A 120-V Rms Voltage At 1000 Hz Is Applied To An Inductor, A 2.00-F Capacitor And A 100- Resistor, All

A 120-V Rms Voltage At 1000 Hz Is Applied To An Inductor, A 2.00-F Capacitor And A 100- Resistor, All as part of an AC circuit, it forms a fundamental example in understanding the behavior of reactive and resistive components in alternating current systems. This setup allows us to analyze complex impedance, phase relationships, and the overall power consumption within the circuit. In this comprehensive guide, we will explore the electrical characteristics of each component, how they interact at a frequency of 1000 Hz, and the key concepts necessary to analyze such a circuit effectively.

Understanding the Circuit Components

To analyze this AC circuit, it is essential to understand the individual properties of the inductor, capacitor, and resistor, especially how they respond to an applied AC voltage at 1000 Hz.

The Resistor (R = 100 Ω)

A resistor opposes current flow proportionally to the applied voltage, following Ohm's law:

- **Impedance (Z):** For a resistor, impedance is purely real and equal to its resistance R, so Z_R = 100 Ω.
- **Phase relationship:** Voltage and current are in phase.
- **Power dissipation:** Resistors convert electrical energy into heat, with real power calculated as $P = V^2/R$.

The Inductor (L): 120-V Rms at 1000 Hz

An inductor opposes changes in current through its property called inductance:

- **Inductive reactance (X_L):** $X_L = 2\pi fL$
- **Given:** Voltage Rms (V_{RMS}) = 120 V, frequency f = 1000 Hz, inductance L (unspecified)

- **Note:** The inductance L is not specified in the problem statement, so assuming the inductor is ideal, we analyze the circuit based on the reactive impedance concept.

Note: Since the inductance L is not provided, we will approach the problem by considering the general form of the inductive reactance and its effects on the circuit.

The Capacitor (C = 2.00 F)

A capacitor opposes changes in voltage, storing energy in the electric field:

- **Capacitive reactance (X_C):** $X_C = 1 / (2\pi fC)$
- **Given:** C = 2.00 F, f = 1000 Hz
- **Calculating:** $X_C = 1 / (2\pi\ 1000\ 2.00) \approx 1 / (12566.37) \approx 7.96\ 10^{-5}\ \Omega$

Note: The capacitor's reactance is extremely small due to its large capacitance value, which significantly influences the circuit's overall impedance.

Analyzing the Circuit's Total Impedance

The total impedance (Ztotal) in a series RLC circuit is a combination of resistance (R), inductive reactance (XL), and capacitive reactance (XC):

Impedance Expression

$Z_{total} = R + j(X_L - X_C)$

Where:

- R = 100 Ω
- XL = 2πfL (unknown without L)
- XC ≈ 7.96 10-5 Ω

Since L is not specified, the impedance's reactive part depends on L.

Calculating the Magnitude of Z_{total}

$$|Z_{total}| = \sqrt{[R^2 + (X_L - X_C)^2]}$$

Given the negligible value of XC, the dominant reactive component depends on XL.

Determining the Inductive Reactance and Circuit Behavior

Since the inductance L was not provided, we consider two scenarios:

Scenario 1: Inductor has a typical inductance value

Suppose an example where L = 10 mH (0.01 H):

1. Calculate X_L:

$$X_L = 2\pi fL = 2\pi \quad 1000 \quad 0.01 \approx 62.83\ \Omega$$

2. Determine the total impedance magnitude:

$$|Z_{total}| \approx \sqrt{[100^2 + (62.83 - 7.96 \quad 10^{-5})^2]} \approx \sqrt{[10000 + 3943.4]} \approx \sqrt{[13943.4]} \approx 118.1\ \Omega$$

Scenario 2: Inductor with a different inductance

Adjusting L will proportionally change XL and thus the total impedance.

Current and Voltage Distribution in the Circuit

The applied RMS voltage (VRMS) is 120 V. The current in the circuit (IRMS) can be calculated as:

$$I_{RMS} = V_{RMS} / |Z_{total}|$$

For the example with L = 10 mH:

$$I_{RMS} \approx 120\ V / 118.1\ \Omega \approx 1.016\ A$$

Phase Angle (ϕ):

The phase difference between voltage and current is given by:

$$\phi = arctangent[(X_L - X_C) / R]$$

Using the same example:

$$\phi \approx arctangent[(62.83 - 0.0000796) / 100] \approx arctangent[0.6283] \approx 32.1°$$

This indicates the circuit is inductive, with the current lagging the voltage by approximately 32.1°.

Power Analysis in the RLC Circuit

Understanding power consumption involves calculating real, reactive, and apparent power:

Real Power (P):

$$P = V_{RMS}\ \ I_{RMS}\ \ cos(\phi)$$

Using the example:

$$cos(\phi) = R / |Z_{total}| \approx 100 / 118.1 \approx 0.847$$

$$P \approx 120\ V\ \ 1.016\ A\ \ 0.847 \approx 103.1\ W$$

Reactive Power (Q):

$$Q = V_{RMS} \quad I_{RMS} \quad \sin(\phi)$$

$$\sin(\phi) = (X_L - X_C) / |Z_{total}| \approx 0.529$$

So,

$$Q \approx 120 \text{ V} \quad 1.016 \text{ A} \quad 0.529 \approx 64.5 \text{ VAR}$$

Apparent Power (S):

$$S = V_{RMS} \quad I_{RMS} \approx 122.0 \text{ VA}$$

Power Factor:

$$pf = \cos(\phi) \approx 0.847$$

The circuit consumes about 103 W of real power, with reactive power of approximately 64.5 VAR, indicating a predominantly inductive nature.

Implications and Applications

Understanding how each component behaves at 1000 Hz has practical implications in various engineering fields:

- **Filter Design:** RLC circuits are fundamental in creating filters for signals at specific frequencies.
- **Power Systems:** Analyzing reactive components helps in power factor correction and system efficiency improvements.
- **Inductive and Capacitive Reactance:** The frequency-dependent behavior influences impedance matching and resonance phenomena.

Frequently Asked Questions

What is the impedance of the inductor when a 120 V RMS at 1000 Hz is applied?

The impedance of the inductor is $Z_L = j\omega L = j(2\pi \times 1000 \text{ Hz})$(inductor value in Henries). You need to know the inductance value to calculate it precisely.

How do you calculate the capacitive reactance of the capacitor at 1000 Hz?

Capacitive reactance $X_C = 1 / (2\pi fC)$. For $C = 2.00$ F and $f = 1000$ Hz, $X_C = 1 / (2\pi \times 1000 \times 2.00)\ \Omega$.

What is the total impedance of the series R-L-C circuit at 1000 Hz?

Total impedance $Z = R + j(X_L - X_C)$. Here, $R = 100\ \Omega$, $X_L = 2\pi fL$, and $X_C = 1 / (2\pi fC)$.

How do you determine the current flowing through the circuit?

The current $I = V_{rms} / |Z|$, where $|Z|$ is the magnitude of the total impedance calculated from the resistor, inductor, and capacitor.

At what frequency does the circuit reach resonance if the inductor and capacitor values are known?

Resonance occurs when $X_L = X_C$, which gives $f_{res} = 1 / (2\pi\sqrt{LC})$. For the given component values, plug in L and C to find f_{res}.

What is the phase angle between the voltage and current in this circuit?

The phase angle φ = arctangent[(X_L - X_C) / R]. If $X_L > X_C$, the circuit is inductive; if $X_C > X_L$, it is capacitive.

How does increasing the frequency affect the circuit's impedance?

Increasing the frequency increases X_L and decreases X_C, affecting the total impedance and possibly moving the circuit closer to resonance.

What is the power dissipated as heat in the resistor?

The average power $P = V_{rms} \times I_{rms} \times \cos(\varphi)$, where $\cos(\varphi)$ is the power factor. Alternatively, $P = I_{rms}^2 \times R$.

If the inductor's inductance is 0.1 H, what is the inductive reactance at 1000 Hz?

$X_L = 2\pi \times 1000\ Hz \times 0.1\ H = 628.32\ \Omega$.

Can the circuit be used to filter specific frequencies? How?

Yes, by tuning the circuit to its resonant frequency or adjusting component values to create a bandpass or bandstop filter, allowing certain frequencies to pass or be attenuated.

Additional Resources

Electrical Circuit Analysis

Introduction

In the realm of electrical engineering, understanding the behavior of circuits with various components—inductors, capacitors, and resistors—is essential for designing and troubleshooting electronic systems. The scenario under examination involves applying a 120 V RMS voltage at a frequency of 1000 Hz to a series circuit comprising an inductor, a capacitor, and a resistor. This configuration serves as an excellent example for exploring fundamental concepts such as impedance, phase relationships, resonance, and power dissipation in AC circuits.

In this detailed analysis, we will thoroughly dissect the circuit's characteristics, derive relevant parameters, and interpret the implications of the high frequency and component values. Whether you're an engineering student, a practicing professional, or an electronics enthusiast, this review aims to deepen your understanding of AC circuit behavior through practical, step-by-step explanations.

Overview of the Circuit Components

The series circuit involves three primary components:

- Inductor (L): An inductor resists changes in current and introduces inductive reactance.
- Capacitor (C): A capacitor opposes changes in voltage and introduces capacitive reactance.
- Resistor (R): A resistor opposes current flow proportionally, dissipating energy as heat.

Given parameters:

- Voltage (V_{rms}): 120 V
- Frequency (f): 1000 Hz
- Resistance (R): 100 Ω
- Capacitance (C): 2.00 F
- Inductance (L): Not explicitly given; the problem suggests analyzing the circuit with these components. For completeness, we will assume an inductance value or analyze accordingly.

Impedance in AC Circuits

In AC circuit analysis, the key to understanding the circuit's behavior lies in the concept of impedance (Z), which combines resistance and reactance into a single complex quantity. Impedance determines both the amplitude and phase of the current relative to the applied voltage.

Components' Impedances

- Resistor (R): Impedance is purely real:

$$
Z_R = R
$$

- Inductor (L): Impedance is purely imaginary:

$$
Z_L = j \omega L
$$

- Capacitor (C): Impedance is purely imaginary with negative sign:

$$
Z_C = \frac{1}{j \omega C} = -j \frac{1}{\omega C}
$$

where $\omega = 2 \pi f$ is the angular frequency.

Calculating Reactances

At 1000 Hz, the angular frequency:

$$
\omega = 2 \pi \times 1000 \approx 6283.19\, \text{rad/sec}
$$

Capacitive Reactance (X_C)

$$
X_C = \frac{1}{\omega C} = \frac{1}{6283.19 \times 2.00} \approx \frac{1}{12566.38} \approx 7.96\, \Omega
$$

This relatively small reactance indicates the capacitor offers less opposition to current at this high frequency, which is typical since capacitive reactance decreases with increasing frequency.

Inductive Reactance (X_L)

The inductance value is not specified. To proceed, let's analyze the circuit with a typical inductance value or explore how the circuit behaves for various L values. For example, assume L = 1.00 H:

$$
X_L = \omega L = 6283.19 \times 1.00 = 6283.19\, \Omega
$$

This large reactance indicates the inductor strongly opposes current at this frequency, significantly affecting the circuit's behavior.

Total Impedance of the Series Circuit

The total impedance:

$$
Z_{total} = R + j (X_L - X_C)
$$

or explicitly:

$$
Z_{total} = R + j (\omega L - \frac{1}{\omega C})
$$

The magnitude of the impedance:

$$
|Z| = \sqrt{R^2 + (X_L - X_C)^2}
$$

Case 1: With L = 1.00 H

$$
|Z| = \sqrt{(100)^2 + (6283.19 - 7.96)^2} \approx \sqrt{10,000 + (6275.23)^2}
$$

$$
|Z| \approx \sqrt{10,000 + 39,377,274} \approx \sqrt{39,387,274} \approx 6280\, \Omega
$$

This indicates the circuit is dominated by the inductor at this frequency, leading to a high total impedance and a low current.

Calculating Current and Voltage Distribution

Using Ohm's law for RMS quantities:

$$
I_{rms} = \frac{V_{rms}}{|Z|}
$$

$$
I_{rms} = \frac{120\, V}{6280\, \Omega} \approx 0.019\, A
$$

The phase angle ϕ between voltage and current:

$$
\phi = \arctan \left(\frac{X_L - X_C}{R} \right)
$$

$$
\phi = \arctan \left(\frac{6283.19 - 7.96}{100} \right) \approx \arctan (62.75) \approx 89.99^\circ
$$

This near-90° phase shift indicates the circuit is almost purely inductive at this inductance value, with the current lagging voltage by nearly 90°.

Power Considerations

The average power dissipated in the resistor:

$$
P_{avg} = V_{rms} \times I_{rms} \times \cos \phi
$$

Since the circuit is predominantly reactive (inductive), $\cos \phi$ is close to zero, implying minimal real power consumption. For the assumed parameters:

$$
\cos \phi \approx \frac{R}{|Z|} = \frac{100}{6280} \approx 0.0159
$$

Thus:

$$
P_{avg} \approx 120 \times 0.019 \times 0.0159 \approx 0.036\, W
$$

This low power dissipation highlights the reactive nature of the circuit at this high frequency.

Exploring Resonance Conditions

Resonance occurs when:

$$
X_L = X_C
$$

or

$$
\omega L = \frac{1}{\omega C}
$$

which yields:

$$
\omega^2 = \frac{1}{L C}
$$

or

$$
f_{res} = \frac{1}{2 \pi \sqrt{L C}}
$$

\]

Using the assumed $L=1\, H$:

$$
f_{res} = \frac{1}{2 \pi \sqrt{1 \times 2}} = \frac{1}{2 \pi \times 1.414} \approx \frac{1}{8.885} \approx 0.112\, \text{Hz}
$$

This is far below 1000 Hz, indicating the circuit is far from resonance at 1000 Hz. To reach resonance at 1000 Hz, the inductor would need to be:

$$
L = \frac{1}{(2 \pi f)^2 C} = \frac{1}{(6283.19)^2 \times 2} \approx \frac{1}{39,370,000} \approx 2.54 \times 10^{-8}\, H
$$

which is practically negligible. Therefore, at 1000 Hz, the circuit is inductively dominant, with minimal energy transfer to the resistor.

Practical Implications and Applications

This analysis has significant practical implications:

- Filter Design: Such a circuit can serve as a high-frequency filter, blocking or passing signals depending on the reactive components' values.
- Impedance Matching: Understanding the impedance helps in matching sources and loads for minimal reflection or maximum power transfer.
- Resonance Tuning: Adjusting L or C to achieve resonance at a target frequency allows for applications in oscillators and tuners.
- Power Dissipation: The minimal real power indicates high reactive power, which requires consideration in power system design for efficiency.

Final Thoughts

Applying a 120 V RMS voltage at 1000 Hz to a series circuit containing a resistor, inductor, and capacitor reveals the nuanced interplay of resistive and reactive elements in AC systems. The high frequency significantly reduces the capacitor's reactance while amplifying the inductor's reactance (assuming typical inductance values), resulting in a predominantly inductive circuit with a large impedance and minimal real power dissipation.

This scenario underscores the importance of component selection and frequency considerations in AC circuit design, whether for filters, oscillators, or power systems. Engineers must carefully analyze impedance and phase relationships to optimize performance, prevent undesired effects, and achieve desired circuit behaviors.

By understanding the fundamental principles demonstrated here, practitioners can better design and troubleshoot complex AC circuits, ensuring efficiency and functionality in a wide range of electronic applications.

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