#7-#8 PleaseFind An Equation Of A Line Perpendicular To The Given Line That Contains The Given Point.

7-8 PleaseFind An Equation Of A Line Perpendicular To The Given Line That Contains The Given Point.

Understanding how to find the equation of a line perpendicular to a given line that passes through a specific point is a fundamental skill in coordinate geometry. This process involves analyzing the slopes of lines, applying the concept of negative reciprocals, and utilizing the point-slope form of a linear equation. Whether you're solving algebra homework, preparing for exams, or working on real-world problems involving geometry, mastering this topic enhances your analytical and problem-solving skills.

In this comprehensive guide, we'll explore the step-by-step procedure to determine the equation of a line perpendicular to a given line, given a point through which the new line passes. We will also discuss common pitfalls, tips for accuracy, and practical examples to solidify your understanding.

Understanding the Basics of Line Equations

Before delving into perpendicular lines, it's essential to review some foundational concepts related to the equations of lines.

Standard Forms of a Line Equation

- Slope-Intercept Form: $y = mx + b$
- m represents the slope.
- b represents the y-intercept.
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Used when a point (x_1, y_1) on the line and the slope m are known.
- General Form: $Ax + By + C = 0$

The Concept of Slope

- The slope (m) indicates the steepness and direction of the line.
- For two points (x_1, y_1) and (x_2, y_2), the slope is:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

- Parallel lines have identical slopes.
- Perpendicular lines have slopes that are negative reciprocals of each other.

Steps to Find the Equation of a Perpendicular Line

This process involves several key steps:

Step 1: Determine the Slope of the Given Line

- If the line's equation is provided in slope-intercept form, identify m directly.
- If the equation is in standard form, rearrange it into slope-intercept form or directly find the slope.

Example:

Given line: $2x + 3y = 6$

Rearranged:

$$
3y = -2x + 6 \implies y = -\frac{2}{3}x + 2
$$

So, the slope m of the given line is $-\frac{2}{3}$.

Step 2: Find the Slope of the Perpendicular Line

- The slope of the perpendicular line, m_perp, is the negative reciprocal of m:

$$
m_{\text{perp}} = -\frac{1}{m}
$$

Using the previous example:

$$
m = -\frac{2}{3} \implies m_{\text{perp}} = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}
$$

Step 3: Use the Given Point and the Perpendicular Slope to Write the Equation

- The point-slope form is most straightforward:

$$
y - y_1 = m_{\text{perp}}(x - x_1)
$$

- Plug in the point (x_1, y_1) and the perpendicular slope m_{perp}.

Example:

Given point: $(4, 1)$

Perpendicular slope: $\frac{3}{2}$

Equation:

$$
y - 1 = \frac{3}{2}(x - 4)
$$

Step 4: Simplify the Equation to Slope-Intercept or Standard Form

- Distribute the slope:

$$
y - 1 = \frac{3}{2}x - 6
$$

- Add 1 to both sides:

$$
y = \frac{3}{2}x - 5
$$

This is the equation of the line perpendicular to the original line passing through the point $(4, 1)$.

Practical Examples and Applications

Let's explore some real-world problems and examples to reinforce these concepts.

Example 1: Find the Equation of a Perpendicular Line

Given:

- Original line: $y = 4x + 1$
- Point: $(-2, 3)$

Solution:

1. Identify the slope of the original line:

$$
m = 4
$$

2. Find the perpendicular slope:

$$
m_{\text{perp}} = -\frac{1}{4}
$$

3. Use point-slope form:

$$
y - 3 = -\frac{1}{4}(x + 2)
$$

4. Simplify:

$$
y - 3 = -\frac{1}{4}x - \frac{1}{2}
$$
$$
y = -\frac{1}{4}x - \frac{1}{2} + 3
$$
$$
y = -\frac{1}{4}x + \frac{5}{2}
$$

Answer:

$$
\boxed{ y = -\frac{1}{4}x + \frac{5}{2} }
$$

Example 2: Line in Standard Form

Given:

- Original line: $3x - 2y = 7$
- Point: $(1, 4)$

Solution:

1. Convert to slope-intercept form:

$$
-2y = -3x + 7 \implies y = \frac{3}{2}x - \frac{7}{2}
$$

2. Slope:

$$
m = \frac{3}{2}
$$

3. Perpendicular slope:

$$
m_{\text{perp}} = -\frac{2}{3}
$$

4. Use point-slope form:

$$
y - 4 = -\frac{2}{3}(x - 1)
$$

5. Simplify:

$$
y - 4 = -\frac{2}{3}x + \frac{2}{3}
$$
$$
y = -\frac{2}{3}x + \frac{2}{3} + 4
$$
$$
y = -\frac{2}{3}x + \frac{2}{3} + \frac{12}{3}
$$
$$
y = -\frac{2}{3}x + \frac{14}{3}
$$

Answer:

$$
\boxed{ y = -\frac{2}{3}x + \frac{14}{3} }
$$

Common Mistakes to Avoid

While finding the equation of a perpendicular line might seem straightforward, several common pitfalls can lead to errors:

- **Incorrectly computing the negative reciprocal:** Remember to switch numerator and denominator and change the sign.
- **Using the wrong point:** Ensure the point given is accurately substituted into the equation.
- **Confusing standard form and slope-intercept form:** Convert equations properly to find slopes.
- **Misapplying the formulas:** Double-check calculations, especially signs and fractions.

Tips for Success

- Always write the slope of the original line first before calculating the perpendicular slope.
- Use the point-slope form for clarity, especially when working with specific points.
- Simplify fractions carefully to avoid errors.
- Practice with multiple examples to build confidence.

Conclusion

Finding the equation of a line perpendicular to a given line that passes through a specific point is a vital skill in coordinate geometry. By understanding the relationship between slopes of perpendicular lines, applying the negative reciprocal, and correctly using algebraic formulas, you can solve these problems efficiently and accurately. Remember to always verify your calculations, pay attention to signs, and practice with varied examples to master this important mathematical concept.

Keywords: equation of a line perpendicular, slope, negative reciprocal, point-slope form, coordinate geometry, algebra, math problems, slope-intercept form, standard form, geometry practice, algebra practice

Frequently Asked Questions

How do I find the equation of a line perpendicular to a given line that passes through a specific point?

First, determine the slope of the given line, then find the negative reciprocal of that slope to get the perpendicular slope. Use the point-slope form with this new slope and the given point to write the equation.

What is the step-by-step method to find a perpendicular line passing through a point?

1. Find the slope of the original line. 2. Calculate the negative reciprocal of that slope. 3. Use the point-slope form with this new slope and the given point. 4. Simplify to slope-intercept form if needed.

If the original line has the equation 2x + 3y = 6 and the point is (4, 2), how do I find the perpendicular line?

Rewrite the original in slope form to find its slope: y = -2/3 x + 2. The perpendicular slope is 3. Using point (4, 2), the line’s equation is y - 2 = 3(x - 4), which simplifies to y = 3x - 10.

Can a line be perpendicular if it has the same slope as the original line?

No, for two lines to be perpendicular, their slopes must be negative reciprocals. Lines with the same slope are parallel, not perpendicular.

What is the importance of the point in finding a perpendicular line?

The point provides a specific location through which the new perpendicular line must pass, allowing you to determine its unique equation using the point-slope form.

How do I verify that my line is perpendicular to the given line?

Calculate the slopes of both lines; if they are negative reciprocals, then the lines are perpendicular. Alternatively, check their slopes multiply to -1.

Additional Resources

Find An Equation Of A Line Perpendicular To The Given Line That Contains The Given Point

---

Introduction

In the realm of coordinate geometry, understanding how to determine the equation of a line perpendicular to a given line passing through a specific point is an essential skill. This problem arises frequently in both academic contexts and practical applications, such as engineering, physics, and computer graphics. The task involves deciphering the original line's characteristics, particularly its slope, and then utilizing geometric principles to find the perpendicular counterpart that also passes through a designated point.

This comprehensive review aims to dissect this problem thoroughly, exploring fundamental concepts, step-by-step procedures, and common pitfalls. We will also analyze various scenarios, including lines with different forms and slopes, to equip readers with a robust understanding capable of addressing diverse problems.

Fundamental Concepts

1. Slope of a Line

The slope of a line, denoted typically as m, quantifies its steepness and direction. For a line passing through points (x_1, y_1) and (x_2, y_2), the slope is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

The slope determines the angle of inclination relative to the x-axis.

2. Perpendicular Lines and Slopes

Two lines are perpendicular if their slopes are negative reciprocals of each other. This means:

$$
m_{perpendicular} = -\frac{1}{m_{original}}
$$

Provided the original line is not vertical (slope undefined), the perpendicular line's slope can be found directly using this relation.

3. Point-Slope Form of a Line

Once the slope m and a point (x_0, y_0) on the line are known, the equation can be expressed as:

$$
y - y_0 = m(x - x_0)
$$

This form is particularly useful in constructing the required perpendicular line.

Detailed Procedure for Finding the Perpendicular Line

Step 1: Identify the Given Line's Equation

- If the line is given in slope-intercept form ($y = mx + b$), extract the slope directly.
- If given in standard form ($Ax + By + C = 0$), solve for y to find slope:

$$
y = -\frac{A}{B}x - \frac{C}{B}
$$

- If a point and slope are given, proceed to the next step.

Step 2: Calculate the Slope of the Perpendicular Line

- If the original slope is m, then the perpendicular slope is:

$$
m_{perp} = -\frac{1}{m}
$$

- Special cases:
- Vertical lines ($x = c$), where the slope is undefined. The perpendicular line in this case is horizontal ($y = y_0$), passing through the point (x_0, y_0).
- Horizontal lines ($y = c$), where the slope is 0. The perpendicular line is vertical ($x = x_0$).

Step 3: Use the Point-Perpendicular Line Equation

- Plug in the point (x_0, y_0) and the perpendicular slope into the point-slope form:

$$
y - y_0 = m_{perp}(x - x_0)
$$

- Simplify as needed to obtain the slope-intercept or standard form.

Case Studies and Examples

Example 1: Line with a Known Slope and Point

Given:

- Original line: $y = 2x + 3$
- Point: $(4, 5)$

Solution:

- Original slope: $m = 2$
- Perpendicular slope: $m_{perp} = -\frac{1}{2}$
- Equation of the perpendicular line through $(4, 5)$:

$$
y - 5 = -\frac{1}{2}(x - 4)
$$

- Simplify:

$$
y - 5 = -\frac{1}{2}x + 2
$$
$$
y = -\frac{1}{2}x + 7
$$

Answer:

$$
\boxed{y = -\frac{1}{2}x + 7}
$$

Example 2: Original Line is Vertical

Given:

- Original line: $x = 3$
- Point: $(3, 2)$

Solution:

- Original line is vertical: slope undefined.
- The perpendicular line must be horizontal: $y = y_0$

Equation:

$$
y = 2
$$

Answer:

$$

\boxed{y = 2}
\]

Example 3: Original Line is Horizontal

Given:

- Original line: \(y = -1\)
- Point: \((4, -1)\)

Solution:

- The original line's slope: 0.
- Perpendicular line is vertical: \(x = x_0 = 4\)

Equation:

\[
x = 4
\]

Answer:

\[
\boxed{x = 4}
\]

Addressing Complex and Ambiguous Scenarios

1. Line in Standard Form

Suppose the given line is \(3x - 4y + 7 = 0\), and the point \((x_0, y_0)\) is provided.

- Convert to slope-intercept form:

\[
-4y = -3x - 7
\]
\[
y = \frac{3}{4}x + \frac{7}{4}
\]

- Slope: \(m = \frac{3}{4}\)

- Perpendicular slope:

\[

m_{perp} = -\frac{1}{m} = -\frac{4}{3}
\]

- Equation of the perpendicular line:

\[
y - y_0 = -\frac{4}{3}(x - x_0)
\]

- Simplify as necessary.

2. Verifying the Point Lies on the Perpendicular Line

Always substitute the point into the derived line equation to verify correctness.

Common Pitfalls and Misconceptions

- Incorrect slope calculation: Ensure that when converting from standard form, the signs are handled correctly.
- Neglecting special cases: Vertical and horizontal lines require different approaches.
- Assuming the original line is not vertical: If the original line is vertical, the perpendicular line is necessarily horizontal, and vice versa.
- Misapplication of negative reciprocal: Remember that the negative reciprocal of \(m\) is \(-1/m\), not just \(1/m\) or \(-m\).

Practical Applications and Relevance

Understanding how to find perpendicular lines is fundamental in various fields:

- Engineering: Designing components that are perpendicular for structural integrity.
- Physics: Calculating forces perpendicular to a given direction.
- Computer Graphics: Rendering orthogonal projections.
- Navigation and Mapping: Determining paths perpendicular to existing routes.

Conclusion

The process of finding an equation of a line perpendicular to a given line passing through a specified point hinges on a clear grasp of slope relationships and the point-slope form of a line. Whether the original line is given in slope-intercept form or standard form, the core steps involve extracting the slope, computing its negative reciprocal, and then applying the point-slope formula.

By mastering these procedures, students and professionals can confidently solve a wide array of problems involving perpendicular lines, thereby strengthening their geometric intuition and analytical skills. The key lies in careful calculation, attention to special cases, and thorough verification.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Brooks Cole.
- Anton, H., Rorres, C. (2013). Elementary Linear Algebra. Wiley.
- Lay, D. C. (2012). Linear Algebra and Its Applications. Pearson.
- Online resources such as Khan Academy, MathWorld, and Paul's Online Math Notes provide additional tutorials and practice problems.

Final Remarks

This detailed exploration underscores that, while the task of finding a perpendicular line through a point may seem straightforward, it requires careful consideration of various line forms and special cases. Practicing diverse examples and understanding the underlying principles will lead to proficiency in this fundamental geometric problem.

7 8 Pleasefind An Equation Of A Line Perpendicular To The Given Line That Contains The Given Point

7 8 Pleasefind An Equation Of A Line Perpendicular To The Given Line That Contains The Given Point

Related Articles

- Which Of The Following Might Prefer Working In A Community Center?O A. An Employee Who Likes To Work
- What Would Be The Final Result If In The Extractions Of The Mixture With Base The Solutions Were Not
- A Chromosome Initially Has The Following Segments: A B C D E F G Draw The Chromosome, Identifying Its

Back to Home