A 150Hz Cosine Wave Is Sampled At A Rate Of 200Hz.a. Draw The Wave And Show The Temporal Locations At

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Introduction to Sampling of Cosine Waves

Understanding the sampling process of continuous signals is fundamental in signal processing, telecommunications, and audio engineering. When dealing with sinusoidal signals such as cosine waves, sampling determines how well the original wave can be reconstructed from discrete points. In this article, we explore the specific case of a 150Hz cosine wave sampled at a rate of 200Hz, analyze the implications of this sampling rate, and illustrate the waveform with relevant temporal locations.

Fundamentals of Cosine Wave Sampling

What is a Cosine Wave?

A cosine wave is a type of periodic signal characterized by its amplitude, frequency, and phase. Mathematically, it is expressed as:

$$ x(t) = A \cos(2\pi f t + \phi) $$

where:
- A is the amplitude,
- f is the frequency in Hertz (Hz),
- t is time in seconds,
- ϕ is the phase offset.

For our case:
- A can be any constant (commonly normalized to 1),
- $f = 150\, \text{Hz}$,
- ϕ is typically zero unless specified.

Sampling Frequency and Nyquist Criterion

Sampling converts a continuous-time signal into a discrete sequence by taking measurements at regular intervals. The sampling frequency f_s must be at least twice the highest frequency component in the signal to prevent aliasing, as per the Nyquist-Shannon sampling theorem:

$$ f_s \geq 2f_{max} $$

In our scenario:
- Signal frequency, $f = 150\, \text{Hz}$,
- Sampling rate, $f_s = 200\, \text{Hz}$.

Since $200\, \text{Hz} < 2 \times 150\, \text{Hz} = 300\, \text{Hz}$, the sampling rate is below the Nyquist rate, which implies potential aliasing.

Analyzing the Sampling of a 150Hz Cosine Wave at 200Hz

Understanding Aliasing

Aliasing occurs when the sampling rate is insufficient to capture the true frequency content of the signal. When aliasing happens, the reconstructed signal appears to have a different frequency, which can cause confusion in analysis.

The apparent frequency f_a (or alias frequency) when sampling a true frequency f at a rate f_s is given by:

$$ f_a = |f - n \times f_s| $$

where n is an integer chosen such that f_a lies within the range 0 to $f_s/2$.

In our case, since $f = 150\, \text{Hz}$ and $f_s = 200\, \text{Hz}$:

- Calculate $f - f_s$:

$$ 150\, \text{Hz} - 200\, \text{Hz} = -50\, \text{Hz} $$

- Taking the absolute value:

$$ f_a = 50\, \text{Hz} $$

This indicates that the sampled signal will appear as a 50Hz wave due to aliasing, not the original 150Hz. The perceived wave will be a lower-frequency oscillation, which is a critical consideration in signal analysis.

Practical Implication of Sampling at 200Hz

Because the sampling rate is below the Nyquist threshold, the reconstructed wave from these samples will not accurately represent the original 150Hz cosine wave. Instead, the sampled data points will mimic a 50Hz oscillation, leading to potential misinterpretation if the aliasing is not recognized.

Drawing the Wave and Showing Temporal Locations

Waveform Representation

To visualize the sampled wave and understand the temporal locations, we need to:

- Plot the original 150Hz cosine wave.
- Mark the sampling points at intervals of $1/f_s = 1/200\, \text{seconds} = 0.005\, \text{s}$.
- Show how the samples align with the wave over multiple cycles.

Steps to Draw the Wave

1. Define the time span:

To capture multiple periods, consider a time window of at least 20 milliseconds (since $T = 1/f = 1/150 \approx 6.67\, \text{ms}$), say, from 0 to 0.1 seconds.

2. Generate the continuous wave:

$$ x(t) = \cos(2\pi \times 150\, t) $$

3. Plot the wave:

- Use a fine time resolution (e.g., 0.0001 seconds).
- Plot $x(t)$ over the chosen time span.

4. Mark sampling points:

- At every $t = n \times 0.005\, \text{s}$, with $n = 0, 1, 2, \dots$.
- Record the sample values $x(n \times 0.005)$.

5. Show the sampled points on the wave:

- Use dots or circles at the sampling locations.
- Connect these points to visualize the apparent wave based on sampled data.

Temporal Locations of Sampling Points

The sampling points occur at:

$$t_n = n \times 0.005\, \text{s} \quad \text{for} \quad n=0,1,2,...$$

Example:

Sample Number (n)	Time (t_n)	Sampled Value $x(t_n)$
0	0.00 s	$\cos(0) = 1$
1	0.005 s	$\cos(2\pi \times 150 \times 0.005)$
2	0.010 s	$\cos(2\pi \times 150 \times 0.010)$
...	...	...

Calculating the first few samples:

- $t=0\, \text{s}$: $x(0) = 1$
- $t=0.005\, \text{s}$:

$$x(0.005) = \cos(2\pi \times 150 \times 0.005) = \cos(2\pi \times 0.75) = \cos(1.5 \pi) = 0$$

- $t=0.010\, \text{s}$:

$$x(0.010) = \cos(2\pi \times 150 \times 0.010) = \cos(3 \pi) = -1$$

Plotting these points reveals how the sampled data aligns with the original wave, and helps visualize the aliasing effect.

Understanding the Impact of Sampling Rate on Signal Reconstruction

Aliasing and Its Consequences

Since the sampling rate is below the Nyquist frequency, the original 150Hz wave cannot be reconstructed accurately from the samples. Instead, the samples correspond to a 50Hz wave, which can lead to:

- Misinterpretation of the original frequency.
- Artifacts in digital signal processing.
- Loss of high-frequency information.

Aliasing Example in Practice

Suppose you record a 150Hz audio tone but sample it at 200Hz. The resulting digital signal would appear as a 50Hz tone to the system, possibly leading to confusion or distortion in sound reproduction or data analysis.

How to Prevent Aliasing

To avoid aliasing, engineers typically:

- Use a sampling rate at least twice the highest frequency component.
- Apply anti-aliasing filters before sampling to remove frequencies above the Nyquist limit.
- Select appropriate sampling hardware based on the signal's frequency content.

Summary and Key Takeaways

- Sampling a 150Hz cosine wave at 200Hz results in aliasing, making the wave appear as a 50Hz oscillation.
- The sampling points occur every 5 milliseconds, which can be visualized on the waveform.
- Proper sampling rates are critical to accurately reconstruct signals; the Nyquist theorem guides this choice.
- Visualizing the waveform and temporal locations helps understand the aliasing effect and the importance of adequate sampling.

Conclusion

The process of sampling continuous signals is a cornerstone of digital signal processing. In the specific case of a 150Hz cosine wave sampled at 200Hz, the sampling rate is insufficient for perfect reconstruction, leading to aliasing and a distorted perception of the original frequency. Visual representations of the waveform and the sampling points highlight the importance of selecting appropriate sampling rates. Engineers and signal processors must carefully consider these factors to ensure accurate representation, analysis, and reconstruction of signals in various applications, from audio recording to communications systems.

Further Reading

- "Signals and Systems" by Alan V. Oppenheim and Alan S. Willsky

Frequently Asked Questions

What is the frequency of the original cosine wave in the given problem?

The original cosine wave has a frequency of 150Hz.

At what sampling rate is the 150Hz cosine wave sampled?

It is sampled at a rate of 200Hz.

Is the sampling rate sufficient to accurately sample the 150Hz cosine wave without aliasing?

No, since the Nyquist rate for a 150Hz signal is at least 300Hz, sampling at 200Hz causes aliasing.

What is the apparent or aliased frequency observed when sampling a 150Hz wave at 200Hz?

The aliased frequency can be calculated as |f - n f_s|, which results in approximately 50Hz in this case.

How do you draw the sampled wave and indicate the temporal locations?

Plot the original cosine wave over time, then mark discrete points at intervals of 1/200 seconds, showing the sampled points at these time locations.

What is the period of the original 150Hz cosine wave?

The period is 1/150 seconds, approximately 6.67 milliseconds.

What is the period between sampling points at 200Hz?

The sampling interval is 1/200 seconds, or 5 milliseconds.

How does the sampling rate affect the representation of the wave in the time domain?

Sampling at 200Hz captures only discrete points which may not accurately represent the 150Hz wave due to aliasing, leading to a distorted or lower-frequency apparent wave.

Can the original 150Hz cosine wave be perfectly reconstructed from samples taken at 200Hz?

No, because the sampling rate is below the Nyquist rate for 150Hz, causing aliasing and preventing perfect reconstruction.

What is the importance of showing temporal locations on the wave diagram?

It helps visualize when samples are taken, illustrating how sampling affects the wave's shape and highlights potential aliasing effects.

Additional Resources

Sampling Theory and the 150Hz Cosine Wave at 200Hz Sampling Rate

Understanding the fundamental principles of digital signal processing is crucial for engineers, technicians, and enthusiasts working with analog-to-digital conversion. One of the most pivotal concepts in this domain is the sampling of continuous signals, which involves measuring the signal's amplitude at discrete time intervals. In this article, we delve into a specific example: a 150Hz cosine wave sampled at a rate of 200Hz, exploring the implications of this sampling rate, visualizing the waveform, and analyzing the temporal sampling locations.

Introduction to Signal Sampling and Its Significance

Sampling serves as the bridge between the analog world—continuous signals—and the digital realm—discrete data points. When an analog signal is sampled, it is measured at specific moments in time, converting the continuous wave into a sequence of numbers that can be processed, stored, and analyzed digitally.

Key Concepts in Sampling:

- Sampling Rate (Frequency): The number of samples taken per second, measured in Hertz (Hz).
- Nyquist Frequency: Half of the sampling rate, representing the maximum frequency that can be accurately reconstructed without aliasing.
- Aliasing: The distortion or misrepresentation of a signal caused when sampling occurs below the Nyquist rate.

For the specific case of a 150Hz cosine wave sampled at 200Hz, understanding these concepts is vital to interpret the resulting digital representation and potential artifacts.

Fundamentals of the 150Hz Cosine Wave

A cosine wave at 150Hz can be mathematically expressed as:

$$
x(t) = A \cos(2\pi f t + \phi)
$$

where:
- A is the amplitude,
- $f = 150\, \text{Hz}$,
- ϕ is the phase offset,
- t is time in seconds.

For simplicity, assume $A=1$ and $\phi=0$, leading to:

$$
x(t) = \cos(2\pi \times 150 t)
$$

Characteristics:

- Period T: The duration of one cycle is $T = 1/f = 1/150 \approx 6.67\, \text{ms}$.
- Frequency Content: The wave oscillates 150 times per second, producing a high-frequency signal relative to many sampling rates.

Sampling at 200Hz: Implications and Analysis

Sampling a 150Hz signal at 200Hz introduces specific challenges and interesting phenomena, especially considering the Nyquist theorem.

Nyquist Frequency and Its Significance:

$$
f_N = \frac{f_s}{2} = \frac{200\, \text{Hz}}{2} = 100\, \text{Hz}
$$

Since the signal frequency (150Hz) exceeds the Nyquist frequency (100Hz), aliasing will occur.

What is Aliasing?

Aliasing manifests when higher frequency signals are indistinguishable from lower frequency signals after sampling. The sampled signal appears as a lower frequency sine wave, which can lead to misinterpretation or distortion.

Calculating the Alias Frequency:

Using the aliasing formula:

$$
f_{alias} = |f - n \times f_s|
$$

where n is an integer that minimizes the difference.

Find n:

$$
n = \text{round}\left(\frac{f}{f_s}\right) = \text{round}\left(\frac{150}{200}\right) = 1
$$

Calculate:

$$

f_{alias} = |150 - 1 \times 200| = |150 - 200| = 50\, \text{Hz}
\]

Result: Sampling the 150Hz wave at 200Hz results in an alias wave of approximately 50Hz. This means that in the digital representation, the wave appears as a 50Hz cosine wave rather than the original 150Hz.

Visualizing the Wave: Drawing and Temporal Sampling Locations

To better comprehend the phenomena, visual representation is crucial. Although a static article can't display images directly, detailed descriptions and instructions will guide you through drawing and understanding the waveform and sampling points.

Drawing the Continuous 150Hz Cosine Wave

Steps to draw the original wave:

1. Time Axis: Draw a horizontal axis representing time, marking intervals of 1 ms to cover at least two periods (~13.33 ms).
2. Amplitude Axis: Draw a vertical axis representing amplitude, centered at zero.
3. Plotting the Wave:
- Since \(T = 6.67\, \text{ms}\), the wave completes one cycle every 6.67 ms.
- Mark points at multiples of \(T\), such as 0 ms, 6.67 ms, 13.33 ms, noting the cosine's value at these points:
- \(t=0\): \(\cos(0) = 1\)
- \(t=6.67\, \text{ms}\): \(\cos(2\pi \times 150 \times 6.67 \times 10^{-3}) = \cos(2\pi \times 150 \times 0.00667) = \cos(2\pi \times 1) = 1\)
- \(t=3.33\, \text{ms}\): \(\cos(\pi) = -1\)
- Sketch a smooth cosine wave passing through these points, showing the periodic oscillation.

Sampling at 200Hz: Temporal Locations

Sampling Interval:

\[
T_s = \frac{1}{f_s} = \frac{1}{200} = 5\, \text{ms}
\]

Sampling points occur every 5 ms:

- 0 ms
- 5 ms
- 10 ms
- 15 ms
- 20 ms
- ... and so on.

Sampling Points and Their Corresponding Signal Values:

Calculate the sampled amplitude $x[n]$:

$$
x[n] = \cos(2\pi \times 150 \times n T_s)
$$

where $n=0,1,2,...$.

At each sampling point:

- n=0 (0 ms): $\cos(0) = 1$
- n=1 (5 ms): $\cos(2\pi \times 150 \times 0.005) = \cos(2\pi \times 0.75) = \cos(1.5\pi)=0$
- n=2 (10 ms): $\cos(2\pi \times 150 \times 0.01) = \cos(3\pi) = -1$
- n=3 (15 ms): $\cos(2\pi \times 150 \times 0.015) = \cos(4.5\pi) = 0$
- n=4 (20 ms): $\cos(6\pi) = 1$

Plot these points on the graph, showing that the sampled points alternate between 1, 0, -1, 0, 1, etc.

Visual Interpretation:

- The sampled points do not align perfectly with the original wave's peaks and troughs.
- The pattern of sampled points suggests a lower frequency wave of 50Hz (as derived earlier), confirming aliasing.

Understanding the Alias Effect Through the Wave and Sampling Points

The critical insight from this analysis is the phenomenon of aliasing:

- The original 150Hz wave, when sampled at 200Hz, appears as a 50Hz wave.
- The sampled points’ pattern reflects this lower frequency, with the wave seemingly oscillating more slowly than it actually does.

Practical Implications:

- Reconstruction issues: Attempting to reconstruct the original 150Hz wave from these samples would fail without additional filtering because the samples contain misrepresented frequency content.
- Design considerations: To accurately capture a 150Hz wave, the sampling rate should be at least twice the frequency (i.e., ≥ 300Hz). Sampling at 200Hz violates the Nyquist criterion, leading to aliasing.

Conclusion: The Criticality of Appropriate Sampling Rates

This case study vividly illustrates the importance of selecting an appropriate sampling rate in digital signal processing. Sampling a 150Hz cosine wave at 200Hz results in aliasing, transforming the original high-frequency signal into a lower-frequency artifact that can mislead analysis and processing.

Key Takeaways:

- Sampling rate must be at least twice the highest frequency component (Nyquist rate) to prevent aliasing.
- Sampling below the Nyquist rate causes high-frequency signals to appear as lower-frequency waves.
- Visualization of sampling points reveals the distortions and aliasing effects, emphasizing the need for proper sampling strategies.

Final Note:

In real-world applications, engineers employ anti-aliasing filters before sampling to eliminate high-frequency components beyond the Nyquist limit, ensuring accurate digital representations of analog signals. This example underscores the importance of understanding sampling theory fundamentals for effective signal processing and system design.

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