4239-42 A Particle Is Moving With The Given Data. Find The Position Of The Particle. 39. V(t) = Sin T

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Understanding the motion of particles is a fundamental aspect of kinematics, a branch of physics that deals with the description of motion without considering forces. In many problems, given data such as velocity functions, initial positions, and time intervals, students and professionals can determine the position of a particle at any given time. This article provides a comprehensive guide to solving such problems, specifically focusing on the case where the velocity function is given as V(t) = Sin T.

By the end of this guide, readers will learn how to interpret velocity functions, perform integrations to find displacement, and determine the position of a particle at any desired time, considering initial conditions. This knowledge is essential for physics students, engineering professionals, and anyone interested in the mathematical modeling of motion.

Understanding the Problem

The problem statement is as follows:

> "A particle is moving with the given data. Find the position of the particle. 39. V(t) = Sin T"

This indicates that the velocity of the particle as a function of time is given by V(t) = Sin T, where T is the time variable. The task is to find the position of the particle at a specific time, given the initial conditions.

Key Concepts in Particle Motion

Before diving into calculations, it's crucial to understand some foundational concepts:

Velocity and Displacement

- Velocity (V(t)): The rate of change of position with respect to time. It can be positive or negative depending on the direction of motion.
- Displacement (s(t)): The change in position of the particle over a period of time.
- Initial Position (s(0)): The position of the particle at time t=0, which is often given or assumed.

Relationship Between Velocity and Position

The position of a particle at any time t can be found by integrating the velocity function:

$$
s(t) = s(0) + \int_{0}^{t} V(\tau) \, d\tau
$$

where $s(0)$ is the initial position and τ is a dummy variable of integration.

Assumptions and Initial Conditions

- Unless specified, it is often assumed that the initial position $s(0)$ is zero.
- The function $V(t)$ = Sin T is valid for $t \geq 0$, or within the domain where the sine function is defined.

Step-by-Step Solution to Find the Position

Let's proceed step-by-step to determine the position of the particle at any time t.

1. Write Down the Velocity Function

Given:

$$
V(t) = \sin t
$$

This is a standard sine function that oscillates between -1 and 1.

2. Integrate the Velocity Function to Find Displacement

Since displacement $s(t)$ is the integral of velocity:

$$
s(t) = s(0) + \int_{0}^{t} \sin \tau \, d\tau
$$

Assuming initial position $s(0) = 0$, the integral simplifies to:

$$
s(t) = \int_{0}^{t} \sin \tau \, d\tau
$$

The integral of $\sin \tau$ is $-\cos \tau$, so:

$$
s(t) = [-\cos \tau]_{0}^{t} = -\cos t + \cos 0
$$

Since $\cos 0 = 1$:

$$
s(t) = -\cos t + 1
$$

3. Final Expression for Position

The position of the particle at any time t is:

$$
\boxed{
s(t) = 1 - \cos t
}
$$

This equation gives the displacement relative to the initial position, which we've taken as zero.

Interpreting the Result

The derived position function $s(t) = 1 - \cos t$ has several important implications:

- The position oscillates between 0 and 2 because $\cos t$ varies between -1 and 1.
- When $\cos t = 1$, i.e., at $t = 0, 2\pi, 4\pi, \ldots$, the position $s(t) = 0$.
- When $\cos t = -1$, i.e., at $t = \pi, 3\pi, 5\pi, \ldots$, the position $s(t) = 2$.

This indicates a periodic oscillation typical of harmonic motion.

Additional Considerations

1. Effect of Initial Position

If the initial position $s(0)$ is not zero, then the displacement would be adjusted accordingly:

$$
s(t) = s(0) + \int_{0}^{t} V(\tau) \, d\tau
$$

For example, if $s(0) = s_0$, then:

$$
s(t) = s_0 - \cos t + \cos 0 = s_0 + 1 - \cos t
$$

2. Velocity at a Given Time

Since the velocity is $V(t) = \sin t$, its value at any time can be directly calculated.

3. Total Displacement Over an Interval

To find the total displacement from $t = t_1$ to $t = t_2$:

$$
\Delta s = s(t_2) - s(t_1) = [-\cos t_2 + 1] - [-\cos t_1 + 1] = -\cos t_2 + \cos t_1
$$

Practical Applications of the Problem

Understanding how to determine particle position given a velocity function is critical in many real-world scenarios:

- **Mechanical Systems**: Analyzing oscillations in pendulums or springs.
- **Electrical Engineering**: Studying oscillating currents and signals.
- **Physics Simulations**: Modeling wave motion and harmonic oscillators.
- **Robotics**: Calculating positions of moving parts over time.

Summary and Key Takeaways

- The velocity function $V(t) = \sin t$ indicates harmonic motion.
- Integrating velocity over time provides the displacement or position.
- Assuming initial position $s(0) = 0$, the position function becomes $s(t) = 1 - \cos t$.
- The position oscillates between 0 and 2, characteristic of simple harmonic motion.
- Adjusting for different initial positions involves adding the initial position to the integral result.

Conclusion

In this detailed analysis, we've demonstrated how to find the position of a particle moving with a given velocity function $V(t) = \sin t$. The key steps involved integrating the velocity function and applying initial conditions to arrive at the position function $s(t) = 1 - \cos t$. This process exemplifies fundamental concepts in kinematics and integral calculus, providing a solid foundation for solving similar problems involving particle motion.

Understanding these principles equips students and professionals to analyze complex motion scenarios, model oscillatory systems, and interpret real-world physical phenomena accurately. Whether dealing with simple harmonic motion or more complex trajectories, the techniques outlined here serve as essential tools in the study of dynamics.

Frequently Asked Questions

How do we determine the position of a particle when given its velocity function V(t) = sin(t)?

To find the position, we integrate the velocity function $V(t) = \sin(t)$ with respect to time t, adding an initial position constant. The position function is then $x(t) = -\cos(t) + C$.

What is the significance of the constant of integration when calculating the particle's position from velocity?

The constant of integration represents the initial position of the particle at $t = 0$. It ensures the position function accurately reflects the particle's starting point.

If the initial position of the particle at t=0 is known to be x(0) = x0, what is the specific position function for V(t) = sin(t)?

The position function is $x(t) = -\cos(t) + x0$, where x0 is the initial position at $t=0$.

How does the periodic nature of the velocity function V(t) = sin(t) affect the particle's motion?

Since $V(t) = \sin(t)$ is periodic with period 2π, the particle's velocity oscillates, causing the particle to move back and forth. Its position repeats in a cyclical pattern, resulting in oscillatory motion.

What are the key steps to find the position of a particle given its velocity function V(t) = sin(t)?

The key steps are: (1) Integrate $V(t) = \sin(t)$ to find the general position function $x(t) = -\cos(t) + C$, (2) use initial conditions to determine C, and (3) interpret the resulting function to understand the particle's motion over time.

Additional Resources

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Understanding the motion of particles is a fundamental aspect of classical mechanics, often encountered in physics and engineering problems. When provided with a velocity function, such as V(t) = sin t, it becomes possible to determine the particle's position at any given time, provided initial conditions are known. In this comprehensive guide, we will explore how to analyze such problems step-by-step, illustrating the process of integrating velocity functions, applying initial conditions, and deriving the particle’s position function. Whether you're a student preparing for exams or a professional seeking clarity on motion analysis, this article aims to demystify the process thoroughly.

Introduction to Particle Motion and Basic Concepts

Before diving into the specific problem V(t) = sin t, it’s essential to review some fundamental concepts:

- Position Function, s(t): Describes the location of the particle at time t.
- Velocity Function, V(t): The rate of change of position with respect to time, i.e., V(t) = ds/dt.
- Acceleration Function, A(t): The rate of change of velocity, i.e., A(t) = dV/dt.

Given the velocity function, the primary goal is to find the position function. This involves integrating the velocity function and applying initial conditions to determine the constant of integration.

Step-by-Step Breakdown of the Problem

1. Given Data and Known Quantities

- Velocity function:
V(t) = sin t, where t ≥ 0 (assuming the time starts at zero unless specified).

- Initial position:
Usually denoted as s(0) = s_0.
Note: The initial position is essential; if not provided, it’s typical to assume s(0) = 0.

2. Find the General Position Function s(t)

Since velocity is the derivative of position:

$$
V(t) = \frac{ds}{dt}
$$

Rearranged as:

$$
ds = V(t) dt
$$

To find $s(t)$:

$$
s(t) = \int V(t) dt + C
$$

where C is the constant of integration, determined by initial conditions.

3. Integrate the Velocity Function

Given:

$$
V(t) = \sin t
$$

The indefinite integral:

$$
s(t) = \int \sin t \, dt + C
$$

Recall the integral:

$$
\int \sin t \, dt = -\cos t + D
$$

Since D is an arbitrary constant, it is absorbed into C when performing indefinite integration. Therefore:

$$
s(t) = -\cos t + C
$$

Applying Initial Conditions to Find the Constant of Integration

To determine the specific position function, the initial position $s(0)$ must be known.

Case 1: Initial Position is Zero ($s(0) = 0$)

Plugging $t = 0$ into the position function:

$$
s(0) = -\cos 0 + C = -1 + C
$$

\]

Set equal to zero:

$$
0 = -1 + C \Rightarrow C = 1
$$

Thus, the position function is:

$$
s(t) = -\cos t + 1
$$

Case 2: Initial Position $s(0) = s_0$

If initial position is s_0, then:

$$
s(0) = -\cos 0 + C = -1 + C = s_0
$$

Solve for C:

$$
C = s_0 + 1
$$

So the position function becomes:

$$
s(t) = -\cos t + s_0 + 1
$$

Final Expression for the Position of the Particle

Based on the above, the general solution for the particle's position when $V(t) = \sin t$ is:

$$
\boxed{
s(t) = -\cos t + C
}
$$

where C is determined by initial conditions. For common initial conditions:

- If $s(0) = 0$, then $s(t) = -\cos t + 1$.
- If $s(0) = s_0$, then $s(t) = -\cos t + s_0 + 1$.

Graphical Interpretation of the Motion

Understanding the behavior of the particle over time can be enhanced via graphs:

- Velocity graph: Since $V(t) = \sin t$, it oscillates between -1 and 1 with a period of 2π.
- Position graph: Derived from integrating velocity, resulting in a cosine-based wave shifted vertically.

The position function:

$$
s(t) = -\cos t + C
$$

is a cosine wave shifted vertically by C, oscillating between:

$$
s_{max} = 1 + C \quad \text{and} \quad s_{min} = -1 + C
$$

Practical Applications and Examples

Example 1: Calculating Position After $\pi/2$ Seconds

Suppose the initial position is zero:

$$
s(t) = -\cos t + 1
$$

Calculate at $t = \pi/2$:

$$
s\left(\frac{\pi}{2}\right) = -\cos \frac{\pi}{2} + 1 = -0 + 1 = 1
$$

The particle is at position 1 unit at $t = \pi/2$ seconds.

Example 2: Determining the Time When Particle Reaches a Specific Position

Find t when $s(t) = 0$, assuming $s(0) = 0$:

$$
0 = -\cos t + 1 \Rightarrow \cos t = 1
$$

Solution:

$$
t = 2n\pi, \quad n \in \mathbb{Z}
$$

The particle reaches position zero at multiples of 2π seconds, i.e., $t = 0, 2\pi, 4\pi$, etc.

Additional Considerations and Variations

1. When Initial Velocity Is Known

If initial velocity is given, say $V(0) = V_0$, it can be used to verify or determine the constant of integration or initial conditions.

For example:

$$
V(0) = \sin 0 = 0
$$

which aligns with initial velocity zero in this case.

2. Acceleration and Further Dynamics

If acceleration is needed, differentiate velocity:

$$
A(t) = \frac{dV}{dt} = \cos t
$$

This provides insight into the particle's increasing or decreasing speed at different times.

3. Multiple Particles or Complex Motion

For more complex scenarios, superposition of multiple velocity functions or additional forces may be considered.

Summary and Key Takeaways

- The position function of the particle is obtained by integrating the velocity function.
- The indefinite integral of $V(t) = \sin t$ is $s(t) = -\cos t + C$.
- The constant C is determined using initial position data.
- The particle's motion can be analyzed graphically and numerically, providing insights into its oscillatory behavior.
- Understanding these fundamental techniques is vital for solving a broad spectrum of motion problems in physics and engineering.

Final Words

Analyzing particle motion given a velocity function like V(t) = sin t offers a clear pathway to understanding the particle's position over time. By integrating the velocity function and applying initial conditions, you can precisely determine how the particle moves within its environment. This process underscores the power of calculus in translating rate-of-change data into meaningful positional information, a cornerstone concept in the mathematical modeling of dynamic systems.

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