(6) The Figure Shows A Rectangle And 6 Identical Semicircles In A Circle. Find The Area Of The Shaded

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Understanding geometric figures and their properties is fundamental in solving complex area-related problems. In this article, we delve into a challenging yet intriguing problem involving a rectangle inscribed within a circle, with six identical semicircles arranged around it, and a shaded region whose area we need to find. By breaking down the problem systematically, exploring relevant geometric principles, and applying appropriate formulas, we aim to guide you through a comprehensive solution.

Understanding the Problem: The Figure and Its Components

The problem statement describes a geometric figure composed of several elements:

- A circle that forms the outer boundary.
- A rectangle inscribed within or associated with this circle.
- Six identical semicircles arranged around or on the rectangle, possibly on its sides or in a specific pattern.
- A shaded region within the figure, whose area is to be calculated.

To visualize this, imagine the circle as the primary boundary. Inside, there's a rectangle, which might be inscribed (touching the circle at certain points). Around or on this rectangle, six semicircles are placed—most likely on its sides or vertices—forming a symmetrical pattern. The shaded region could be the area between the rectangle and the semicircles, or perhaps a specific segment.

Understanding the arrangement is crucial before proceeding to calculations. Since the figure isn't explicitly provided, we will make logical assumptions based on standard geometric configurations and problem-solving conventions.

Key Geometric Concepts Involved

Before diving into calculations, let's review some fundamental concepts relevant to this problem:

1. Area of a Rectangle

- Formula: $\text{Area} = \text{length} \times \text{breadth}$

2. Area of a Semicircle

- Formula: $\frac{1}{2} \pi r^2$, where r is the radius.

3. Properties of Circles and Inscribed Figures

- The circle's radius often relates to the dimensions of inscribed figures.
- The diameter of a semicircle aligns with the side it is constructed upon.

4. Symmetry and Arrangement

- Symmetry simplifies calculations, especially when multiple identical semicircles are involved.

Step-by-Step Approach to Find the Area of the Shaded Region

To systematically solve the problem, follow these steps:

Step 1: Establish Known Dimensions and Variables

- Assign variables to unknown lengths, such as:
- l for the rectangle's length.
- b for the rectangle's breadth.
- R for the radius of the circle.
- r for the radius of each semicircle.

Step 2: Analyze the Geometric Relationships

- Determine how the semicircles are arranged relative to the rectangle.
- Identify whether the semicircles are on sides or vertices.
- Use the given information or typical configurations to relate these elements.

Step 3: Find the Radius of the Circle and Semicircles

- If the rectangle is inscribed in the circle, the diagonal of the rectangle equals the diameter of the circle:

$$
\text{Diagonal} = \sqrt{l^2 + b^2}
$$

$$
R = \frac{\sqrt{l^2 + b^2}}{2}
$$

- For semicircles, their radii are often equal to some segment length, perhaps half of the rectangle's side or based on other given data.

Step 4: Calculate the Area of the Semicircles

- For each semicircle:

$$
\text{Area} = \frac{1}{2} \pi r^2
$$

- Total area for six semicircles:

$$
6 \times \frac{1}{2} \pi r^2 = 3 \pi r^2
$$

Step 5: Determine the Area of the Rectangle

- Use the known or assumed lengths:

$$
\text{Rectangle area} = l \times b
$$

Step 6: Find the Area of the Entire Circle

- Using the radius R:

$$
\text{Circle area} = \pi R^2
$$

Step 7: Identify the Shaded Region

- The shaded region could be:
- The part of the circle excluding the semicircles and rectangle.
- The region between the semicircles and the rectangle.
- Or another specific segment.

- Clarify which region is shaded based on the figure, then subtract the non-shaded parts from the total.

Step 8: Final Calculation

- Combine all computed areas to find the shaded area:

\[
\text{Shaded Area} = \text{Area of circle} - \text{(areas of semicircles + rectangle)}
\quad \text{or as per the specific configuration}
\]

Assumptions and Typical Configurations

Since the problem does not specify the exact arrangement, common assumptions include:

- The rectangle is inscribed within the circle, with its diagonal equal to the diameter.
- The six semicircles are placed on the sides of the rectangle, three on each longer side, or on all sides symmetrically.
- Each semicircle has the same radius, possibly related to the rectangle's dimensions.

This approach aligns with standard geometric problems involving circles, rectangles, and semicircles.

Sample Calculation Based on Assumed Dimensions

Let's consider a specific example for illustration:

- Suppose the rectangle has length \(l = 6 \) units and breadth \(b = 4 \) units.
- The rectangle is inscribed within the circle, so:

\[
R = \frac{\sqrt{6^2 + 4^2}}{2} = \frac{\sqrt{36 + 16}}{2} = \frac{\sqrt{52}}{2} =

$$
\frac{2 \sqrt{13}}{2} = \sqrt{13} \approx 3.605
$$

- The circle's area:

$$
\pi R^2 = \pi \times 13 \approx 40.84
$$

- If the semicircles are on each side of length 6 (or possibly on the sides of the rectangle), and their radii are r, determined by the half-lengths:

- For semicircles on the longer sides, $r = \frac{l}{2} = 3$.
- For semicircles on the shorter sides, $r = 2$.

- Total semicircular area (assuming semicircles on all four sides):

$$
\text{Total semicircle area} = 6 \times \frac{1}{2} \pi r^2
$$

- For sides with $r=3$:

$$
3 \times \pi \times 9 = 27 \pi \approx 84.78
$$

- For sides with $r=2$:

$$
3 \times \pi \times 4 = 12 \pi \approx 37.70
$$

- Summing these gives the total semicircular area, which might be compared with the circle's area to find the shaded region.

Note: The actual calculation depends on the precise figure, which should be referenced for exact determination.

Conclusion and Final Remarks

The problem of finding the shaded area in a figure involving a rectangle, semicircles, and a circle is a classic example of applying multiple geometric principles. The key steps involve:

- Correctly interpreting the figure and making reasonable assumptions.
- Establishing relationships between the elements based on known geometric properties.
- Calculating areas systematically, leveraging formulas for rectangles, semicircles, and

circles.
- Understanding the arrangement to identify the shaded region accurately.

Important Tips for Solving Similar Problems:
- Always carefully analyze the diagram and note all given dimensions.
- Use symmetry and known properties to simplify calculations.
- When dimensions are missing, consider the relationships that can be derived from the figure.
- Break complex shapes into simpler components whose areas are straightforward to compute.
- Double-check the geometric relationships to avoid calculation errors.

In conclusion, solving such geometry problems requires a methodical approach, clarity in assumptions, and precise application of formulas. With practice, recognizing patterns and relationships becomes intuitive, enabling you to quickly determine areas and solve complex figures efficiently.

Keywords: Geometry, Circle, Rectangle, Semicircle, Area calculation, Geometric figures, Inscribed shapes, Semicircular area, Circle area, Problem-solving in geometry

Frequently Asked Questions

What is the key geometric concept involved in calculating the shaded area in the figure with a rectangle and six semicircles inside a circle?

The key concept is understanding how to find the combined area of the rectangle and semicircles, as well as how they relate within the circle, typically involving area formulas for rectangles and semicircles and subtracting or adding areas accordingly.

How do you determine the radius of each semicircle when they are placed inside a circle along with a rectangle?

The radius of each semicircle can often be determined from the dimensions of the rectangle or the circle, depending on how they are inscribed. If the semicircles are on the sides of the rectangle, their radius is usually half the length of those sides, which may be related to the circle's radius if the semicircles fit perfectly inside.

What formula is used to find the area of a semicircle in this problem?

The area of a semicircle is given by (1/2) π r^2, where r is the radius of the semicircle.

How do you approach calculating the shaded area in a figure with multiple shapes like a rectangle and semicircles inside a circle?

You typically find the area of the larger shape (the circle), then subtract the areas of the non-shaded parts or add the areas of shaded parts if they are separate. In this case, you would calculate the areas of the rectangle and semicircles, then determine which parts are shaded based on the figure, combining their areas accordingly.

Can symmetry or special properties of the figure simplify the calculation of the shaded area?

Yes, symmetry can simplify calculations by allowing you to compute the area of one part and then multiply or replicate it for other parts. Additionally, properties like the semicircles being inscribed or tangent to the rectangle and circle can help relate their dimensions, making the calculation more straightforward.

Additional Resources

Understanding the Area of the Shaded Region in a Circle with a Rectangle and Six Semicircles

When exploring geometric figures, especially those involving combinations of rectangles, circles, and semicircles, the problem of determining the area of a shaded region often presents both a challenge and an opportunity for deeper understanding of geometric principles. The figure shows a rectangle and 6 identical semicircles inscribed within or arranged around a circle. Find the area of the shaded region, is a typical example where careful analysis, strategic decomposition, and application of formulas are key to arriving at the correct solution. This type of problem not only tests your grasp of basic geometry but also encourages spatial visualization and creative problem-solving.

In this comprehensive guide, we will break down the problem step-by-step, explore relevant geometric concepts, and provide a clear pathway to solving the problem. Whether you are a student preparing for competitive exams or a math enthusiast sharpening your skills, this detailed analysis aims to clarify every aspect involved.

Understanding the Problem

Before diving into calculations, it’s crucial to understand the components of the figure and what exactly is asked:

- The figure contains a circle, which acts as the boundary.
- Inside the circle, there is a rectangle.
- Additionally, there are six identical semicircles arranged in some configuration related to the rectangle and the circle.
- The shaded region is a specific part of the overall figure, whose area we need to

determine.

While the problem statement refers to a figure, typical configurations include semicircles inscribed on the sides of the rectangle, semicircles placed externally, or semicircles arranged along the circumference. The key is to interpret the figure carefully and identify the dimensions and relationships.

Step 1: Visualizing the Figure

Since the problem mentions a rectangle and six identical semicircles within a circle, a common configuration might be:

- The rectangle is inscribed or inscribed in the circle.
- Semicircles are drawn on the sides of the rectangle, either internally or externally, with the six semicircles possibly arranged on the sides or vertices.

Possible arrangement:

- The rectangle is positioned within the circle such that its vertices lie on the circle.
- Semicircles are drawn on the sides of the rectangle (either internally or externally), with their diameters equal to the sides of the rectangle.
- The six semicircles could be arranged on four sides, with some sides having two semicircles (possibly on opposite sides), totaling six.

Note: Since the problem references six identical semicircles, they could be:

- Semicircles on the four sides, with two semicircles sharing sides or placed on different sides.
- Or, semicircles placed on the vertices, but given the number six, the configuration involving sides is more plausible.

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2. Key Geometric Concepts and Formulas

To solve the problem, several fundamental concepts and formulas are essential:

a. Area of a Rectangle

$$ \text{Area} = \text{length} \times \text{breadth} $$

b. Area of a Semicircle

Given the radius r:

$$ \text{Area of semicircle} = \frac{1}{2} \pi r^2 $$

c. Properties of Circles and Semicircles

- Diameter d relates to radius as $r = d/2$.
- When semicircles are drawn on the sides of a rectangle, their diameters typically match the side lengths.

d. Area of the Circle

$$\text{Area} = \pi R^2$$

where R is the radius of the main circle.

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3. Establishing the Relationship Between Dimensions

The crux of the problem lies in understanding the relationships between the rectangle, the semicircles, and the circle:

- Position and size of the rectangle: Is it inscribed in the circle? If so, the diagonal of the rectangle equals the diameter of the circle.
- Size of the semicircles: Since they are identical, their radii are equal, and their diameters are known or related to the rectangle's sides.

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4. Step-by-Step Solution Approach

Step 1: Identify the dimensions of the rectangle

Suppose the rectangle has length l and breadth b. The key is to find l and b in terms of the radius of the circle or other known quantities.

If the rectangle is inscribed in the circle:

$$\text{Diagonal} = \text{Diameter of the circle}$$

$$\sqrt{l^2 + b^2} = 2 R$$

where R is the radius of the main circle.

Step 2: Relate the semicircles to the rectangle

Assuming the semicircles are drawn on the sides of the rectangle:

- Semicircles on sides l and b with radii $r_l = l/2$ and $r_b = b/2$.

Given six identical semicircles, possible configurations include:

- Two semicircles on each pair of sides (total four), and two semicircles on some other sides or vertices.
- Alternatively, semicircles could be arranged along the perimeter to form a pattern.

Identify the configuration based on the figure, but for simplicity, assume:

- The semicircles are drawn on the sides of length l, with two semicircles per side (one on each side), totaling four.
- The remaining two semicircles are drawn on the other sides or placed symmetrically.

Given the six semicircles are identical, their radii are equal, say r:

$$r = \frac{l}{2} = \frac{b}{2}$$

Thus, the sides are:

$$l = 2r, \quad b = 2r$$

Step 3: Express the rectangle in terms of r

Since the rectangle's sides are $2r$, then:

$$l = 2r, \quad b = 2r$$

and the diagonal:

$$\sqrt{(2r)^2 + (2r)^2} = 2 R$$

$$\sqrt{4r^2 + 4r^2} = 2 R$$

$$\sqrt{8r^2} = 2 R$$

$$2r \sqrt{2} = 2 R$$

$$R = r \sqrt{2}$$

Step 4: Find the area of the shaded region

The shaded area could be:

- The area of the circle minus the areas occupied by the rectangle and semicircles.
- Or, the area of the circle minus the combined areas of the rectangle and semicircles, depending on the configuration.

Assuming the shaded region is the part of the circle not occupied by the rectangle and the semicircles:

$$\text{Shaded Area} = \text{Area of circle} - \left(\text{Area of rectangle} + \text{Area of semicircles} \right)$$

Calculate each component:

- Circle area:

$$\pi R^2 = \pi (r \sqrt{2})^2 = \pi \times 2 r^2 = 2 \pi r^2$$

- Rectangle area:

$$ l \times b = 2r \times 2r = 4 r^2 $$

- Six semicircles total area:

Each semicircle has area:

$$ \frac{1}{2} \pi r^2 $$

Total:

$$ 6 \times \frac{1}{2} \pi r^2 = 3 \pi r^2 $$

- Total area occupied:

$$ \text{Rectangle} + \text{Semicircles} = 4 r^2 + 3 \pi r^2 $$

- Shaded area:

$$ 2 \pi r^2 - (4 r^2 + 3 \pi r^2) $$

$$ = 2 \pi r^2 - 4 r^2 - 3 \pi r^2 $$

$$ = (2 \pi r^2 - 3 \pi r^2) - 4 r^2 $$

$$ = - \pi r^2 - 4 r^2 $$

Since the area cannot be negative, this indicates that the shaded area is the remaining part of the circle after subtracting the areas occupied by the rectangle and semicircles if the figure is arranged differently.

Alternatively, if the shaded region is the part enclosed within the semicircles or a different configuration, the calculation adjusts accordingly.

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5. Clarifying the Final Result

Given the assumptions above, the precise calculation depends on the actual configuration in the figure. Typically, in such problems:

- The key is to find the relation between r and R.
- Use the geometric relationships to express all areas in terms of known quantities.
- Subtract the non-shaded parts from the total circle area to find the shaded region.

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6. Summary of Key Steps for Similar Problems

- Identify the components of the figure and their relationships.

- Determine the dimensions of the rectangle and semicircles based on the figure.
- Relate all dimensions to the radius of the main circle.
- Use formulas for areas of rectangles, semicircles

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