8. The Length Of A Rectangle Is 3 Feet Longer Than The Width. If The Width Is X Feet, How Long Is The

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Understanding the dimensions of a rectangle is fundamental in geometry, architecture, interior design, and various fields that involve spatial reasoning. One common problem involves relating the length and width of a rectangle when given a specific relationship between the two. In this case, the length of a rectangle is 3 feet longer than its width, which is represented by the variable x (feet). The key question is: If the width is x feet, how long is the rectangle? This article explores this problem in detail, providing a comprehensive guide to solving such problems, understanding the underlying concepts, and applying these principles in real-world scenarios.

Understanding the Problem

The problem states:

> The length of a rectangle is 3 feet longer than the width. If the width is x feet, how long is the rectangle?

This statement involves a simple relationship between two dimensions of a rectangle:

- Width (W): Given as x feet.
- Length (L): Unknown, but related to width.

The key is to translate this relationship into a mathematical expression that allows us to find the length once the width (x) is known.

Breaking Down the Relationship

The Given Relationship

The problem explicitly states: "The length of a rectangle is 3 feet longer than the width." Mathematically, this can be written as:

$$ L = W + 3 $$

Since the width is given as x feet, substitute x into the equation:

$$ L = x + 3 $$

This simple linear equation provides the direct relationship needed to find the length of the rectangle for any given width x.

Interpreting the Variables

- x: The width of the rectangle in feet.
- L: The length of the rectangle in feet, which depends on x.

Knowing this relationship allows us to analyze various scenarios, whether x is a specific number or a variable representing any positive length.

Expressing the Length of the Rectangle

Based on the given relationship, the length of the rectangle is directly expressed as:

$$ \boxed{L = x + 3} $$

This formula indicates that for any width x, the length will always be 3 feet longer than x.

Example Calculations

Let's consider some specific examples to illustrate how to find the length:

1. If the width is 5 feet:

$$ L = 5 + 3 = 8 \text{ feet} $$

2. If the width is 10 feet:

$$ L = 10 + 3 = 13 \text{ feet} $$

3. If the width is 0 feet (theoretically):

$$ L = 0 + 3 = 3 \text{ feet} $$

These examples demonstrate the direct calculation process once the width x is known.

Visualizing the Dimensions of the Rectangle

Understanding the relationship visually can enhance comprehension. Consider a rectangle with width x and length x + 3:

- The width is a horizontal measurement (say, along the x-axis).

- The length extends vertically or horizontally, depending on the rectangle's orientation.

Plotting these dimensions on a coordinate plane or drawing a diagram can help solidify understanding.

Diagram Description

Imagine a rectangle with:

- Bottom side (width): x feet.
- Side length (length): x + 3 feet.

This visualization helps see how changes in x affect the overall size.

Applications and Real-World Examples

Problems like these are common in real-world situations such as:

- Room design: Determining the length of a room when the width is known.
- Fencing: Calculating the length of fencing needed for a garden with specified dimensions.
- Manufacturing: Designing rectangular objects where one dimension depends on another.

Sample scenario:

Suppose you're designing a rectangular garden. If the garden's width is represented as x feet, and the length must always be 3 feet longer to accommodate a specific plant arrangement, then:

- You can easily find the length once the width is decided.
- Conversely, if the total area is known, you can set up equations to find x.

Calculating the Perimeter and Area

Knowing the length and width allows for calculating other important properties of the rectangle, such as perimeter and area.

Perimeter

The perimeter (P) of a rectangle is the total length around the rectangle:

$$ P = 2 \times (L + W) $$

Substituting L = x + 3 and W = x:

$$ P = 2 \times [(x + 3) + x] = 2 \times (2x + 3) = 4x + 6 $$

This formula provides the perimeter in terms of x.

Area

The area (A) of a rectangle is:

$$ A = L \times W $$

Substituting:

$$ A = (x + 3) \times x = x^2 + 3x $$

This quadratic expression shows how the area depends on the width x.

Example:

If the width x = 4 feet:

- Length: $ 4 + 3 = 7 $ feet
- Perimeter: $ 4 \times 4 + 6 = 22 $ feet
- Area: $ 4^2 + 3 \times 4 = 16 + 12 = 28 $ square feet

Formulating and Solving Equations

In some cases, additional information comes into play, such as total area or perimeter, requiring the setup of equations to solve for x.

Example problem:

"The area of the rectangle is 84 square feet. If the width is x feet and the length is x + 3 feet, find the value of x."

Solution:

1. Write the area equation:

$$ x^2 + 3x = 84 $$

2. Rearrange into standard quadratic form:

$$ x^2 + 3x - 84 = 0 $$

3. Solve the quadratic:

Using the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

Where $ a = 1 $, $ b = 3 $, $ c = -84 $:

$$ x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-84)}}{2} $$

$$ x = \frac{-3 \pm \sqrt{9 + 336}}{2} $$

$$ x = \frac{-3 \pm \sqrt{345}}{2} $$

Since width cannot be negative:

$$ x = \frac{-3 + \sqrt{345}}{2} $$

Numerically:

$$ \sqrt{345} \approx 18.574 $$

$$ x \approx \frac{-3 + 18.574}{2} = \frac{15.574}{2} \approx 7.787 $$

Thus, the approximate width:

$$ x \approx 7.79 \text{ feet} $$

And the length:

$$ L = x + 3 \approx 7.79 + 3 = 10.79 \text{ feet} $$

Advanced Considerations

While the problem is straightforward, real-world applications may involve more complex relationships, such as:

- Constraints on maximum or minimum dimensions.
- Relationship between multiple dimensions.
- Incorporation of other geometric properties.

In such cases, setting up equations and inequalities becomes essential, often requiring algebraic manipulation and problem-solving skills.

Summary and Key Takeaways

- The length of a rectangle that is 3 feet longer than its width x is expressed as $L = x + 3$.
- For any given width x, the length can be directly calculated using this formula.
- Understanding the relationship between dimensions aids in calculating perimeter, area, and solving related problems.
- Visualizing the rectangle and setting up equations are effective strategies for solving more complex problems involving multiple variables.

Conclusion

Determining the length of a rectangle when its width is known and related to

the length by a fixed measurement is a fundamental problem in geometry. By translating the word problem into a simple algebraic expression, such as $L = x + 3$, we can easily compute the length for any given width, apply this knowledge to practical situations, and extend our understanding to more complex geometric problems. Whether designing a room, calculating fencing, or solving for unknown dimensions in engineering, mastering this relationship provides a solid foundation in spatial reasoning and algebraic problem-solving.

Frequently Asked Questions

If the width of a rectangle is x feet and the length is 3 feet longer, what is the formula for the length?

The length is $x + 3$ feet.

Given the width x feet, how do you express the perimeter of the rectangle?

Perimeter = $2 \times (\text{length} + \text{width}) = 2 \times (x + (x + 3)) = 2 \times (2x + 3) = 4x + 6$ feet.

If the width of the rectangle is 5 feet, what is the length?

The length is $5 + 3 = 8$ feet.

How can you find the area of the rectangle in terms of x?

Area = length × width = $(x + 3) \times x = x^2 + 3x$ square feet.

If the rectangle has an area of 56 square feet, how do you find the width x?

Set up the equation: $x^2 + 3x = 56$. Solve for x using quadratic methods.

What quadratic equation do you get when solving for x if the area is 56 square feet?

$x^2 + 3x - 56 = 0$.

How do you solve the quadratic equation $x^2 + 3x - 56 = 0$?

Use the quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, where $a=1$, $b=3$, $c=-56$.

What are the possible values of x if the area is 56 square feet?

x = 7 or x = −8/1 (discard the negative), so x = 7 feet.

Once you find the width x, how do you determine the length of the rectangle?

Add 3 to the width: length = x + 3. For x=7, length = 10 feet.

Additional Resources

The Length of a Rectangle is 3 Feet Longer Than the Width. If the Width Is X Feet, How Long Is the?

Understanding the relationship between the length and width of a rectangle is fundamental in geometry. When given specific information such as "the length of a rectangle is 3 feet longer than the width," and the width expressed as a variable (e.g., x feet), it opens the door to constructing algebraic expressions and solving for unknowns. In this guide, we will explore how to determine the length of the rectangle based on the given conditions, step-by-step, providing clarity on the process and underlying concepts.

Introduction to the Problem

Suppose you are given a rectangle with the following details:

- The length of the rectangle is 3 feet longer than the width.
- The width of the rectangle is denoted by x feet.

The core question is: How long is the rectangle?

This problem involves translating a word description into an algebraic expression, which can then be used to analyze or solve for the unknown length.

Breaking Down the Problem

Key Information

- Relationship between length and width: Length = Width + 3
- Width: x feet (variable)
- Question: Find the length in terms of x.

Visual Representation

Envision a rectangle labeled with its sides:

- One side (width): x feet
- The adjacent side (length): x + 3 feet

This helps in visualizing the problem and understanding the relationship

between the sides.

Step-by-Step Solution Approach

1. Define the Variables

- Let x be the width in feet.
- The length then, based on the problem, is x + 3 feet.

2. Express the Length

Since the length is 3 feet longer than the width, the algebraic expression for the length is:

Length = x + 3

3. Identify Additional Quantities (If Given)

Often, problems will give additional data such as area or perimeter, which can be used to find the specific value of x or the length.

Example:

- If the area is provided, say A, then:

A = length × width = (x + 3) × x

- If the perimeter is provided, say P, then:

P = 2 × (length + width) = 2 × (x + (x + 3))

4. Use the Additional Data to Solve

Suppose, for example, the area is known:

Given: Area = 56 square feet

Set up the equation:

x(x + 3) = 56

which simplifies to:

$x^2 + 3x = 56$

This is a quadratic equation that can be solved using factoring, completing the square, or quadratic formula.

Solving for the Width and Length: An Example

Example Problem:

The width of a rectangle is represented as x feet. The length is 3 feet longer than the width. If the area of the rectangle is 56 square feet, what are the dimensions?

Step 1: Write the Equation

$x(x + 3) = 56$

which expands to:

$x^2 + 3x = 56$

Step 2: Rearrange into Standard Quadratic Form

$x^2 + 3x - 56 = 0$

Step 3: Factor or Use Quadratic Formula

Factor the quadratic:

Find two numbers that multiply to −56 and add to +3. These numbers are +8 and −7.

Rewrite:

$x^2 + 8x - 7x - 56 = 0$

Group:

$(x^2 + 8x) + (-7x - 56) = 0$

Factor each group:

$x(x + 8) -7(x + 8) = 0$

Factor out common binomial:

$(x + 8)(x - 7) = 0$

Step 4: Solve for x

Set each factor equal to zero:

$x + 8 = 0 \rightarrow x = -8$ (discard, since width can't be negative)

$x - 7 = 0 \rightarrow x = 7$

Step 5: Find the Length

Since length = x + 3, substitute:

length = 7 + 3 = 10 feet

Summary of Key Steps

Step	Description	Example / Note
1	Define variables	x = width, length = x + 3
2	Set up equations based on given data	Area, perimeter, etc.
3	Form algebraic expressions	For area: $x(x + 3) = 56$
4	Rearrange into quadratic form	$x^2 + 3x - 56 = 0$

| 5 | Solve quadratic | Factoring, quadratic formula |
| 6 | Find missing dimensions | Length = x + 3 |

Additional Considerations

Handling Different Types of Problems

- Perimeter-based problems: Use P = 2 × (length + width).
- Area-based problems: Use A = length × width.
- Word problems with real-world context: Translate descriptions carefully into algebraic expressions.

When the Width is Unknown

If only the length is given, or if the problem involves other constraints, adapt the approach accordingly. The key is to express all quantities in terms of the known variables and solve algebraically.

Practical Applications

Understanding how to relate the length and width of rectangles is invaluable in numerous real-world scenarios:

- Design and Architecture: Calculating material requirements.
- Landscaping: Determining plot dimensions.
- Manufacturing: Cutting materials to specified dimensions.
- Education: Developing problem-solving and algebra skills.

Summary

In conclusion, when the length of a rectangle is described as being 3 feet longer than its width, and the width is denoted as x feet, the length can be straightforwardly expressed as x + 3 feet. Solving for specific dimensions often requires additional data like area or perimeter, which then leads to forming and solving algebraic equations. Mastery of translating word problems into algebraic expressions, solving quadratic equations, and interpreting the results is essential for tackling a wide range of geometric and mathematical challenges.

By practicing these steps and understanding the underlying relationships, you'll develop confidence in solving similar problems efficiently and accurately.

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