A Basket Contains 15 Apples, Of Which Two Are Rotten. A Sample Of Three Apples Is Selected At Random.

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Understanding the probability of selecting rotten versus fresh apples from a basket is a fascinating topic that combines elements of combinatorics and probability theory. This scenario involves a basket with a total of 15 apples, two of which are rotten, and the process of randomly selecting a sample of three apples. Such problems are common in statistics and are often used to illustrate fundamental concepts like probability calculations, sampling methods, and the impact of composition on outcomes.

In this article, we will explore the various aspects of this problem, including how to calculate the probability of selecting a certain number of rotten apples, the total possible samples, and the practical applications of these calculations. We will also discuss related concepts and provide step-by-step methods to approach similar problems.

Understanding the Composition of the Basket

Before diving into probability calculations, it’s essential to understand the composition of the basket:

- Total apples: 15
- Rotten apples: 2
- Good apples: 13

This basic breakdown sets the foundation for calculating various probabilities related to random sampling.

Sampling Without Replacement

In this scenario, sampling is done without replacement, meaning once an apple is selected, it is not put back into the basket before the next selection. This affects the probabilities at each step, as the composition of the basket changes with each pick.

Why Sampling Without Replacement Matters

- It reduces the total number of apples after each draw.
- Probabilities depend on previous selections.
- It makes calculations slightly more complex compared to sampling with replacement.

Calculating Total Number of Possible Samples

The total number of ways to select 3 apples from 15 can be calculated using combinations:

$$ \text{Total samples} = \binom{15}{3} $$

Calculating this:

$$ \binom{15}{3} = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455 $$

Thus, there are 455 possible different samples of three apples.

Calculating Probabilities of Different Outcomes

The key question is: What is the probability of selecting a sample with a certain number of rotten apples? The possible outcomes include:

- 0 rotten apples (all good)
- 1 rotten apple
- 2 rotten apples

We will analyze each case separately.

Probability of Selecting 0 Rotten Apples

To select 0 rotten apples, all three apples must be from the 13 good apples.

Number of favorable samples:

$$ \binom{13}{3} = \frac{13 \times 12 \times 11}{3 \times 2 \times 1} = 286 $$

Total possible samples: 455

Probability:

$$ P(0 \text{ rotten}) = \frac{\binom{13}{3}}{\binom{15}{3}} = \frac{286}{455} \approx 0.628 $$

Probability of Selecting 1 Rotten Apple

For exactly one rotten apple:

- Select 1 rotten apple from the 2 rotten ones: $\binom{2}{1} = 2$
- Select 2 good apples from the 13 good ones: $\binom{13}{2} = 78$

Number of favorable samples:

$$ 2 \times 78 = 156 $$

Probability:

$$ P(1 \text{ rotten}) = \frac{156}{455} \approx 0.342 $$

Probability of Selecting 2 Rotten Apples

For exactly two rotten apples:

- Select both rotten apples: $\binom{2}{2} = 1$
- Select 1 good apple from the 13 good ones: $\binom{13}{1} = 13$

Number of favorable samples:

$$ 1 \times 13 = 13 $$

Probability:

$$ P(2 \text{ rotten}) = \frac{13}{455} \approx 0.029 $$

Summary of Probabilities

Outcome	Number of Favorable Samples	Probability
0 rotten apples	286	$\frac{286}{455} \approx 0.628$
1 rotten apple	156	$\frac{156}{455} \approx 0.342$
2 rotten apples	13	$\frac{13}{455} \approx 0.029$

This breakdown provides a clear understanding of the likelihood of each possible outcome when selecting three apples at random.

Expected Number of Rotten Apples in a Sample

Another interesting aspect is calculating the expected number of rotten apples in a sample of three.

- The expected value E can be computed as:

$$ E = \sum_{k=0}^{2} k \times P(\text{k rotten}) $$

Calculating:

$$ E = 0 \times 0.628 + 1 \times 0.342 + 2 \times 0.029 $$

$$ E = 0 + 0.342 + 0.058 = 0.4 $$

Thus, on average, among three randomly selected apples, there is a 0.4 chance (or expected value) of drawing a rotten apple.

Implication: In repeated sampling, roughly 40% of the time, a selected sample of three apples will contain at least one rotten apple.

Practical Applications of This Problem

Understanding such probability scenarios has real-world applications:

- Quality Control: Manufacturing plants can assess the likelihood of defective products in a batch.
- Food Industry: Estimating the chances of picking rotten produce in a shipment.
- Sampling Techniques: Designing experiments where samples are drawn without replacement.
- Risk Assessment: Calculating probabilities of undesirable outcomes in various sampling contexts.

Extensions and Related Problems

This problem can be extended or modified to explore other scenarios:

- Sampling with Replacement: How do probabilities change if apples are replaced after each draw?
- Different Sample Sizes: What if instead of three, a different number of apples is sampled?
- Varying Rottenness: What if the number of rotten apples increases or decreases?
- Multiple Baskets: Analyzing probabilities when selecting from multiple baskets with different compositions.

Conclusion

The problem of selecting apples from a basket with known quantities of rotten and good apples demonstrates fundamental concepts of probability, combinatorics, and sampling without replacement. By calculating the total number of possible samples and favorable outcomes, we can determine the likelihood of various outcomes and understand the expected number of rotten apples in a sample.

These principles are not only academically interesting but also practically valuable in various industries and fields requiring statistical analysis and decision-making under uncertainty. Mastering these basic probability calculations equips individuals with essential skills for analyzing real-world problems involving randomness and sampling.

Meta Description:
Learn how to calculate probabilities in a scenario where a basket contains 15 apples, including 2 rotten ones, and 3 are randomly selected. Explore combinatorial methods, outcomes, and practical

applications.

Frequently Asked Questions

What is the probability that a randomly selected sample of three apples contains exactly one rotten apple?

The probability is calculated as the number of ways to select 1 rotten and 2 good apples divided by the total number of ways to select any 3 apples. That is (2C1 13C2) / 15C3 = (2 78) / 455 = 156 / 455 ≈ 0.3429.

What is the probability that all three apples in the sample are good?

The probability is the number of ways to select 3 good apples divided by total selections: 13C3 / 15C3 = 286 / 455 ≈ 0.6286.

What is the probability that the sample contains at least one rotten apple?

The probability is 1 minus the probability that all apples are good: 1 - (13C3 / 15C3) = 1 - (286 / 455) ≈ 0.3714.

How many different samples of 3 apples can be selected from the basket?

The total number of ways is 15C3 = 455.

What is the probability that the sample contains exactly two rotten apples?

Since there are only 2 rotten apples, selecting both rotten and 1 good apple: 2C2 13C1 / 15C3 = (1 13) / 455 = 13 / 455 ≈ 0.0286.

If the sample contains at least one rotten apple, what is the probability it contains exactly one rotten apple?

Using conditional probability: (probability of exactly one rotten apple) / (probability of at least one rotten apple) = (156/455) / (169/455) = 156 / 169 ≈ 0.921.

What is the expected number of rotten apples in the sample?

Expected value = sample size probability of selecting a rotten apple = 3 (2/15) = 6/15 = 0.4.

If two apples are rotten, what is the probability that both are included in the sample?

Since there are only 2 rotten apples, the probability that both are selected in a sample of 3 is 2C2 13C1 / 15C3 = 13 / 455 ≈ 0.0286.

What is the probability that the sample contains no rotten apples?

The probability that all three selected apples are good is 13C3 / 15C3 = 286 / 455 ≈ 0.6286.

Additional Resources

A Basket Contains 15 Apples, Of Which Two Are Rotten. A Sample Of Three Apples Is Selected At Random.
This classic problem in probability and combinatorics provides a fascinating look into the likelihood of selecting certain types of items from a finite set. Whether you're a student studying probability theory, a teacher designing classroom exercises, or simply a curious mind exploring the nuances of chance, understanding how to analyze such scenarios is both intellectually rewarding and practically useful. In this comprehensive guide, we'll break down the problem step-by-step, explore different probability approaches, and discuss related concepts to deepen your understanding.

Introduction to the Problem

Imagine a basket filled with 15 apples, out of which 2 are rotten and the remaining 13 are fresh. You randomly select 3 apples from this basket without replacement—that is, once an apple is chosen, it's not put back into the basket before the next pick. The question is: what is the probability that the sample contains certain combinations of rotten and fresh apples?

This problem offers an excellent opportunity to delve into combinatorial probability, understanding how the composition of a population affects the chances of various outcomes, and applying concepts like hypergeometric distribution.

Understanding the Scenario

Before jumping into calculations, it helps to clarify the key elements:

- Total apples in the basket: 15
- Number of rotten apples: 2
- Number of fresh apples: 13
- Sample size: 3 apples
- Sampling method: Random, without replacement (each apple can only be selected once)

With these in mind, the primary goals are to compute:

- The probability that the sample contains exactly a certain number of rotten apples.
- The probability that the sample contains at least a certain number of rotten apples.
- The probability of other specific combinations, such as all fresh apples, or exactly one rotten apple.

Key Concepts in Probability and Combinatorics

1. Sample Space and Outcomes

The total number of ways to select 3 apples from 15 is given by the combination:

C(15, 3) = the number of combinations of 15 items taken 3 at a time:

$$C(15, 3) = \frac{15!}{3! \times 12!} = 455$$

This means there are 455 equally likely possible samples, assuming each sample is equally likely.

2. Favorable Outcomes

When calculating specific probabilities, identify the number of favorable outcomes — the number of samples fitting the criteria of interest.

3. Hypergeometric Distribution

The problem naturally aligns with the hypergeometric distribution, which describes the probability of drawing a certain number of "successes" (e.g., rotten apples) in a sample drawn without replacement from a finite population containing a known number of successes.

The hypergeometric probability formula is:

$$P(X = k) = \frac{C(R, k) \times C(N - R, n - k)}{C(N, n)}$$

where:
- N = total population size (15)
- R = total number of success states in the population (2 rotten apples)
- n = number of draws (3)
- k = number of successes in the sample (number of rotten apples in the sample)

Calculating Probabilities Step-by-Step

1. Probability of Selecting Exactly 0 Rotten Apples

This scenario involves choosing all 3 apples from the 13 fresh ones:

- Favorable outcomes: $C(13, 3)$
- Total outcomes: $C(15, 3)$

Calculations:

$$ C(13, 3) = \frac{13!}{3! \times 10!} = 286 $$

Probability:

$$ P(\text{0 rotten}) = \frac{286}{455} \approx 0.628 $$

2. Probability of Selecting Exactly 1 Rotten Apple

- Number of ways to choose 1 rotten apple from 2:

$$ C(2, 1) = 2 $$

- Number of ways to choose 2 fresh apples from 13:

$$ C(13, 2) = 78 $$

- Total favorable outcomes:

$$ 2 \times 78 = 156 $$

Probability:

$$ P(\text{1 rotten}) = \frac{156}{455} \approx 0.342 $$

3. Probability of Selecting Exactly 2 Rotten Apples

- Number of ways to choose 2 rotten apples from 2:

$$ C(2, 2) = 1 $$

- Number of ways to choose 1 fresh apple from 13:

$$ C(13, 1) = 13 $$

- Total favorable outcomes:

$$ 1 \times 13 = 13 $$

Probability:

$$ P(\text{2 rotten}) = \frac{13}{455} \approx 0.0286 $$

4. Probability of Selecting All 3 Apples Rotten (Impossible in this scenario)

Since there are only 2 rotten apples, selecting 3 rotten apples is impossible:

$$ P(\text{3 rotten}) = 0 $$

Summary Table of Probabilities

Number of Rotten Apples in Sample	Number of Ways	Probability
0	286	286/455 ≈ 0.628
1	156	156/455 ≈ 0.342
2	13	13/455 ≈ 0.0286
3	0	0

Exploring Additional Probabilities and Concepts

1. Probability of At Least One Rotten Apple

To find the probability that at least one rotten apple is included in the sample:

$$ P(\text{at least 1 rotten}) = 1 - P(\text{0 rotten}) $$

$$ = 1 - \frac{286}{455} = \frac{455 - 286}{455} = \frac{169}{455} \approx 0.371 $$

2. Probability of No Rotten Apples

Already calculated as approximately 0.628, which indicates that most samples are likely to contain only fresh apples.

3. Expected Number of Rotten Apples in the Sample

In probability theory, the expected value (mean) for the number of rotten apples in the sample can be calculated as:

$$ E = n \times \frac{R}{N} $$

$$ E = 3 \times \frac{2}{15} = \frac{6}{15} = 0.4 $$

This suggests that, on average, each random sample of three apples will contain about 0.4 rotten apples.

Practical Implications and Real-World Applications

Understanding this problem isn't just an academic exercise. It has practical implications in fields such as quality control, inventory management, and risk assessment. For instance:

- Quality Control: Manufacturers may want to know the probability that a batch contains a certain number of defective items.
- Sampling Inspection: When randomly inspecting a subset of products, estimating the likelihood of defects can inform decisions about accepting or rejecting batches.
- Risk Management: In food safety, understanding the probability of contaminated items within a subset guides inspection protocols.

Extending the Analysis

The problem can be expanded or modified in various ways, such as:

- Changing the sample size: What if 4 apples are selected instead of 3?
- Different proportions of rotten apples: How do probabilities shift if the basket contains more or fewer rotten apples?
- Multiple sampling: What's the probability of finding rotten apples across multiple samples?

Each variation introduces new calculations, often involving hypergeometric or binomial distributions, and helps develop a deeper understanding of probabilistic modeling.

Conclusion

The problem of selecting apples from a basket containing a known number of rotten and fresh apples serves as a fundamental example of combinatorial probability. By applying basic principles, such as combinations and the hypergeometric distribution, we can accurately compute the likelihood of various outcomes.

Understanding these concepts enhances our ability to analyze real-world situations involving finite populations and sampling without replacement. Whether in manufacturing, quality assurance, or everyday decision-making, mastering such probabilistic reasoning equips us with tools to make informed, data-driven choices.

Key Takeaways

- The total number of possible samples is determined by combinations: $C(N, n)$.
- Probabilities of specific outcomes are calculated by counting favorable combinations over total combinations.
- The hypergeometric distribution is essential when sampling without replacement.
- Most samples tend to contain only fresh apples in this scenario, as reflected by the high probability of zero rotten apples.
- Expected value provides a quick estimate of the average number of rotten apples in random samples.

By mastering these foundational concepts, you are better prepared to approach more complex probability problems and interpret the implications of sampling in various contexts.

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