A Circle Has A Center At (-3, - 2) And Passes Through The Point (1, 4). What Is The Standard Equation

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Understanding the fundamental equations of circles is essential in both algebra and geometry. In this article, we will explore how to determine the standard form of a circle's equation given its center and a point it passes through. Specifically, we will analyze the problem: a circle with a center at (-3, -2) passing through the point (1, 4). By the end of this guide, you will understand the step-by-step process to derive the standard equation of such a circle, along with useful tips and related concepts.

Introduction to the Standard Equation of a Circle

The standard form of a circle's equation in a Cartesian coordinate system is expressed as:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:
- (h, k) represents the center of the circle.
- r is the radius of the circle.

This form is particularly useful because it directly relates the geometry of the circle to its algebraic representation. To write the equation, we need two critical pieces of information:
1. The coordinates of the center (h, k).
2. The radius r.

Given the center and a point on the circle, we can find the radius by calculating the distance between the center and that point.

Step-by-Step Solution to Find the Standard Equation

Let's follow a systematic approach to derive the standard equation for our specific problem.

Step 1: Identify the Center Coordinates

From the problem statement:
- Center: $(-3, -2)$

Thus, $h = -3$ and $k = -2$.

Step 2: Find the Radius r

The radius is the distance from the center to any point on the circle. Given that the circle passes through $(1, 4)$, we can determine r using the distance formula:

$$ r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

where:

- $(x_1, y_1) = (-3, -2)$
- $(x_2, y_2) = (1, 4)$

Calculating:

$$
r = \sqrt{(1 - (-3))^2 + (4 - (-2))^2}
$$

$$
r = \sqrt{(1 + 3)^2 + (4 + 2)^2}
$$

$$
r = \sqrt{(4)^2 + (6)^2}
$$

$$
r = \sqrt{16 + 36}
$$

$$
r = \sqrt{52}
$$

Simplify $\sqrt{52}$:

$$
r = \sqrt{4 \times 13} = 2 \sqrt{13}
$$

Therefore, the radius $r = 2 \sqrt{13}$.

Step 3: Write the Equation Using the Standard Form

Plugging the values into the standard circle equation:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

becomes:

$$
(x - (-3))^2 + (y - (-2))^2 = (2 \sqrt{13})^2
$$

Simplify the signs:

$$
(x + 3)^2 + (y + 2)^2 = (2 \sqrt{13})^2
$$

Calculate r^2:

$$
(2 \sqrt{13})^2 = 4 \times 13 = 52
$$

Hence, the equation of the circle is:

$$
(x + 3)^2 + (y + 2)^2 = 52
$$

Final Standard Equation of the Circle

$$
\boxed{
(x + 3)^2 + (y + 2)^2 = 52
}
$$

This is the standard form of the circle with the given center and passing through the specified point.

Understanding the Components of the Equation

Let's analyze each part of this equation to deepen your understanding.

Center Coordinates (h, k)

- The terms $(x + 3)$ and $(y + 2)$ indicate the circle's center at $(-3, -2)$. This is because in the standard form, the signs are inverted from the coordinate of the center.

Radius r

- The right side of the equation, 52, represents r^2. Since $r = 2 \sqrt{13}$, this confirms the radius.

Additional Concepts and Related Topics

To expand your understanding further, here are some related concepts and topics:

1. Distance Formula

- Used to find the length between two points in the coordinate plane.

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

- Essential for calculating the radius when a point on the circle and the center are known.

2. Graphing the Circle

- To graph the circle:
- Plot the center at $(-3, -2)$.

- Draw a circle with radius $2 \sqrt{13}$, which is approximately 7.21 units.
- Use the radius to mark points around the center.

3. Equation in General Form

- The standard form can be expanded to the general form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

- Converting between forms involves expansion and simplification.

4. Applications of Circle Equations

- Used in various fields like engineering, physics, architecture, and computer graphics.
- Critical for designing circular objects or analyzing circular motion.

Tips for Solving Similar Problems

- Always identify the center coordinates first.
- Use the distance formula to find the radius when a point on the circle is given.
- Remember to square the radius when writing the standard form.
- Keep track of signs when substituting the center coordinates into the equation.
- Practice converting between different forms of the circle's equation.

Practice Problems for Mastery

Try solving these problems to reinforce your understanding:

1. A circle has a center at (4, -1) and passes through (7, 3). Find its standard equation.
2. Given the equation $(x - 2)^2 + (y + 5)^2 = 25$, identify the center and radius of the circle.
3. Write the equation of a circle with a center at (-6, 4) and radius 3.

Solutions:

1. Find the radius using the distance formula:

$$
r = \sqrt{(7-4)^2 + (3 - (-1))^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5
$$

Equation:

$$
(x - 4)^2 + (y + 1)^2 = 25
$$

2. Center: $(2, -5)$, Radius: $\sqrt{25} = 5$.

3. Equation:

$$(x + 6)^2 + (y - 4)^2 = 3^2 = 9$$

Conclusion

Determining the standard equation of a circle given its center and a point it passes through is a fundamental skill in algebra and geometry. By applying the distance formula to find the radius and then substituting the center coordinates and radius into the standard form, you can accurately derive the circle's equation. Remember, the key steps involve identifying the center, calculating the radius, and then writing the equation. Mastery of this process enhances your problem-solving skills and deepens your understanding of the geometric properties of circles.

With practice, you'll become proficient at translating geometric scenarios into algebraic equations, a valuable skill applicable in various mathematical and real-world contexts.

Frequently Asked Questions

What is the formula for the standard equation of a circle?

The standard equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

How do we find the radius of the circle given the center and a point on the circle?

The radius r is the distance between the center $(-3, -2)$ and the point $(1, 4)$, calculated using the distance formula: $r = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

What is the step-by-step process to find the standard equation of this circle?

First, find the radius using the distance formula between the center $(-3, -2)$ and point $(1, 4)$. Then, substitute the center coordinates and the radius into the standard circle equation $(x - h)^2 + (y - k)^2 = r^2$.

What is the calculated radius for this circle?

The radius r is $\sqrt{[(1 + 3)^2 + (4 + 2)^2]} = \sqrt{[4^2 + 6^2]} = \sqrt{[16 + 36]} = \sqrt{52} = 2\sqrt{13}$.

What is the standard form equation of the circle with center $(-3, -2)$ passing through $(1, 4)$?

The equation is $(x + 3)^2 + (y + 2)^2 = (2\sqrt{13})^2$, which simplifies to $(x + 3)^2 + (y + 2)^2 = 52$.

Why is the standard form important in understanding the circle's properties?

Because it clearly shows the center and radius, allowing us to easily graph the circle and analyze its geometric properties.

Can you verify the radius by plugging in the point (1, 4) into the equation?

Yes, substituting (1, 4) into $(x + 3)^2 + (y + 2)^2 = 52$ gives $(1 + 3)^2 + (4 + 2)^2 = 4^2 + 6^2 = 16 + 36 = 52$, confirming the point lies on the circle.

Additional Resources

A Circle Has A Center At (-3, -2) And Passes Through The Point (1, 4). What Is The Standard Equation? When working with circles in coordinate geometry, understanding how to derive the standard equation of a circle based on given points and centers is fundamental. In this guide, we'll explore how to find the standard form of a circle's equation when provided with a specific center and a point on the circle. This process involves calculating the radius using the distance formula and then substituting the values into the standard circle equation. Whether you're a student preparing for exams or someone interested in the geometric properties of circles, this comprehensive walkthrough will clarify each step.

Understanding the Standard Equation of a Circle

Before diving into calculations, it's essential to understand what the standard equation of a circle looks like and what each component represents.

The Standard Equation of a Circle

The standard form of a circle's equation in the coordinate plane is:

$(x - h)^2 + (y - k)^2 = r^2$

Where:
- (h, k) is the center of the circle.
- r is the radius of the circle.

Components Breakdown:

- The center is a fixed point that defines the position of the circle.
- The radius is the distance from the center to any point on the circle.
- The equation expresses all points (x, y) that are exactly r units away from the center.

Step-by-Step Guide to Find the Standard Equation

Given:
- Center C(-3, -2)
- Passes through P(1, 4)

The goal: Find the standard equation of the circle.

Step 1: Understand What is Given and What is Needed

- You are given the center of the circle, which is (-3, -2).
- You are provided with a point on the circle, (1, 4), which allows us to determine the radius.
- Using these, you can write the full standard equation.

Step 2: Calculate the Radius Using the Distance Formula

The radius r is the distance between the center C(-3, -2) and the point P(1, 4).

The distance formula between two points (x_1, y_1) and (x_2, y_2) is:

$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Applying this formula:

- $x_1 = -3$, $y_1 = -2$
- $x_2 = 1$, $y_2 = 4$

Calculating:

$r = \sqrt{[(1 - (-3))^2 + (4 - (-2))^2]}$
$r = \sqrt{[(1 + 3)^2 + (4 + 2)^2]}$
$r = \sqrt{[4^2 + 6^2]}$
$r = \sqrt{[16 + 36]}$
$r = \sqrt{52}$

Since $\sqrt{52}$ can be simplified:

$r = \sqrt{(4\ 13)} = \sqrt{4}\sqrt{13} = 2\sqrt{13}$

So, the radius $r = 2\sqrt{13}$.

Step 3: Write the Standard Equation

With the center (-3, -2) and radius $2\sqrt{13}$, substitute into the standard form:

$(x - h)^2 + (y - k)^2 = r^2$

Plugging in:

- $h = -3$, $k = -2$
- $r^2 = (2\sqrt{13})^2 = 4\ 13 = 52$

Therefore, the standard equation is:

$(x + 3)^2 + (y + 2)^2 = 52$

Final Equation and Interpretation

The standard equation of the circle with center at (-3, -2) passing through (1, 4) is:

$(x + 3)^2 + (y + 2)^2 = 52$

This equation allows you to determine whether any point lies on the circle by substituting the point's coordinates into the equation and checking if it satisfies the equality.

Additional Insights and Tips

Why Use the Distance Formula?

The distance formula is crucial because the radius is simply the distance between the circle's center and any point on its circumference. Knowing the center and a point on the circle allows for straightforward calculation of the radius.

Simplify the Equation When Necessary

If you're asked to write the equation in a different form or to interpret it graphically, having the simplified form with the radius as a perfect square is helpful. In this case, the radius squared is 52, which is already in simplified radical form.

Visualizing the Circle

Plotting the center at (-3, -2) and marking the point (1, 4) can help visualize the circle. The distance between those points confirms the size of the circle, and plotting other points that satisfy the equation can further confirm its accuracy.

Summary

- Identify the center and point on the circle.
- Use the distance formula to compute the radius.
- Square the radius to find r^2.
- Substitute the center coordinates and r^2 into the standard circle equation.

In our example:

- Center: (-3, -2)
- Point on circle: (1, 4)
- Calculated radius: 2⍰ 13
- Standard equation: $(x + 3)^2 + (y + 2)^2 = 52$

This process exemplifies how to find the standard form of a circle's equation given its center and a point on its circumference, an essential skill in coordinate geometry and algebra.

Additional Practice

To strengthen understanding, try the following:

1. Find the equation of a circle with center at (2, -5) passing through (5, -1).
2. Given the equation $(x - 4)^2 + (y + 3)^2 = 25$, identify the center and radius.
3. For a circle centered at (-1, 2) with radius 3, write its standard equation.

Mastering these steps will enable you to confidently analyze and write equations of circles in various contexts.

Understanding the geometric principles behind the standard equation of a circle helps bridge the gap between algebraic manipulation and spatial visualization, an essential component of mathematical literacy.

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