

A 5.0 Kg Solid Cylinder Of Radius 12 Cm Rotates With $\omega = 3.7$ Rad/s About An Axis Through Its Center.

A 5.0 Kg Solid Cylinder Of Radius 12 Cm Rotates With $\omega = 3.7$ Rad/s About An Axis Through Its Center. Understanding the rotational dynamics of solid cylinders is fundamental in physics and engineering applications. Whether designing mechanical components, analyzing rotational motion, or exploring fundamental physics principles, grasping how a cylinder's mass, radius, and angular velocity interact provides valuable insights. In this comprehensive article, we'll delve into the physics behind a solid cylinder rotating about its central axis, exploring key concepts such as moment of inertia, angular momentum, torque, and energy considerations. Let's explore the fascinating world of rotational motion step-by-step.

Fundamentals of Rotational Motion

What Is a Solid Cylinder?

A solid cylinder is a three-dimensional object characterized by a circular base and uniform mass distribution throughout its volume. In our case:

- Mass (m): 5.0 kg
- Radius (r): 12 cm (or 0.12 meters)
- Angular velocity (ω): 3.7 rad/s

These parameters form the basis for analyzing the rotational behavior of the cylinder.

Understanding Angular Velocity (ω)

Angular velocity (ω) describes how fast an object rotates around an axis. Measured in radians per second (rad/s), it indicates the rate of change of the angular position over time. For our cylinder:

- $\omega = 3.7$ rad/s implies that the cylinder completes a certain number of rotations per second, which can be calculated as:

$$\text{Rotations per second} = \frac{\omega}{2\pi} \approx \frac{3.7}{6.2832} \approx 0.589 \text{ rotations/sec}$$

This indicates the cylinder completes roughly half a rotation each second.

Moment of Inertia of a Solid Cylinder

One of the most critical aspects in rotational dynamics is the moment of inertia (I), which quantifies an object's resistance to changes in its rotational motion. For a solid cylinder rotating about its central axis:

- The moment of inertia is given by:

$$I = \frac{1}{2} m r^2$$

- Substituting the known values:

$$I = \frac{1}{2} \times 5.0 \text{ kg} \times (0.12 \text{ m})^2$$

$$I = 2.5 \times 0.0144$$

$$I = 0.036 \text{ kg}\cdot\text{m}^2$$

This value indicates the cylinder's resistance to angular acceleration.

Significance of Moment of Inertia

- The larger the I , the harder it is to change the cylinder's rotational speed.
- For a solid cylinder, I depends solely on its mass and radius, assuming uniform density.

Calculating Angular Momentum

What Is Angular Momentum?

Angular momentum (L) is a vector quantity representing the rotational equivalent of linear momentum. It describes the quantity of rotation an object has, considering both its inertia and angular velocity:

$$L = I \omega$$

- For our cylinder:

$$L = 0.036 \text{ kg}\cdot\text{m}^2 \times 3.7 \text{ rad/sec}$$

$$L \approx 0.1332 \text{ kg}\cdot\text{m}^2/\text{s}$$

This angular momentum indicates how much "rotational inertia" the cylinder possesses at its current angular velocity.

Importance in Physics and Engineering

- Conservation of angular momentum explains how a rotating cylinder maintains its angular velocity unless acted upon by external torque.
- Knowing L helps in designing systems involving rotational motion, such as flywheels and rotating machinery.

Rotational Kinetic Energy

Understanding Rotational Energy

Rotational kinetic energy (KE_{rot}) is the energy stored in a rotating object due to its angular motion:

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

- Substituting the known values:

$$KE_{\text{rot}} = \frac{1}{2} \times 0.036 \times (3.7)^2$$

$$KE_{\text{rot}} = 0.018 \times 13.69$$

$$KE_{\text{rot}} \approx 0.2464 \text{ Joules}$$

This small value indicates how much energy is involved in maintaining the cylinder's rotation at the given angular velocity.

Applications of Rotational Kinetic Energy

- Energy calculations are critical in systems like turbines, flywheels, and gyroscopes.
- Understanding energy transfer during acceleration or deceleration is essential in mechanical system design.

Torque and Angular Acceleration

What Is Torque?

Torque (τ) is the measure of the force causing an object to rotate. It is related to angular acceleration (α) via the rotational form of Newton's second law:

$$\tau = I \alpha$$

- If an external torque is applied, the angular velocity will change over time.

Calculating Torque for Given Angular Acceleration

Suppose an external torque causes an angular acceleration of 2 rad/sec^2 :

$$\tau = 0.036 \times 2$$

$$\tau = 0.072 \text{ Nm}$$

This torque is required to change the cylinder's angular velocity at that rate.

Practical Examples of Torque Application

- Starting or stopping rotating machinery.
- Accelerating or decelerating rotating components in engines or turbines.

Energy and Power in Rotational Systems

Power in Rotational Motion

Power (P) is the rate at which work is done or energy is transferred. For rotational systems, power can be expressed as:

$$P = \tau \omega$$

- For a torque of 0.072 Nm and angular velocity of 3.7 rad/s:

$$P = 0.072 \times 3.7$$

$$P \approx 0.2664 \text{ Watts}$$

This indicates how much energy per second is involved in maintaining the rotation under these conditions.

Energy Considerations in Mechanical Design

- Efficient energy transfer minimizes losses.
- Proper understanding of rotational energy helps in designing energy-efficient motors and rotors.

Real-World Applications of Rotating Solid Cylinders

Industrial Machinery

Solid cylinders are foundational in various machines:

- Flywheels to store rotational energy.
- Rotors in turbines and motors.
- Axles in rotating shafts.

Physics Education

Studying the rotation of cylinders helps students grasp core concepts of rotational dynamics, moment of inertia, and energy conservation.

Engineering and Design

Engineers leverage knowledge of rotational physics to optimize the design of:

- Mechanical systems requiring precise control of rotation.
- Components subjected to dynamic forces.

Summary and Key Takeaways

- The moment of inertia for a solid cylinder about its central axis is $I = \frac{1}{2} m r^2$.
- For a 5.0 kg cylinder with a radius of 12 cm, $I \approx 0.036 \text{ kg}\cdot\text{m}^2$.
- The angular momentum at 3.7 rad/s is approximately $0.1332 \text{ kg}\cdot\text{m}^2/\text{s}$.
- Rotational kinetic energy at this angular velocity is about 0.246 Joules.
- External torque influences angular acceleration, governed by $\tau = I \alpha$.
- Power associated with rotation can be calculated as $P = \tau \omega$.

Understanding these principles is essential across physics, mechanical engineering, and applied sciences, providing a foundation for analyzing and designing systems involving rotational motion.

Additional Resources

- Textbooks on classical mechanics and rotational dynamics.
- Online simulators demonstrating rotational motion.
- Engineering guides on designing rotating machinery.

By mastering the physics of a solid cylinder's rotation, students and engineers can better predict system behavior, optimize designs, and innovate in fields relying on rotational mechanics.

Frequently Asked Questions

What is the moment of inertia of a 5.0 kg solid cylinder rotating about its central axis?

The moment of inertia I for a solid cylinder about its central axis is given by $I = \frac{1}{2} M R^2$. Substituting $M=5.0 \text{ kg}$ and $R=0.12 \text{ m}$, $I = \frac{1}{2} 5.0 (0.12)^2 = 0.036 \text{ kg}\cdot\text{m}^2$.

How do you calculate the rotational kinetic energy of the cylinder?

The rotational kinetic energy is given by $KE = \frac{1}{2} I \omega^2$. Using $I = 0.036 \text{ kg}\cdot\text{m}^2$ and $\omega = 3.7 \text{ rad/s}$, $KE = \frac{1}{2} 0.036 (3.7)^2 \approx 0.25 \text{ Joules}$.

What is the angular velocity of the cylinder in revolutions per

minute (rpm)?

Convert ω from rad/s to rpm: $\text{rpm} = (\omega / 60) \cdot (2\pi)$. So, $\text{rpm} = (3.7 / 60) \cdot (2 \cdot 3.1416) \approx 35.4$ rpm.

If the cylinder slows down to 2.0 rad/s, what is its new rotational kinetic energy?

Using $KE = (1/2) I \omega^2$, with $\omega = 2.0$ rad/s: $KE = 0.5 \cdot 0.036 \cdot (2.0)^2 = 0.072$ Joules.

What is the torque required to maintain the initial angular velocity of 3.7 rad/s if frictional torque is 0.005 Nm?

The torque τ needed to counteract friction is $\tau = \text{frictional torque} = 0.005$ Nm.

How much work is done to accelerate the cylinder from rest to 3.7 rad/s?

Work done $W = \text{change in rotational kinetic energy} = 0.25$ Joules (initial) - 0 Joules (initial rest) = 0.25 Joules.

What is the linear tangential speed at the outer surface of the cylinder?

Tangential speed $v = \omega R = 3.7 \cdot 0.12 \approx 0.444$ m/s.

If the cylinder rolls without slipping, what is its linear acceleration if it accelerates angularly at 0.5 rad/s²?

Since $v = \omega R$, linear acceleration $a = \alpha R = 0.5 \cdot 0.12 = 0.06$ m/s².

What is the significance of the moment of inertia in this context?

The moment of inertia determines how much torque is needed to change the cylinder's rotational speed and reflects its resistance to angular acceleration.

Can you verify the units used in calculating the moment of inertia and kinetic energy?

Yes, the moment of inertia units are $\text{kg} \cdot \text{m}^2$, and kinetic energy units are Joules ($\text{kg} \cdot \text{m}^2/\text{s}^2$), ensuring consistency and correctness in calculations.

Additional Resources

A 5.0 Kg Solid Cylinder of Radius 12 Cm Rotates with Angular Velocity 3.7 Rad/s About an Axis Through Its Center

Introduction

Understanding the rotational dynamics of solid bodies is fundamental in fields ranging from mechanical engineering to physics education. A quintessential example illustrating these principles involves a solid cylinder—a common shape in machinery, industrial applications, and everyday objects—rotating about its central axis. In this analysis, we examine a solid cylinder with a mass of 5.0 kilograms, a radius of 12 centimeters, rotating with an angular velocity of 3.7 radians per second. Through a detailed exploration of its physical properties, moments of inertia, rotational kinetic energy, and related concepts, we aim to provide a comprehensive understanding of the dynamics involved.

Physical Characteristics of the Cylinder

Geometry and Mass Distribution

The object under consideration is a solid cylinder characterized by:

- Mass (m): 5.0 kg
- Radius (r): 12 cm = 0.12 meters

The geometry of the cylinder significantly influences its rotational behavior. As a solid object, its mass is uniformly distributed throughout its volume, which affects calculations of its moment of inertia and kinetic energy during rotation.

Significance of the Dimensions

The radius of 12 cm places this cylinder within a typical size range seen in mechanical components such as rollers, pulleys, or flywheels. Its mass, combined with the radius, determines how much rotational energy it can store and how resistant it is to changes in its rotational state.

Moment of Inertia: The Rotational Analogue of Mass

Definition and Importance

The moment of inertia (I) quantifies an object's resistance to changes in its rotational motion about a specified axis. It depends on both the mass distribution and the axis of rotation. For a solid cylinder rotating about its central axis, the moment of inertia is given by:

$$I = \frac{1}{2} m r^2$$

This formula applies because of the uniform mass distribution in a solid cylinder.

Calculation for the Given Cylinder

Substituting the known values:

$$I = \frac{1}{2} \times 5.0 \, \text{kg} \times (0.12 \, \text{m})^2$$

$$I = 0.5 \times 5.0 \times 0.0144$$

$$I = 2.5 \times 0.0144 = 0.036 \, \text{kg}\cdot\text{m}^2$$

Result: The moment of inertia of the cylinder about its central axis is $0.036 \, \text{kg}\cdot\text{m}^2$.

Significance of the Moment of Inertia

This value reflects how the distribution of mass affects rotational inertia. A larger radius or mass would increase the moment of inertia, making the body more resistant to changes in rotational velocity. Conversely, a smaller moment indicates a body that can accelerate or decelerate more easily when subjected to torques.

Rotational Kinetic Energy

Fundamental Concept

Rotational kinetic energy (K) is the energy stored in an object due to its rotation, analogous to linear kinetic energy. It is given by:

$$K = \frac{1}{2} I \omega^2$$

where:

- I is the moment of inertia
- ω is the angular velocity

Calculation for the Given Cylinder

Using the previously calculated I and the given angular velocity ($\omega = 3.7 \, \text{rad/s}$):

$$K = \frac{1}{2} \times 0.036 \, \text{kg}\cdot\text{m}^2 \times (3.7)^2$$

$$K = 0.018 \times 13.69$$

$$K \approx 0.246 \, \text{Joules}$$

Result: The rotational kinetic energy of the cylinder is approximately 0.246 Joules.

Interpretation and Implications

While 0.246 Joules might seem modest, for such a small object, it signifies a notable amount of stored energy. This energy can be harnessed in applications like flywheels designed to smooth out power

delivery or to store energy temporarily. The energy capacity scales with both the moment of inertia and the square of the angular velocity, emphasizing the importance of rotational speed in energy storage systems.

Angular Momentum: The Rotational Momentum of the Cylinder

Definition and Calculation

The angular momentum (L) of a rotating body is given by:

$$L = I \omega$$

For our cylinder:

$$L = 0.036 \, \text{kg} \cdot \text{m}^2 \times 3.7 \, \text{rad/s}$$

$$L \approx 0.1332 \, \text{kg} \cdot \text{m}^2/\text{s}$$

This quantity reflects the cylinder's resistance to changes in its rotational state and is conserved in the absence of external torques.

Dynamics and External Influences

Torque and Angular Acceleration

If an external torque (τ) is applied, the angular velocity changes according to:

$$\tau = I \alpha$$

where (α) is the angular acceleration. Understanding the moment of inertia allows engineers and physicists to predict how quickly an object responds to applied forces.

Effect of Friction and Resistance

In real-world applications, factors such as bearing friction, air resistance, and other damping forces gradually decrease the cylinder's angular velocity unless energy is supplied externally. Analyzing such effects requires incorporating damping coefficients and energy loss mechanisms into the dynamics.

Practical Applications and Broader Context

Engineering Applications

- Flywheels: The cylinder's rotational inertia makes it suitable as a flywheel component, storing kinetic energy and releasing it to smooth out fluctuations in power supply.
- Rotational Machinery: Understanding inertia and energy storage helps in designing efficient motors,

turbines, and pulleys.

- Sports Equipment: Rotating cylinders are common in sports gear like weights attached to spinning apparatus, where rotational inertia influences performance.

Scientific Relevance

- Physics Education: The example serves as an excellent pedagogical tool for illustrating fundamental concepts like moments of inertia, kinetic energy, and angular momentum.
- Research: Precise calculations of rotational properties underpin experiments in material science, mechanical design, and advanced physics research.

Advanced Considerations

Rotational Dynamics in Non-ideal Conditions

In real systems, factors such as non-uniform mass distribution, material imperfections, and external forces complicate the simple models. Advanced analysis involves:

- Analyzing torque-induced angular acceleration with non-uniform density models.
- Studying energy dissipation mechanisms like air drag and bearing friction.
- Applying conservation laws in complex systems with multiple interacting bodies.

Rotational Stability and Control

- Engineers often analyze the stability of rotating bodies to prevent wobbling or imbalance.
- Control systems may be implemented to regulate angular velocity, especially in precision instruments or aerospace applications.

Conclusion

The rotation of a solid cylinder, characterized by its mass, radius, and angular velocity, encapsulates fundamental principles of rotational mechanics. This analysis underscores the importance of understanding moments of inertia, kinetic energy, and angular momentum in practical and theoretical contexts. Whether in designing efficient machinery, advancing physics education, or innovating energy storage solutions, the insights derived from such fundamental calculations are invaluable. As technology progresses, mastering rotational dynamics remains central to engineering innovations and scientific discoveries, with the simple yet profound example of a rotating cylinder serving as a cornerstone of this knowledge.

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