40) A Person Tries To Heat Up Her Bath Water By Adding 5.0 L Of Water At 80C To 60 L Of Water At 30C.

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Understanding how temperature changes when mixing different quantities of water at varying temperatures is fundamental in thermodynamics. In this scenario, a person adds 5.0 liters of hot water at 80°C to 60 liters of cooler water at 30°C in an attempt to raise the overall temperature of her bath water. This process involves concepts such as heat transfer, specific heat capacity, and thermal equilibrium. In this comprehensive guide, we will analyze the problem, perform necessary calculations, and explore related concepts to deepen understanding.

Overview of the Problem

This problem involves mixing two quantities of water at different temperatures and determining the final temperature after they are combined. The key aspects include:

- The volume of hot water added: 5.0 liters
- Temperature of hot water: 80°C
- The volume of cooler water: 60 liters
- Temperature of cooler water: 30°C
- The goal: Find the final equilibrium temperature of the combined water.

Understanding the fundamental principles involved requires knowledge of:

- Specific heat capacity of water
- Thermal energy transfer during mixing
- Conservation of energy principle

Fundamental Concepts and Principles

Specific Heat Capacity of Water

The specific heat capacity (c) of water is a measure of how much heat energy is needed to raise the temperature of a unit mass of water by one degree Celsius. For water:

- c ≈ 4.18 J/g°C

This property is crucial because it allows us to quantify the heat transfer during mixing processes.

Mass and Volume Relationship

Since the specific heat capacity is typically given per gram, it is essential to convert volumes to masses when performing calculations:

- 1 liter of water ≈ 1000 grams
- Therefore,

Volume (L)	Mass (g)
5.0	5000
60	60,000

Conservation of Energy

The principle that energy cannot be created or destroyed applies here. When hot water is added to cooler water, heat will transfer from the hot to the cold until thermal equilibrium is reached. The heat lost by hot water equals the heat gained by cold water:

$$ Q_{hot} = Q_{cold} $$

which can be expressed as:

$$ m_{hot} c (T_{hot\_initial} - T_{final}) = m_{cold} c (T_{final} - T_{cold\_initial}) $$

where:

- m = mass
- c = specific heat capacity
- $T_{initial}$ = initial temperature of respective water
- T_{final} = final equilibrium temperature

Step-by-Step Calculation of Final Temperature

To determine the final temperature, we assume perfect mixing and no heat loss to surroundings.

Step 1: Convert Volumes to Masses

- Hot water mass: $m_{hot} = 5.0\,L \times 1000\,g/L = 5000\,g$
- Cold water mass: $m_{cold} = 60\,L \times 1000\,g/L = 60,000\,g$

Step 2: Write the Energy Balance Equation

Using the conservation of energy:

$$ m_{hot} c (T_{hot} - T_{final}) = m_{cold} c (T_{final} - T_{cold}) $$

Since c is the same for both, it cancels out:

$$ m_{hot} (T_{hot} - T_{final}) = m_{cold} (T_{final} - T_{cold}) $$

Plugging in known values:

$$ 5000 (80 - T_{final}) = 60000 (T_{final} - 30) $$

Step 3: Solve for T_{final}

Expanding both sides:

$$ 5000 \times 80 - 5000 T_{final} = 60000 T_{final} - 60000 \times 30 $$

Calculating constants:

$$ 400,000 - 5000 T_{final} = 60,000 T_{final} - 1,800,000 $$

Bring all terms to one side:

$$ 400,000 + 1,800,000 = 60,000 T_{final} + 5000 T_{final} $$

Simplify:

$$ 2,200,000 = 65,000 T_{final} $$

Solve for T_{final}:

$$ T_{final} = \frac{2,200,000}{65,000} \approx 33.85^\circ C $$

Interpretation of Results

The calculation indicates that after mixing, the temperature of the water will stabilize at approximately 33.85°C. This is only slightly higher than the initial temperature of the cooler water (30°C) because the hot water mass (5 L) is relatively small compared to the larger volume (60 L), resulting in a modest temperature increase.

Additional Considerations and Real-World

Factors

While the calculations assume ideal conditions, real-world scenarios introduce additional complexities:

Heat Loss to Surroundings

- Actual temperature may be lower due to heat transfer to the environment.
- Factors such as insulation, bath material, and ambient temperature influence heat retention.

Non-Uniform Mixing

- In practice, temperature might not be perfectly uniform initially.
- Stirring or agitation helps achieve a uniform temperature more rapidly.

Impact of Container Material

- Materials with different thermal conductivities can influence heat transfer efficiency.

Time to Reach Equilibrium

- The process isn't instantaneous.
- Heat transfer dynamics involve conduction, convection, and possibly radiation.

Practical Applications and Related Concepts

Understanding the principles demonstrated in this problem has broader applications:

1. Cooking and Food Preparation

- Adjusting water temperatures for optimal cooking conditions.
- Mixing hot and cold water to reach desired temperatures efficiently.

2. Heating Systems Design

- Designing efficient water heating and distribution systems.
- Calculating energy requirements for heating water in residential and

commercial settings.

3. Environmental and Energy Conservation

- Minimizing energy usage when heating water.
- Implementing insulation to reduce heat loss.

4. Thermodynamics Education

- Demonstrating heat transfer principles.
- Applying conservation of energy in practical problems.

Summary and Key Takeaways

- The final temperature after mixing 5.0 L of hot water at 80°C with 60 L of cooler water at 30°C is approximately 33.85°C.
- The calculation relies on the conservation of energy, converting volumes to masses, and understanding specific heat capacity.
- Real-world factors such as heat loss and mixing efficiency can influence actual outcomes.
- This problem exemplifies fundamental thermodynamic principles and their applications in everyday life.

Conclusion

Mixing hot and cold water to achieve a desired temperature is a common task that illustrates core thermodynamic concepts. By understanding heat transfer, specific heat capacity, and conservation of energy, one can accurately predict the final temperature in such scenarios. Whether in household activities or industrial processes, these principles help optimize energy use and improve efficiency. This problem serves as a practical example of physics in action, highlighting the importance of thermodynamics in our daily routines.

Keywords: heat transfer, specific heat capacity, thermal equilibrium, water mixing, temperature calculation, thermodynamics, energy conservation, hot water, cold water, bath water heating

Frequently Asked Questions

What is the final temperature of the bath water after mixing 5.0 L at 80°C with 60 L at 30°C?

The final temperature can be calculated using the principle of conservation of heat: (mass1 × temperature1 + mass2 × temperature2) / (mass1 + mass2). Converting liters to kilograms (assuming water density of 1 kg/L), the final temperature is approximately 35°C.

Why does the temperature of the mixture fall between the initial temperatures of the two water samples?

Because heat flows from the hotter water to the colder water until thermal equilibrium is reached, resulting in a final temperature between the initial temperatures, depending on their respective masses and initial temperatures.

How does the mass of water affect the final temperature when mixing two water samples?

The larger the mass of a water sample at a given temperature, the greater its influence on the final temperature; the sample with more mass will dominate the final temperature after mixing.

What assumptions are made when calculating the final temperature of the mixed water?

Assumptions include no heat loss to the surroundings, perfect mixing, and that the specific heat capacity of water remains constant at different temperatures.

If heat loss occurs during mixing, how would that affect the final temperature calculation?

Heat loss would cause the final temperature to be lower than the calculated value assuming no heat loss, meaning the actual final temperature would be slightly less due to energy escaping to the environment.

Can this method be used to determine the final temperature when mixing other liquids, and what factors need to be considered?

Yes, but the specific heat capacities of the other liquids must be known, and differences in thermal properties should be accounted for, as they affect heat transfer and final temperature calculations.

Additional Resources

40) A Person Tries To Heat Up Her Bath Water By Adding 5.0 L Of Water At 80°C To 60 L Of Water At 30°C

In a typical scenario familiar to many, a person attempts to warm up her bath water by mixing different quantities of hot and cold water. Specifically, she adds 5.0 liters of water at 80°C to 60 liters of water at 30°C. While this might seem like a straightforward task, underlying thermodynamic principles govern the resulting temperature of the mixture. Understanding how heat transfer, conservation of energy, and specific heat capacities work together can shed light on what the final bath temperature might be. This article explores the science behind this common household activity, providing a detailed, technical yet accessible explanation of the processes involved.

The Basic Concept: Conservation of Energy in Mixing Fluids

At the heart of this scenario lies the principle of conservation of energy. When two quantities of water at different temperatures are mixed, heat flows from the hotter to the colder, eventually reaching thermal equilibrium. The key assumption here is that no heat is lost to the environment during the mixing process, which simplifies calculations but may not perfectly reflect real-world conditions.

Core premise: The heat lost by the hot water equals the heat gained by the cold water, assuming an isolated system.

Mathematically, this is expressed as:

$$ Q_{hot} + Q_{cold} = 0 $$

or equivalently,

$$ m_{hot} c_{water} (T_{hot,initial} - T_{final}) = m_{cold} c_{water} (T_{final} - T_{cold,initial}) $$

where:

- m = mass of water
- c_{water} = specific heat capacity of water (~4.18 J/g·°C)
- $T_{initial}$ = initial temperature of the water
- T_{final} = final equilibrium temperature of the mixture

Since the specific heat capacity of water is constant over typical temperature ranges, it simplifies calculations.

Quantitative Analysis: Calculating the Final Temperature

Let's apply the principles to the specific case:

- Hot water volume: 5.0 liters at 80°C
- Cold water volume: 60 liters at 30°C

Step 1: Convert volumes to masses

Since the density of water is approximately 1 kg/L:

- $m_{hot} = 5.0\, \text{L} \times 1\, \text{kg/L} = 5.0\, \text{kg}$
- $m_{cold} = 60\, \text{L} \times 1\, \text{kg/L} = 60\, \text{kg}$

Step 2: Write the heat balance equation

\[

$$
m_{hot} c (T_{hot,initial} - T_{final}) = m_{cold} c (T_{final} - T_{cold,initial})
$$

Since c cancels out:

$$
m_{hot} (T_{hot,initial} - T_{final}) = m_{cold} (T_{final} - T_{cold,initial})
$$

Plugging in the known values:

$$
5.0\, \text{kg} \times (80^\circ C - T_{final}) = 60\, \text{kg} \times (T_{final} - 30^\circ C)
$$

Step 3: Solve for T_{final}

$$
5.0 (80 - T_{final}) = 60 (T_{final} - 30)
$$

Expanding:

$$
400 - 5.0 T_{final} = 60 T_{final} - 1800
$$

Bring all terms to one side:

$$
400 + 1800 = 60 T_{final} + 5.0 T_{final}
$$

$$
2200 = 65 T_{final}
$$

Finally, isolate T_{final}:

$$
T_{final} = \frac{2200}{65} \approx 33.85^\circ C
$$

Result: The final temperature of the bath after mixing is approximately 33.9°C.

Interpreting the Result: Practical Implications and Limitations

The calculation indicates that adding 5 liters at 80°C to 60 liters at 30°C raises the bath temperature only modestly, from roughly 30°C to about 34°C. This is a significant insight, especially for those expecting a warmer bath by adding hot water. The relatively small increase is primarily due to the large volume (and thus mass) of the colder water compared to the hot water.

Key takeaways:

- Volume ratio matters: Adding a small volume of hot water to a larger volume of cold water results in a minimal temperature increase.
- Heat capacity considerations: Since water has high specific heat capacity, large volumes require substantial heat input to change temperature significantly.
- Thermal equilibrium: The final temperature is always between the initial temperatures, weighted by the masses involved.

Limitations and real-world factors:

- Heat loss: In practice, some heat will escape to surroundings during mixing, slightly reducing the final temperature.
- Temperature measurement accuracy: Slight variations in initial temperatures or measurement errors can influence the final outcome.
- Temperature uniformity: Proper mixing ensures uniform temperature; otherwise, temperature gradients may exist temporarily.

Broader Context: How This Principle Applies to Everyday Life

Understanding the thermodynamics of mixing hot and cold water isn’t just an academic exercise—it has practical implications in home heating, industrial processes, and environmental science.

Applications include:

- Designing efficient water heating systems: Knowing how much hot water is needed to reach a desired temperature can optimize energy use.
- Cooking and food preparation: Recipes often depend on precise temperature control achieved through mixing.
- Environmental management: Understanding how water bodies equilibrate temperatures informs climate models and ecological studies.

Exploring Variations and Related Concepts

While the scenario presents a straightforward case, several variations can influence the outcome:

- Adding more hot water: Increasing the volume at 80°C will raise the final temperature proportionally.
- Different initial temperatures: Varying initial temperatures of the inputs alters the final equilibrium.
- Heat losses to environment: Real systems tend to lose heat, especially during transfer, making actual final temperatures lower than ideal calculations suggest.

Furthermore, concepts such as specific heat capacity and thermal conductivity extend beyond water, influencing material selection and thermal management in engineering.

Conclusion: The Science Behind a Simple Bath

The scenario of mixing hot and cold water to achieve a desired temperature is a classic example of thermodynamics in everyday life. Through conservation of energy, it becomes clear that adding a small amount of hot water to a large volume of cold water results in only a modest temperature increase. Calculations grounded in the principles of heat transfer reveal that to significantly warm a bath, either larger quantities of hot water or higher initial temperatures are necessary.

This understanding underscores the importance of volume ratios and thermal properties in practical applications. Whether in household routines or industrial processes, the fundamental laws of physics govern how heat flows and equilibrates, shaping our interactions with the thermal world around us.

By appreciating these underlying principles, individuals can make more informed decisions—be it in preparing a comfortably warm bath or designing energy-efficient heating systems. The science of heat transfer, while complex in its full scope, often manifests in simple, everyday actions, bridging the gap between theory and daily life.

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