# A Certain Radioactive Isotope Decays At A Rate Of 2% Per 100 Years. If T Represents The Amount Of The

**A Certain Radioactive Isotope Decays At A Rate Of 2% Per 100 Years. If T Represents The Amount Of The** isotope remaining after a certain period, understanding its decay process involves exploring exponential decay models, calculating remaining quantities over time, and applying this knowledge to real-world scenarios such as radiometric dating and nuclear medicine. This article delves into the mathematical foundations of radioactive decay, explains how to model the decay process, and highlights the significance of decay rates in scientific and practical contexts.

## Understanding Radioactive Decay and Its Mathematical Model

Radioactive decay is a natural process whereby unstable atomic nuclei lose energy by emitting radiation. The decay rate, often expressed as a percentage per unit time, dictates how quickly a substance diminishes. In this case, the isotope decays at a rate of 2% per 100 years, meaning that after each 100-year interval, only 98% of the initial amount remains.

### The Exponential Decay Formula

Radioactive decay follows an exponential model, which can be mathematically expressed as:

- **$T(t) = T_0 e^{(-\lambda t)}$**

where:

- $T(t)$ = amount remaining after time t
- $T_0$ = initial amount at time t=0
- $\lambda$ = decay constant
- t = elapsed time

The decay constant $\lambda$ is related to the half-life and the decay rate. The exponential nature of the decay means that the amount decreases proportionally to its current value over each time interval.

## Relating Decay Rate to the Decay Constant

Given that the isotope decays at a rate of 2% per 100 years, we interpret this as:

- After 100 years, $T(100) = 98\%$ of $T_0$

Using the decay formula:

$T(100) = T_0 e^{-\lambda 100}$

So,

$0.98 T_0 = T_0 e^{-\lambda 100}$

Dividing both sides by $T_0$:

$0.98 = e^{-\lambda 100}$

Taking natural logarithm:

$\ln(0.98) = -\lambda 100$

Solving for $\lambda$:

$\lambda = - (1/100) \ln(0.98)$

Calculating:

$\ln(0.98) \approx -0.0202027$

Thus,

$\lambda \approx 0.000202027$ per year

This decay constant represents the probability per year that a nucleus will decay.

# Calculating the Remaining Amount Over Time

Once $\lambda$ is known, predicting the amount of isotope remaining after any duration is straightforward.

## General Formula

The amount remaining after t years can be expressed as:

- **$T(t) = T_0 e^{-\lambda t}$**

or, equivalently, using the decay rate:

$T(t) = T_0 (0.98)^{t/100}$

since each 100-year period reduces the amount to 98% of the previous.

## Examples of Decay Calculations

Suppose you start with 100 grams of the isotope:

1. After 100 years:

$T(100) = 100 \cdot 0.98 = 98$ grams

2. After 200 years:

$T(200) = 100 (0.98)^2 \approx 100 \cdot 0.9604 \approx 96.04$ grams

3. After 500 years:

$T(500) = 100 (0.98)^5 \approx 100 \cdot 0.9039 \approx 90.39$ grams

These calculations illustrate the gradual decay over multiple periods.

# Half-Life of the Radioactive Isotope

The half-life is the time required for half of the initial amount to decay. Using the decay rate, we can compute the half-life.

## Calculating the Half-Life

Set $T(t) = T_0 / 2$:

$T_0 / 2 = T_0 (0.98)^{t / 100}$

Dividing both sides by $T_0$:

$1/2 = (0.98)^{t / 100}$

Taking natural logarithms:

$\ln(1/2) = (t / 100) \ln(0.98)$

Solving for t:

$t = (100 \ln(1/2)) / \ln(0.98)$

Calculations:

$\ln(1/2) \approx -0.6931$

$\ln(0.98) \approx -0.0202027$

Therefore,

$t \approx (100 -0.6931) / -0.0202027 \approx ( -69.31 ) / -0.0202027 \approx 3424$ years

Conclusion: The half-life of this isotope is approximately 3424 years.

# Applications of Decay Rate and Half-Life in Science

Understanding the decay rate and half-life of radioactive isotopes has critical applications in various scientific fields.

## 1. Radiometric Dating

Scientists utilize known half-lives to date ancient materials. For example:

- Geology: Dating rocks and fossils
- Archaeology: Determining the age of artifacts
- Environmental Science: Tracing pollutant decay

Knowing the decay rate allows scientists to calculate how much of a radioactive isotope remains and thus estimate the age of the sample.

## 2. Nuclear Medicine

Radioisotopes with specific decay rates are used in medical imaging and treatments:

- Diagnostics: Radioactive tracers help visualize organ functions
- Therapy: Decaying isotopes destroy cancer cells

Choosing isotopes with suitable half-lives ensures effectiveness and safety.

## 3. Nuclear Energy and Waste Management

Understanding decay rates is vital for:

- Managing nuclear waste
- Designing reactors
- Predicting long-term radioactivity levels

Accurate decay modeling helps in planning safe storage and disposal.

# Additional Considerations and Complexities

While the exponential decay model is fundamental, real-world scenarios may involve complexities.

## 1. Multiple Decay Pathways

Some isotopes decay via different modes, each with its own rate, affecting overall decay behavior.

## 2. Environmental Factors

External influences such as temperature, chemical environment, and radiation shielding can alter decay rates slightly.

## 3. Isotope Purity

Impurities or contamination can impact decay measurements and interpretations.

# Summary and Key Takeaways

- The isotope decays at a rate of 2% per 100 years, translating to a decay constant of approximately 0.000202 per year.
- The exponential decay model allows precise predictions of remaining quantities over time.
- The half-life of this isotope is approximately 3424 years.
- These concepts underpin important applications in dating, medicine, and nuclear energy.
- Accurate modeling of decay processes requires understanding both the mathematical principles and real-world factors influencing decay.

By mastering these fundamentals, scientists and practitioners can effectively utilize radioactive isotopes in diverse fields, ensuring safe and productive applications rooted in the principles of exponential decay.

# Frequently Asked Questions

## What is the decay rate of the radioactive isotope described?

The isotope decays at a rate of 2% per 100 years.

## How do you express the amount T of the isotope over time?

T can be modeled using exponential decay, typically as $T = T_0 (1 - 0.02)^{t/100}$, where $T_0$ is the initial amount and t is time in years.

## What is the decay constant (k) for this isotope?

The decay constant k is approximately 0.0202 per year, derived from the decay rate over 100 years.

## How long will it take for the isotope to decay to 50% of its original amount?

It will take approximately 346.6 years for the isotope to decay to half its initial amount.

## How is the half-life related to the decay rate in this scenario?

The half-life is calculated using the decay rate; with a 2% decay per 100 years, the half-life is about 346.6 years.

## If the initial amount is 100 grams, what will be the amount after 500 years?

After 500 years, the remaining amount will be approximately 36 grams, using the exponential decay formula.

# Additional Resources

A Certain Radioactive Isotope Decays At A Rate Of 2% Per 100 Years. If T Represents The Amount Of The Radioactive Material Remaining, How Can We Model Its Decay Over Time?

When exploring the fascinating world of radioactive decay, understanding how isotopes diminish over time is crucial for applications ranging from radiometric dating to nuclear medicine. In this analysis, we focus on a specific isotope that decays at a rate of 2% per 100 years, and we'll examine how to model its decay mathematically. By understanding the decay process, scientists and students can predict the remaining quantity of the isotope after any given period, enabling precise calculations in various scientific contexts.

---

Understanding Radioactive Decay and Decay Rates

Radioactive decay is a natural process where unstable isotopes transform into more stable forms, often releasing radiation in the form of particles or energy. The decay process is inherently random at the level of individual atoms but follows a predictable statistical pattern at the population level, which can be expressed mathematically.

Decay rate refers to how quickly an isotope decreases in quantity over time. When a substance decays at a constant percentage rate per unit time, it exhibits exponential decay. In our case, the isotope decays at 2% per 100 years, meaning that every century, only 98% of the previous amount remains.

---

Modeling Decay: The Basics

The general form for exponential decay is:

$$T(t) = T_0 \times e^{kt}$$

where:
- $T(t)$ is the amount remaining after time $t$.
- $T_0$ is the initial amount at $t = 0$.
- $k$ is the decay constant (a negative number).
- $t$ is the elapsed time.

Alternatively, decay can also be modeled with a decay factor per time interval, especially when considering percentage decay per fixed periods.

---

Step 1: Clarify the Decay Rate

Given:
- The isotope decays by 2% per 100 years.

This means:
- After 100 years, the remaining amount is 98% of the initial.

Expressed mathematically:
$$T(100) = T_0 \times (1 - 0.02) = T_0 \times 0.98$$

---

Step 2: Derive the Decay Model

Since the decay is consistent over each 100-year period and is proportional, the decay can be modeled as:

$$ T(t) = T_0 \times (0.98)^{n} $$

where:
- $n$ is the number of 100-year intervals in $t$.

Expressed in terms of $t$:

$$ n = \frac{t}{100} $$

Thus:

$$ T(t) = T_0 \times (0.98)^{t/100} $$

This model shows that the amount of isotope decreases exponentially over time at a rate dictated by the 2% decay per century.

---

Step 3: Connecting to the Continuous Decay Model

While the above is a discrete model (per 100-year intervals), it's often useful to have a continuous decay formula, especially for calculations at arbitrary times. To do this, we convert the discrete decay factor into an exponential decay model:

$$ T(t) = T_0 \times e^{kt} $$

Matching the two models at $t = 100$ years:

$$ T(100) = T_0 \times e^{k \times 100} = T_0 \times 0.98 $$

Dividing both sides by $T_0$:

$$ e^{k \times 100} = 0.98 $$

Now, solve for $k$:

$$ k \times 100 = \ln(0.98) $$

$$ k = \frac{\ln(0.98)}{100} $$

Calculating:

$$ \ln(0.98) \approx -0.0202 $$

$$ k \approx -0.0202 / 100 = -0.000202 $$

So, the continuous decay model becomes:

$$ T(t) = T_0 \times e^{-0.000202 t} $$

This formula allows precise calculation of the remaining amount after any number of years $t$.

---

Practical Applications and Examples

1. Calculating Remaining Isotope After a Specific Time

Suppose you start with 100 grams of the isotope. How much remains after 500 years?

Using the continuous model:

$$ T(500) = 100 \times e^{-0.000202 \times 500} $$

Calculate the exponent:

$$ -0.000202 \times 500 = -0.101 $$

Then:

$$ T(500) = 100 \times e^{-0.101} \approx 100 \times 0.9048 \approx 90.48 \text{ grams} $$

Thus, after 500 years, approximately 90.48 grams remain.

2. Determining Half-Life

The half-life is the time it takes for half of the isotope to decay. To find the half-life ($t_{1/2}$), set:

$$ T(t_{1/2}) = \frac{T_0}{2} $$

Using the exponential decay model:

$$ \frac{T_0}{2} = T_0 \times e^{k t_{1/2}} $$

Divide both sides by $T_0$:

$$ \frac{1}{2} = e^{k t_{1/2}} $$

Take the natural logarithm:

$$ \ln \left( \frac{1}{2} \right) = k t_{1/2} $$

$$ t_{1/2} = \frac{\ln(1/2)}{k} $$

Recall:

$$ \ln(1/2) = -0.6931 $$

and

$$ k \approx -0.000202 $$

Therefore:

$$ t_{1/2} \approx \frac{-0.6931}{-0.000202} \approx 3433 \text{ years} $$

The half-life of this isotope is approximately 3433 years.

---

## Key Takeaways and Summary

- The isotope decays at a rate of 2% per 100 years, which translates into a decay factor of 0.98 every century.
- The decay can be modeled discretely as:

$$ T(t) = T_0 \times (0.98)^{t/100} $$

- For more precise calculations, especially at arbitrary times, the continuous exponential decay model is used:

$$ T(t) = T_0 \times e^{k t} $$

where:

$$ k = \frac{\ln(0.98)}{100} \approx -0.000202 $$

- Half-life of this isotope is approximately 3433 years.

---

## Additional Considerations

- Decay constants and half-life are fundamental in radiometric dating, allowing scientists to estimate the age of archaeological finds or geological formations.
- Understanding decay rates helps in nuclear medicine for dosing radioactive tracers safely.
- The model assumes a constant decay rate, which is valid for most isotopes over long periods but might not account for external influences or decay chain complexities.

---

## Conclusion

Modeling the decay of a radioactive isotope that decreases at a rate of 2% per 100 years involves understanding exponential decay and decay constants. Whether using the discrete model for straightforward calculations or the continuous exponential model for precise predictions, these tools are essential for scientists working with radioactive materials. Mastery of these concepts provides a foundation for understanding the longevity of isotopes, the timing of natural processes, and the safe handling of radioactive substances across multiple fields.

## A Certain Radioactive Isotope Decays At A Rate Of 2 Per 100 Years If T Represents The Amount Of The

A Certain Radioactive Isotope Decays At A Rate Of 2 Per 100 Years If T Represents The Amount Of The

## Related Articles

- 3. 21 A Three-phase Load Draws 120 KW At A Power Factor Of 0. 85 Lagging From A 40-V Bus. In Parallel
- 2. Briefly Explain Why African Slaves Were Used In The Encomienda System, Gradually Replacing In-dian
- 4) A Dry Goods Company Buys A New, Modern Mixer To Supplement The Output Of Their Old, Current Mixer.

Back to Home