A 4.6pf Capacitor Initially Uncharged Is Connected In Series With 7.50kohm Resistor And An Emf Source

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When a 4.6 picofarad (pf) capacitor initially uncharged is connected in series with a 7.50 kilo-ohm (kΩ) resistor and an electromotive force (emf) source, a fascinating electrical process begins. This setup exemplifies the fundamental principles of RC (resistor-capacitor) circuits, which are widely studied for their transient response characteristics. Understanding how the capacitor charges over time, the behavior of the current, and the voltage distribution across the circuit components provides valuable insights into both theoretical and practical aspects of electrical engineering.

In this article, we delve into the detailed analysis of this circuit, exploring the initial conditions, the charging process, the mathematical modeling, and real-world applications. Whether you're a student, educator, or professional, grasping these concepts enhances your comprehension of transient circuits and their significance in various electronic systems.

Understanding the Circuit Components and Initial Conditions

Components Overview

The circuit consists of three primary components:

- Capacitor (C): 4.6 pf, initially uncharged, which means the voltage across it is zero at the start.
- Resistor (R): 7.50 kΩ, which limits the current flow and influences the charging rate.
- Emf Source (V): A constant voltage source that drives the charging process.

Initial Conditions

At the moment of connection (t = 0), the capacitor is uncharged, so:

- Voltage across the capacitor, $V_C(0) = 0$
- Initial current, $I(0) = \frac{V}{R}$, assuming the emf source voltage V is known
- The capacitor begins to accumulate charge, leading to a gradual increase in voltage across it

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Analyzing the Charging Process in the RC Circuit

Theoretical Background

The charging of a capacitor in an RC circuit follows an exponential pattern. When connected in series with a resistor and a voltage source, the voltage across the capacitor, $V_C(t)$, increases over time until it equals the emf source voltage, V.

The fundamental differential equation governing this process is:

$$
V = V_C(t) + R \cdot I(t)
$$

Since $I(t) = C \frac{dV_C(t)}{dt}$, the equation becomes:

$$
V = V_C(t) + R C \frac{dV_C(t)}{dt}
$$

Rearranged to solve for $V_C(t)$:

$$
\frac{dV_C(t)}{dt} + \frac{1}{R C} V_C(t) = \frac{V}{R C}
$$

This is a first-order linear differential equation with the solution:

$$
V_C(t) = V \left(1 - e^{-\frac{t}{R C}}\right)
$$

The current as a function of time is:

$$
I(t) = \frac{V}{R} e^{-\frac{t}{R C}}
$$

Calculating the Time Constant

The time constant τ characterizes how quickly the capacitor charges:

\[

$$
\tau = R \times C
$$

Given:

- $R = 7.50\, \text{k}\Omega = 7,500\, \Omega$
- $C = 4.6\, \text{pf} = 4.6 \times 10^{-12}\, \text{F}$

Calculating:

$$
\tau = 7,500 \times 4.6 \times 10^{-12} = 3.45 \times 10^{-8}\, \text{s}
$$

This extremely small time constant indicates that the capacitor charges very rapidly, practically instantaneously from a typical measurement perspective.

Practical Implications of the Circuit Behavior

Voltage Across the Capacitor Over Time

The voltage across the capacitor increases exponentially:

$$
V_C(t) = V \left(1 - e^{-\frac{t}{\tau}}\right)
$$

- Initially (at $t=0$), $V_C(0) = 0$
- After a time much greater than τ, $V_C(t) \to V$

Because τ is extremely small in this case, the capacitor charges to nearly the emf source voltage almost immediately.

Current Decay in the Circuit

The current decreases exponentially:

$$
I(t) = \frac{V}{R} e^{-\frac{t}{\tau}}
$$

- Starts at $I(0) = \frac{V}{R}$
- Approaches zero as $t \to \infty$

This rapid decay signifies that the initial surge of current diminishes quickly as the capacitor becomes fully charged.

Energy Stored in the Capacitor

The energy stored when fully charged is:

$$
U = \frac{1}{2} C V^2
$$

Given the tiny capacitance, the stored energy is minimal, but the principles remain the same.

Real-World Applications and Significance

High-Frequency Circuit Applications

The extremely small time constant indicates that such a circuit responds very quickly, making it suitable in high-frequency applications, such as:

- RF (Radio Frequency) circuits
- Fast switching circuits
- Pulse shaping and timing circuits

Measurement and Instrumentation

Understanding transient responses helps in designing accurate measurement systems for capacitance, voltage, and current, especially in systems requiring rapid data acquisition.

Educational Demonstrations

This circuit serves as an excellent demonstration of the fundamental principles of capacitor charging, exponential decay, and transient analysis for students learning circuit theory.

Summary and Key Takeaways

- A 4.6 pf capacitor initially uncharged, when connected in series with a 7.50 kΩ resistor and an emf source, exhibits rapid charging behavior due to its tiny capacitance.

- The charging process follows an exponential pattern governed by the time constant $\tau = R C$.
- The voltage across the capacitor approaches the emf source voltage almost instantaneously because of the small τ.
- The current in the circuit decays exponentially from its initial maximum value.
- Practical applications of such circuits are found in high-frequency electronics, rapid switching, and transient response analysis.
- The principles demonstrated are fundamental for understanding more complex electronic systems involving capacitors.

Conclusion

Understanding the dynamics of a tiny capacitor like the 4.6 pf in series with a resistor and emf source illustrates core concepts of transient circuit behavior. Despite its minuscule size, this setup encapsulates the fundamental exponential charging process and highlights the importance of parameters like resistance, capacitance, and voltage in determining circuit response times. Mastering these principles is essential for designing and analyzing advanced electronic devices and systems across various technological fields.

Keywords: RC circuit, capacitor charging, exponential response, transient analysis, time constant, high-frequency electronics, electrical engineering, transient response, circuit analysis

Frequently Asked Questions

What is the initial behavior of a 4.6 pF capacitor connected in series with a 7.50 kΩ resistor and an emf source at the moment of connection?

Initially, the capacitor is uncharged, so it acts like a short circuit, causing a current to flow rapidly through the resistor until the capacitor begins to charge.

How does the voltage across the capacitor evolve over time in this RC circuit?

The voltage across the capacitor increases exponentially from zero toward the emf source voltage, following the relation $V(t) = V_0(1 - e^{(-t/RC)})$, where R is the resistance and C is the capacitance.

What is the time constant of this circuit, and what does it signify?

The time constant $\tau = RC = 7{,}500\ \Omega \times 4.6\ pF \approx 34.5$ nanoseconds. It signifies the time it takes for the capacitor to charge up to about 63.2% of the emf source voltage.

How long does it take for the capacitor to reach near full charge in this circuit?

Since the time constant is approximately 34.5 ns, the capacitor reaches over 99% of the emf source voltage after about 5τ, which is roughly 172.5 ns.

What is the final voltage across the capacitor once fully charged?

The voltage across the capacitor will eventually be equal to the emf source voltage, as it becomes fully charged, with no further current flow.

How does the small capacitance value of 4.6 pF affect the charging process compared to larger capacitors?

A very small capacitance like 4.6 pF results in a very short time constant, meaning the capacitor charges almost instantaneously relative to larger capacitors, and the voltage reaches near the source voltage very quickly.

Additional Resources

A 4.6pf Capacitor Initially Uncharged Is Connected In Series With 7.50kohm Resistor And An Emf Source

In the realm of electrical engineering, understanding the transient behavior of RC (resistor-capacitor) circuits is fundamental to designing and analyzing a wide array of electronic systems. One particularly instructive scenario involves connecting a tiny capacitor—specifically, a 4.6 picofarad (pf) capacitor—that initially holds no charge in series with a resistor and an electromotive force (emf) source. Such an arrangement provides insight into the processes of charging, discharging, and the dynamic response of capacitive components in circuits.

This article delves into the principles governing this setup, exploring the fundamental equations, the temporal evolution of voltage and current, and the physical implications of the circuit's behavior. Whether you're a student seeking clarity or a seasoned engineer refining your understanding, this comprehensive discussion aims to illuminate the intricacies of this classic RC configuration.

Understanding the Circuit Components and Configuration

The Capacitor: A Tiny Reservoir of Charge

At the heart of this circuit is a capacitor with a capacitance of 4.6 picofarads (pf). To contextualize, 1 picofarad is 10-12 farads, making this capacitor exceptionally small. Such minute capacitance values are typical in high-frequency applications and integrated circuits, where parasitic capacitances can influence circuit behavior significantly.

A capacitor's primary role is to store electrical energy in an electric field between its plates. When uncharged, it has zero potential difference across its terminals. As charge accumulates, the voltage across the capacitor increases proportionally, according to the relation:

VC(t) = Q(t)/C

where Q(t) is the charge at time t, and C is the capacitance.

The Resistor: The Limiter of Current

Connected in series with the capacitor is a resistor with a resistance of 7.50 kilo-ohms (kΩ). Resistors oppose the flow of current, and their value directly affects how quickly a capacitor charges or discharges. The resistor's presence introduces a time-dependent behavior characterized by the circuit's time constant.

The EMF Source: Providing the Driving Voltage

The circuit includes an emf source, such as a battery or a DC power supply, which maintains a constant voltage throughout the process. The emf's value influences the maximum charge the capacitor can eventually attain and the initial current flow when the circuit is closed.

The Series Connection: A Pathway for Charge Flow

When these components are connected in series, the current that flows from the emf source passes through the resistor and then charges the capacitor. Initially, with the capacitor uncharged, the current is at its maximum, gradually decreasing as the capacitor accumulates charge and its voltage approaches the emf's voltage.

Initial Conditions and Circuit Behavior at t=0

Initial Uncharged State

Before closing the circuit, the capacitor is uncharged, meaning $Q(0) = 0$, and the voltage across it is zero, $V_C(0) = 0$. At this moment, the entire emf voltage appears across the resistor, resulting in the maximum initial current.

Applying Kirchhoff's Voltage Law

At t=0, the sum of voltage drops across the resistor and capacitor must equal the emf (E):

$$E = V_R(t) + V_C(t)$$

Initially, $V_C(0) = 0$, so the initial current is:

$$I(0) = E / R$$

This initial current determines how quickly the capacitor begins to charge.

The Mathematical Model of Charging: Deriving the Equations

Fundamental Differential Equation

The dynamic behavior of the circuit can be captured by applying Kirchhoff's voltage law (KVL):

$$E = V_R(t) + V_C(t)$$

Since $V_R(t) = I(t) \times R$, and $I(t) = dQ/dt$, with $Q(t) = C \times V_C(t)$, we get:

$$E = R \times dQ/dt + Q(t)/C$$

Rearranged:

$$R \times dQ/dt + Q(t)/C = E$$

This is a first-order linear differential equation describing how the charge on the capacitor evolves over time.

Solution to the Differential Equation

The differential equation:

dQ/dt + (1/RC) × Q(t) = E / R

has the general solution:

Q(t) = Qfinal + [Q(0) - Qfinal] × e-t / (RC)

where:

- Qfinal = C × E (the steady-state charge when the capacitor is fully charged)
- Q(0) = 0 (initial charge)

Substituting:

Q(t) = C × E × [1 - e-t / (RC)]

Correspondingly, the voltage across the capacitor:

VC(t) = E × [1 - e-t / (RC)]

And the current:

I(t) = dQ/dt / C = (E / R) × e-t / (RC)

Time Constant and Circuit Dynamics

Calculating the Time Constant (τ)

The RC time constant, τ, is a key parameter dictating how quickly the circuit responds:

τ = R × C

Given R = 7.50 kΩ = 7,500 Ω and C = 4.6 pf = 4.6 × 10-12 F:

τ = 7,500 × 4.6 × 10-12 = 3.45 × 10-8 seconds

This extremely small time constant indicates that the capacitor charges almost instantaneously relative to most practical timescales.

Physical Interpretation of τ

- Rapid Charging: Since τ is on the order of tens of nanoseconds, the capacitor reaches near its maximum charge almost immediately after the circuit is closed.
- Voltage Evolution: The voltage across the capacitor approaches the emf exponentially, with approximately 63.2% of the final voltage appearing after one τ.
- Current Decay: The initial current of I(0) = E / R diminishes exponentially to zero as the capacitor becomes fully charged.

Quantitative Analysis: Numerical Insights

Estimating Initial Current

Suppose the emf source provides 12 V:

$I(0) = E / R = 12\ V / 7{,}500\ \Omega \approx 1.6 \times 10^{-3}\ A$ (1.6 mA)

This initial current is substantial given the tiny capacitance, but the rapid charging means it diminishes very quickly.

Voltage Across the Capacitor Over Time

Using:

$V_C(t) = E \times [1 - e^{-t / \tau}]$

At $t = \tau$ (≈ 34.5 ns):

$V_C(\tau) \approx 12\ V \times (1 - e^{-1}) \approx 12\ V \times (1 - 0.368) \approx 7.57\ V$

Within a few τs, the capacitor voltage is very close to the emf voltage, illustrating swift charging.

Physical and Practical Implications

High-Frequency Applications

Due to the extremely small capacitance and rapid response, this circuit exemplifies behavior relevant to high-frequency circuits, such as RF components and parasitic capacitances in integrated circuits.

Limitations and Real-world Considerations

- Parasitic Inductances: At such quick timescales, inductive effects may become non-negligible.
- Component Tolerances: Real capacitors and resistors have manufacturing tolerances that can affect precise behavior.
- Measurement Challenges: Capturing nanosecond-scale voltage changes demands specialized equipment like high-bandwidth oscilloscopes.

Thermal and Dielectric Considerations

- The tiny capacitor may be sensitive to dielectric breakdown or other failure modes at high voltages or currents, though typically negligible at low voltages.

Broader Context and Applications

Design of High-Speed Circuits

Understanding the transient response of small capacitors in series with resistors aids in designing high-speed digital and analog circuits, ensuring signal integrity and timing accuracy.

Signal Filtering and Timing

RC circuits serve as filters, timers, and pulse-shaping devices. The rapid charging characteristic of a 4.6 pf capacitor with a high resistance can be exploited in precise timing applications.

Educational and Research Significance

This configuration offers a textbook example for educational purposes, demonstrating fundamental principles of circuit analysis, exponential charge/discharge, and the importance of the time constant.

Conclusion

Connecting a

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