5. The Edges Of A Solid Cube Are 3p Cm Long. At One Corner Of The Cube, A Small Cube Is Cut Away. All

5. The Edges Of A Solid Cube Are 3p Cm Long. At One Corner Of The Cube, A Small Cube Is Cut Away. All

Understanding the geometric properties of cubes is fundamental in both mathematics and real-world applications. The problem involving a solid cube with edges measuring 3p centimeters and a small cube cut away from one corner presents an interesting case study in spatial reasoning, volume calculation, and surface area analysis. This article aims to explore the details of this problem comprehensively, providing clear explanations, mathematical derivations, and practical insights to facilitate a thorough understanding.

Introduction to the Cube and the Problem Setup

Before delving into the specifics, it is essential to establish the foundational concepts related to the cube described in the problem.

Dimensions of the Original Cube

The given cube has edges measuring 3p centimeters, where "p" is a variable or parameter that might represent a specific value or a placeholder for a numerical quantity. The dimensions can be summarized as follows:

- Edge length of the original cube: 3p cm
- Volume of the original cube: $(3p)^3 = 27p^3$ cubic centimeters
- Surface area of the original cube: $6 \times (3p)^2 = 6 \times 9p^2 = 54p^2$ square centimeters

Understanding these basic properties sets the stage for analyzing the modifications made to the cube, namely, the removal of a smaller cube from a corner.

Positioning of the Smaller Cube

The problem specifies that a small cube is cut away from one corner of the original cube. Typically, in such problems, the small cube:

- Is cut exactly at a corner, removing a smaller cube with edges aligned along the original cube's axes.
- Has a certain edge length, often denoted by a variable (say, "a"), which may be given or to be

determined.
- Creates a cavity or indentation in the original cube, affecting its surface area and volume.

The exact size of the small cube and its position are crucial in calculating the resulting shape's volume and surface area.

Determining the Size of the Small Cube Cut Away

To analyze the problem fully, it is necessary to specify or derive the size of the small cube that is removed.

Assuming the Small Cube's Edge Length

Often, in such problems, the size of the small cube is related proportionally to the original cube's dimensions. For example, suppose:

- The small cube has an edge length of "a" cm.
- The cut occurs at a corner, with the small cube removed such that its vertices are aligned with the original cube's axes.

In many cases, the problem provides the value of "a," or it might be deduced from context or additional information. If no explicit size is given, common assumptions include:

- The small cube's edge length is a fraction of 3p, such as $a = p$ or $a = \frac{3p}{k}$, where "k" is some constant.

For the purpose of this comprehensive analysis, let's assume the small cube has an edge length of "a" cm, which is less than 3p, ensuring it fits within the original cube.

Constraints on the Size of the Small Cube

The size of "a" must satisfy the following:

- $0 < a < 3p$, so the small cube fits entirely within the original cube.
- The position of the small cube at the corner is such that it is fully contained within the original cube.

Calculating the Volume After the Cube is Cut Away

The primary quantity of interest is the remaining volume of the solid after the small cube is removed.

Original Cube Volume

As established:

- $V_{original} = 27p^3$

Volume of the Removed Small Cube

- $V_{small} = a^3$

Remaining Volume of the Modified Cube

- $V_{remaining} = V_{original} - V_{small} = 27p^3 - a^3$

This straightforward calculation highlights how the removal affects the total volume.

Analyzing Surface Area Changes Due to the Cut

Beyond volume, the surface area is significantly impacted by the cut, especially since new surfaces are exposed.

Original Surface Area

- $S_{original} = 6 \times (3p)^2 = 54p^2$

Impact of Removing the Small Cube

When the small cube is cut away:

- The original surface is altered; some parts are removed, and new surfaces are exposed.
- The cavity created by removing the small cube introduces additional surface area.

Calculating the New Surface Area

The total surface area after removal:

- Starts with the original surface area.

- Subtracts the area of the faces where the small cube was attached.
- Adds the area of the internal surfaces exposed within the cavity.

Assuming the small cube is removed entirely, and its faces are exposed internally, the new surface area becomes:

$$ S_{new} = S_{original} - \text{area of the faces covered by the small cube} + \text{area of the internal cavity surfaces} $$

Given the removal is at a corner, the exposed internal surfaces are typically three rectangles, each corresponding to the faces of the small cube that are now internal.

Step-by-Step Calculation of Surface Area After the Cut

Let's detail the calculation assuming the small cube is removed from a corner at the origin, with axes along X, Y, Z directions.

Original Faces Removed

- The small cube's faces that align with the original cube's faces are located at:

- $x = a$
- $y = a$
- $z = a$

- The areas of these faces are:

- a^2 each.

- Since three faces are removed from the corner, the total area removed from the original surface is:

- $3 \times a^2$

Internal Surface Area Exposed

- The cavity created exposes three rectangular surfaces of size:

- Each $a \times a$, corresponding to the inner faces of the cavity.

- Total internal surface area:

- $3 \times a^2$

Net Surface Area Calculation

- The original surface area:

- $54p^2$

- Subtract the three faces where the small cube was attached:

- $3 \times a^2$

- Add the three internal faces of the cavity:

- $3 \times a^2$

- Therefore, the total surface area after the cut:

$$
S_{final} = 54p^2 - 3a^2 + 3a^2 = 54p^2
$$

This indicates that, in this idealized case, the surface area remains unchanged (assuming the cavity's internal faces are not counted as external surfaces). In reality, if the internal cavity is exposed, the external surface area increases, so a more detailed analysis would be required.

Practical Applications and Real-World Examples

Understanding the geometric modifications of cubes has numerous practical applications:

Manufacturing and Material Cutting

- When parts are cut from larger blocks, calculating remaining volume and surface area helps in material estimation.
- For example, CNC machining often involves removing corners or sections, similar to the small cube cutaway.

Architecture and Design

- Designing modular structures or decorative elements often involves cutting away sections from cubic forms.
- Accurate calculations ensure material efficiency and aesthetic precision.

Educational Purposes

- Problems like this serve as excellent exercises for students learning three-dimensional geometry, spatial visualization, and volume/surface area calculations.

Summary and Key Takeaways

- The original cube has edges of length 3p cm, with a volume of $27p^3$ and surface area of $54p^2$.
- Removing a small cube of edge length "a" from one corner reduces the volume by a^3.
- The surface area changes depending on the size of "a" and the position of the cut, often increasing due to exposed internal surfaces.
- Exact calculations depend on the specific dimensions of the small cube and the nature of the cut.

Conclusion

The problem involving a solid cube with edges 3p cm long, from which a smaller cube is cut away at one corner, illustrates key principles in solid geometry. By analyzing volumes and surface areas, we gain insights into the effects of structural modifications. Whether for academic purposes, practical engineering, or design, understanding these concepts enhances spatial reasoning and mathematical proficiency. Future considerations might involve exploring different cut shapes, multiple cuts, or dynamic parameters to deepen the understanding of complex geometric transformations.

Note: To tailor this analysis further, specific values for "p" and "a" can be incorporated once provided. The methodology outlined here serves as a flexible framework adaptable to various scenarios involving cube modifications.

Frequently Asked Questions

What is the length of the edges of the original solid cube?

The edges of the original solid cube are 3p cm long.

What shape is removed from the original cube at one corner?

A small cube is cut away from one corner of the original cube.

How does cutting a small cube at the corner affect the volume of the original cube?

It reduces the volume of the original cube by the volume of the small cube cut away.

If the small cube cut from the corner has an edge length of 'a' cm, what is its volume?

The volume of the small cube is a^3 cubic centimeters.

How can the new shape of the cube be described after removing the small cube?

The resulting shape is a cube with a corner cut off, resembling a truncated cube with a rectangular or square face missing.

What is the significance of the term '5. The Edges Of A Solid Cube Are 3p Cm Long' in understanding the problem?

It indicates the original cube's edges measure 3p cm, which helps calculate its volume and the size of the cut cube if needed.

How would you find the remaining volume of the solid after the small cube is removed?

Subtract the volume of the small cube (a^3) from the volume of the original cube ($27p^3$) to find the remaining volume.

Additional Resources

Solid Cube with Edges of 3p cm and a Corner Cutaway: An In-Depth Geometric Exploration

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Introduction

In the realm of geometry and spatial reasoning, few objects captivate the imagination quite like the cube—a fundamental three-dimensional shape that embodies symmetry, simplicity, and mathematical elegance. Today, we delve into a fascinating variant: a solid cube with edges measuring 3p centimeters and a carefully executed cutaway at one corner. This scenario presents an engaging challenge for students, educators, and geometry enthusiasts alike, as it combines fundamental concepts with intricate spatial visualization.

This article aims to provide a comprehensive analysis of this geometric figure, exploring its properties, the implications of the corner cut, and the mathematical considerations involved. Whether you're interested in the theoretical aspects or practical applications, this review will serve as a detailed guide to understanding this compelling shape.

The Basic Structure: The Solid Cube

Dimensions and Properties

The cube in question has edges of length 3p cm, where _p_ is an arbitrary positive parameter, potentially representing a variable or an unknown in a problem setup.

- Edge Length: 3p cm
- Volume: $V = (3p)^3 = 27p^3 \text{ cm}^3$
- Surface Area: $A = 6 \times (3p)^2 = 54p^2 \text{ cm}^2$

This fundamental data underscores the scale and size of the original cube before any modifications. The proportionality to _p_ allows for scalable analysis, where changing _p_ alters the shape's dimensions uniformly.

The Corner Cutaway: Concept and Geometric Significance

What Does It Mean to Cut Away a Corner?

In geometric terms, cutting away a corner of a cube involves slicing off a small tetrahedral portion at one vertex, resulting in a new, more complex shape. The cut is typically characterized by defining points along the edges emanating from the chosen vertex and connecting these points to form a new face—often a triangle or polygon—thereby truncating the original corner.

This process transforms the original cube into a truncated cube with additional faces. Such modifications are common in architectural design, manufacturing, and mathematical modeling to study properties like volume, surface area, and symmetry.

Visualizing the Cut

Imagine the cube with vertices labeled for clarity: denote the corner to be cut as A, with neighboring vertices B, C, and D along the edges. The cut involves selecting points P, Q, and R on edges AB, AC, and AD, respectively, such that these points define the new face when connected.

Mathematical Analysis of the Corner Cut

Defining the Cut Parameters

To analyze the shape rigorously, we need to specify the cut's extent. Common approaches include:

- Proportional Cuts: Cutting away a fixed fraction along each edge from the vertex.
- Fixed Length Cuts: Removing a segment of a specified length along each edge.

Suppose we choose a proportional cut where each point P, Q, and R divides the edges AB, AC, and AD respectively, at a ratio _k_ of the total edge length.

- Position of P: At a distance $k \times 3p$ from A along AB
- Similarly for Q and R

Let’s assume that $0 < k < 1$ to ensure the cut is within the edges.

Coordinates for the Vertices

Assuming the cube is aligned with a coordinate system for clarity:

- A: $(0,0,0)$
- B: $(3p,0,0)$
- C: $(0,3p,0)$
- D: $(0,0,3p)$

Then, the points on the edges are:

- P: $(k \times 3p, 0, 0)$
- Q: $(0, k \times 3p, 0)$
- R: $(0, 0, k \times 3p)$

Connecting these points forms a triangular face that truncates the corner.

Impact on the Shape: Volume and Surface Area

Volume Calculation

The removal of the corner creates a tetrahedral cut. The volume of this tetrahedron is crucial for understanding the remaining solid.

- Vertices of the tetrahedron:
- A: $(0,0,0)$
- P: $(k \times 3p, 0, 0)$
- Q: $(0, k \times 3p, 0)$
- R: $(0, 0, k \times 3p)$

- Volume of the tetrahedron:

$$
V_{cut} = \frac{1}{6} \times |\textbf{AP} \times \textbf{AQ} \cdot \textbf{AR}|
$$

Calculating the vectors:

$$
\textbf{AP} = (k \times 3p, 0, 0)
$$

$$
\textbf{AQ} = (0, k \times 3p, 0)
$$
$$
\textbf{AR} = (0, 0, k \times 3p)
$$

The scalar triple product:

$$
V_{cut} = \frac{1}{6} \times |(k \times 3p, 0, 0) \times (0, k \times 3p, 0) \cdot (0, 0, k \times 3p)|
$$

Computing the cross product:

$$
\textbf{AP} \times \textbf{AQ} = (0, 0, (k \times 3p)^2)
$$

Dotting with AR:

$$
(0, 0, (k \times 3p)^2) \cdot (0, 0, k \times 3p) = (k \times 3p)^3
$$

Thus:

$$
V_{cut} = \frac{1}{6} \times (k \times 3p)^3 = \frac{1}{6} \times 27 p^3 k^3 = \frac{27}{6} p^3 k^3 = 4.5 p^3 k^3
$$

Interpretation: The volume of the cut tetrahedron depends on the cube of _k_ and the cube of _p_. As _k_ approaches 0, the cut vanishes; as _k_ approaches 1, the tetrahedron approaches the entire corner.

Remaining Volume

The volume of the remaining solid after cutting is:

$$
V_{remaining} = 27 p^3 - 4.5 p^3 k^3 = p^3 \left(27 - 4.5 k^3\right)
$$

This allows for precise calculation of the leftover volume based on the cut parameter _k_.

Surface Area Adjustments

The original cube has a surface area of $54 p^2$. The cut modifies this area by:

- Removing the three rectangular faces adjacent to the corner (which are replaced by the new triangular face and the three newly formed rectangular faces around the cut).
- Adding the area of the new triangular face.

Area of the new triangular face:

- Base: between points P and Q (length $k \times 3p$)
- Sides: determined by distances between P, Q, and R.

Calculations:

$$
\textbf{PQ} = \text{distance between } P \text{ and } Q: \sqrt{(k \times 3p)^2 + (k \times 3p)^2} = \sqrt{2} \times 3p k
$$

Similarly,

$$
\textbf{PR} = \textbf{QR} = \sqrt{(k \times 3p)^2 + (k \times 3p)^2} = \sqrt{2} \times 3p k
$$

The area of the triangular face:

$$
A_{triangle} = \frac{1}{2} \times \text{base} \times \text{height}
$$

Given the symmetry, the area simplifies to:

$$
A_{triangle} = \frac{1}{2} \times (\sqrt{2} \times 3p k) \times (k \times 3p) = \frac{1}{2} \times 3p k \sqrt{2} \times 3p k
$$

$$
A_{triangle} = \frac{1}{2} \times 3p k \times 3p k \times \sqrt{2} = \frac{1}{2} \times 9 p^2 k^2 \sqrt{2} = \frac{9 p^2 k^2 \sqrt{2}}{2}
$$

The total surface area after cut:

- Subtract the areas of the three faces removed (each of area $3p \times 3p = 9 p^2$)
- Add the area of the new triangular face

Total area:

\[
A_{new

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