

- Calculate the lengths of sides AB, BC, and CA.
- Find the area of triangle ABC.
- Possibly determine the perimeter.
- Classify the triangle based on side lengths or angles, such as identifying whether it's scalene, isosceles, or equilateral, or acute, right, or obtuse.

In this review, we will methodically derive these properties, starting with the foundational calculations.

Coordinate Geometry Fundamentals

Before diving into calculations, let's revisit some essential formulas and concepts.

Distance Formula in 3D

Given two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the distance d between them is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
$$

This formula is pivotal for calculating side lengths.

Area of a Triangle in 3D

One efficient way to find the area of a triangle in 3D space is to use the vector cross product:

1. Find two vectors originating from the same vertex, for example:
- $\vec{AB} = \vec{B} - \vec{A}$
- $\vec{AC} = \vec{C} - \vec{A}$

2. Compute the cross product:
$$
\vec{AB} \times \vec{AC}
$$

3. The magnitude of this cross product gives twice the area of the triangle:
$$
\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AC}|
$$

Step-by-Step Calculation

1. Calculating the Side Lengths

Let's compute the lengths of sides AB, BC, and CA.

Vertices:

- $A(-3, -2, 1)$
- $B(-1, 2, -5)$
- $C(2, 4, 1)$

Side AB:

$$
AB = \sqrt{(-1 - (-3))^2 + (2 - (-2))^2 + (-5 - 1)^2}
$$

Calculate each component:

- x-difference: $-1 + 3 = 2$
- y-difference: $2 + 2 = 4$
- z-difference: $-5 - 1 = -6$

Thus:

$$
AB = \sqrt{(2)^2 + (4)^2 + (-6)^2} = \sqrt{4 + 16 + 36} = \sqrt{56} \approx 7.483
$$

Side BC:

$$
BC = \sqrt{(2 - (-1))^2 + (4 - 2)^2 + (1 - (-5))^2}
$$

Calculate each component:

- x-difference: $2 + 1 = 3$
- y-difference: $4 - 2 = 2$
- z-difference: $1 + 5 = 6$

Thus:

$$
BC = \sqrt{(3)^2 + (2)^2 + (6)^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7
$$

Side CA:

$$
CA = \sqrt{(2 - (-3))^2 + (4 - (-2))^2 + (1 - 1)^2}
$$

Calculate each component:

- x-difference: $2 + 3 = 5$
- y-difference: $4 + 2 = 6$
- z-difference: $1 - 1 = 0$

Thus:

$$
CA = \sqrt{(5)^2 + (6)^2 + 0^2} = \sqrt{25 + 36 + 0} = \sqrt{61} \approx 7.81
$$

Summary of side lengths:

Side	Length	Approximate Value
AB	$\sqrt{56}$	7.483
BC	7	7.000
CA	$\sqrt{61}$	7.81

---

2. Calculating the Area of Triangle ABC

Using vector cross product method:

Step 1: Compute vectors $\vec{AB}$ and $\vec{AC}$.

$$

$$
\vec{AB} = \vec{B} - \vec{A} = (-1 - (-3), 2 - (-2), -5 - 1) = (2, 4, -6)
$$

$$
\vec{AC} = \vec{C} - \vec{A} = (2 - (-3), 4 - (-2), 1 - 1) = (5, 6, 0)
$$

Step 2: Compute the cross product $\vec{AB} \times \vec{AC}$.

$$
\vec{AB} \times \vec{AC} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 4 & -6 \\
5 & 6 & 0
\end{vmatrix}
$$

Calculating determinant:

$$
\mathbf{i} (4 \times 0 - (-6) \times 6) - \mathbf{j} (2 \times 0 - (-6) \times 5) + \mathbf{k} (2 \times 6 - 4 \times 5)
$$

Simplify each component:

- $\mathbf{i}$: $0 - (-36) = 36$
- $\mathbf{j}$: $0 - (-30) = 30$
- $\mathbf{k}$: $12 - 20 = -8$

So,

$$
\vec{AB} \times \vec{AC} = (36, 30, -8)
$$

Step 3: Find the magnitude of the cross product:

$$
|\vec{AB} \times \vec{AC}| = \sqrt{36^2 + 30^2 + (-8)^2} = \sqrt{1296 + 900 + 64} = \sqrt{2260} \approx 47.55
$$

Step 4: Calculate the area:

$$
\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AC}| \approx \frac{1}{2} \times 47.55 \approx 23.78
$$

Hence, the area of triangle ABC is approximately 23.78 square units.

Additional Properties and Classifications

3. Perimeter Calculation

Adding side lengths:

\[
\text{Perimeter} = AB + BC + CA \approx 7.483 + 7 + 7.81 \approx 22.293
\]

This value helps in classifying the triangle further.

4. Triangle Classification

- Side-based classification:

- Since all three sides are distinct (\(AB \neq BC \neq CA \)), the triangle is scalene.

- Angle-based classification:

- To determine if the triangle is right-angled, use the Pythagorean theorem on the sides.

Check if any combination satisfies:

\[
\text{(square of longest side)} \approx \text{sum of squares of other two sides}
\]

- Longest side: \(CA \approx 7.81 \)

Compute:

\[
(AB)^2 \approx 56, \quad (BC

4 Given That Points A 3 2 1 B 1 2 5 And C 2 4 1 Are Three Vertices Of Triangle Abc Find 3

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