A Diatomic Ideal Gas Is Heated At Constant Volume Until Its Pressure Becomes Three Times. It Is Again

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Understanding the thermodynamic behavior of gases is fundamental in physics and engineering. Particularly, the behavior of a diatomic ideal gas under various thermal processes provides insights into energy transfer, molecular interactions, and the principles governing state changes. In this article, we explore a specific scenario: a diatomic ideal gas is heated at constant volume until its pressure triples, and then analyze the subsequent implications of this process. We will delve into the principles involved, calculations, and the physical interpretations, providing a comprehensive understanding suitable for students and professionals alike.

Introduction to Diatomic Ideal Gases

Before examining the process in detail, it is essential to understand the basic properties of diatomic ideal gases.

Characteristics of Diatomic Gases

- Consist of molecules composed of two atoms, such as oxygen (O_2), nitrogen (N_2), and hydrogen (H_2).
- Exhibit specific heat capacities due to their rotational and vibrational degrees of freedom.
- Under ideal conditions, their behavior can be approximated by the ideal gas law: $PV = nRT$.

Thermodynamic Properties

- The molar specific heat at constant volume, C_V, for a diatomic ideal gas (assuming rotational but not vibrational modes excited) is approximately $\frac{5}{2} R$.
- The molar specific heat at constant pressure, C_P, is approximately $\frac{7}{2} R$.
- The ratio $\gamma = \frac{C_P}{C_V}$ is about 1.4 for diatomic gases under typical conditions.

Initial Conditions and the Heating Process

Suppose a sample of diatomic ideal gas initially has a pressure P_1, volume V, and temperature T_1. The process involves heating the gas at a constant volume until its pressure becomes three times its initial value, i.e.,

$$P_2 = 3 P_1.$$

Given that the volume remains constant, the process is an isochoric (constant volume) heating.

Applying the Ideal Gas Law

Since the volume V is constant, the ideal gas law simplifies the analysis:

$$P V = n R T,$$

where:
- P is the pressure,
- V is the volume,
- n is the number of moles,
- R is the universal gas constant,
- T is the temperature in Kelvin.

From the initial state:

$$P_1 V = n R T_1,$$

and after heating:

$$P_2 V = n R T_2.$$

Dividing the second equation by the first:

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}.$$

Since $P_2 = 3 P_1$, it follows that:

$$
T_2 = 3 T_1.
$$

This indicates the temperature triples during the heating process.

Energy Changes During Heating at Constant Volume

The process involves an energy transfer into the gas in the form of heat, leading to an increase in internal energy.

Internal Energy of a Diatomic Gas

The internal energy U of an ideal diatomic gas depends only on temperature and can be expressed as:

$$
U = n C_V T,
$$

where C_V is the molar heat capacity at constant volume.

Given $C_V = \frac{5}{2} R$, the change in internal energy as the gas heats from T_1 to T_2 is:

$$
\Delta U = n C_V (T_2 - T_1) = n \times \frac{5}{2} R (T_2 - T_1).
$$

Since $T_2 = 3 T_1$:

$$
\Delta U = n \times \frac{5}{2} R (3 T_1 - T_1) = n \times \frac{5}{2} R \times 2 T_1 = 5 n R T_1.
$$

This represents the energy absorbed by the gas during heating.

Heat Added to the Gas

Because the process occurs at constant volume, the heat added Q equals the change in internal energy:

$$
Q = \Delta U = 5 n R T_1.
$$

The process adheres to the first law of thermodynamics:

$$
\Delta U = Q - W,
$$

where W is the work done. Since the volume is constant, $W = 0$, and therefore $Q = \Delta U$.

Subsequent Process: Isobaric Expansion or Other Scenarios

The phrase "It Is Again" in the original prompt hints at exploring what happens after this heating process or considering additional steps, such as expansion or further heating.

Suppose, after heating at constant volume, the gas undergoes an isobaric (constant pressure) process. The analysis of such a process involves understanding the changes in temperature, volume, and energy.

Heating at Constant Pressure

If the gas is allowed to expand at constant pressure $P_2 = 3 P_1$, the temperature increases from T_2 to a higher temperature T_3. Using the ideal gas law:

$$
\frac{V_3}{V_2} = \frac{T_3}{T_2}.
$$

The work done during the expansion:

$$
W_{exp} = P_{2} (V_3 - V_2) = n R T_3 - n R T_2,
$$

which simplifies to:

$$
W_{exp} = n R (T_3 - T_2).
$$

The heat absorbed during this expansion:

$$
Q_{exp} = n C_P (T_3 - T_2),
$$

where $C_P = \frac{7}{2} R$. This process increases the internal energy as well as performs work on the surroundings.

Summary of Key Results

Process Step	Key Equation	Description
Heating at constant volume	$T_2 = 3 T_1$	Temperature triples as pressure triples
Internal energy change	$\Delta U = 5 n R T_1$	Energy absorbed during heating
Heat added	$Q = 5 n R T_1$	Equal to ΔU at constant volume
Subsequent isobaric expansion	$V_3 / V_2 = T_3 / T_2$	Volume increases with temperature at constant pressure

Physical Interpretations and Applications

Understanding this process provides insights into real-world applications such as:

- Designing engines and refrigerators where gases undergo controlled thermodynamic cycles.
- Analyzing atmospheric phenomena where gases experience pressure and temperature changes.
- Calculating work and energy transfer in chemical reactions involving gases.

This process exemplifies the fundamental principles of thermodynamics: how heat energy affects internal energy, pressure, and temperature, especially in diatomic gases where molecular degrees of freedom influence heat capacities.

Conclusion

The scenario of a diatomic ideal gas heated at constant volume until its pressure becomes three times its initial value illustrates key thermodynamic principles. The temperature increases proportionally, internal energy rises accordingly, and the process exemplifies the relationship between heat transfer and internal energy change. Further processes, such as expansion at constant pressure, build upon this foundation, illustrating the interconnectedness of thermodynamic paths and the importance of understanding these concepts in practical and theoretical contexts.

By mastering these fundamental ideas, students and engineers can better analyze and design systems involving thermal processes, ensuring efficient energy utilization and deeper insight into the behavior of gases under various conditions.

Frequently Asked Questions

What is the relationship between pressure, volume, and temperature for a diatomic ideal gas at constant volume?

At constant volume, the pressure of an ideal gas is directly proportional to its temperature ($P \propto T$).

If a diatomic ideal gas is heated at constant volume until its pressure triples, how does its temperature change?

The temperature increases three times, since $P \propto T$ at constant volume, so $T_{final} = 3 \times T_{initial}$.

What is the significance of the gas being diatomic in this problem?

The diatomic nature affects the specific heat capacities and the degrees of freedom, which influence the internal energy and heat calculations during heating.

How much heat is supplied to the gas during this process if the initial temperature is T_1?

The heat supplied is $\Delta Q = n C_v \Delta T$, where C_v is the molar heat capacity at constant volume. Since $\Delta T = 2T_1$ (from T_1 to $3T_1$), $\Delta Q = n C_v (2T_1)$.

Does the process described involve any work done by the gas?

No, since the volume remains constant, the work done $W = P\Delta V = 0$.

What is the change in internal energy of the gas during this process?

The internal energy change $\Delta U = n\, C_v\, \Delta T$. With the temperature tripling, $\Delta U = n\, C_v\, (2T_1)$.

If the initial pressure is P_1 and the initial temperature is T_1, what is the final pressure after heating?

Since $P \propto T$ at constant volume, the final pressure $P_2 = 3\, P_1$.

Additional Resources

Heating a diatomic ideal gas at constant volume until its pressure triples presents a fascinating exploration of thermodynamic principles, molecular behavior, and the interplay of temperature and pressure in idealized systems. This process not only exemplifies fundamental laws such as Gay-Lussac's law and the ideal gas law but also offers insights into molecular energy distributions, degrees of freedom, and the efficiency of energy transfer in thermodynamic cycles. In this comprehensive review, we delve into the detailed physics underlying this transformation, providing a thorough analysis suitable for students, educators, and professionals interested in thermodynamics and statistical mechanics.

Understanding the Scenario: Heating a Diatomic Ideal Gas at Constant Volume

Fundamental Assumptions and Setup

In classical thermodynamics, an ideal gas is a hypothetical gas composed of non-interacting point particles that obey the ideal gas law:

$$PV = nRT$$

where:
- P is the pressure,
- V is the volume,
- n is the number of moles,
- R is the universal gas constant,
- T is the absolute temperature.

Since the gas in question is diatomic, it consists of molecules with two atoms, such as nitrogen (N_2) or oxygen (O_2). These molecules have more complex internal energy structures compared to monatomic gases, primarily due to rotational and vibrational degrees of freedom.

The key condition in the problem is constant volume, meaning the container holding the gas does not change size during heating. The process involves heating the gas until the pressure becomes three times its initial value, after which the system's properties are analyzed.

Thermodynamic Principles at Play

Ideal Gas Law and Its Implications

At any point during the process, the ideal gas law applies:

$$P V = n R T$$

Since volume V and amount n are constant, the relationship simplifies to:

$$P \propto T$$

This indicates that, under constant volume and fixed mole number, the pressure is directly proportional to the temperature. Therefore, if the pressure triples, the temperature must also triple:

$$P_2 = 3 P_1 \Rightarrow T_2 = 3 T_1$$

This straightforward relationship forms the foundation for analyzing the process.

Energy Considerations in Heating a Diatomic Gas

Internal Energy and Degrees of Freedom

The internal energy U of an ideal gas depends solely on temperature and the degrees of freedom of its molecules. For a diatomic molecule, the total energy includes contributions from translational, rotational, and vibrational motions:

- Translational: molecules move freely in three-dimensional space (3 degrees of freedom).
- Rotational: molecules rotate around axes; diatomic molecules have 2 rotational degrees of freedom (assuming vibrational modes are not excited at moderate temperatures).
- Vibrational: atoms within a molecule vibrate around equilibrium positions; vibrational modes activate at higher temperatures.

The equipartition theorem states that each quadratic degree of freedom contributes an average energy of:

$$ \frac{1}{2} k_B T $$

where k_B is Boltzmann's constant.

Total internal energy per molecule:

$$ U_{per\,molecule} = \frac{f}{2} k_B T $$

where f is the total degrees of freedom. For a diatomic molecule at temperatures where vibrational modes are not excited:

$$ f = 3\, (translational) + 2\, (rotational) = 5 $$

Thus,

$$ U_{per\,molecule} = \frac{5}{2} k_B T $$

and for N molecules,

$$ U = N \times \frac{5}{2} k_B T $$

or, in terms of molar quantities,

$$ U = n C_v T $$

where C_v is the molar heat capacity at constant volume for a diatomic ideal gas:

$$ C_v = \frac{f}{2} R = \frac{5}{2} R $$

This internal energy increases linearly with temperature.

Heating at Constant Volume: Energy Input and Temperature Rise

The process involves adding heat Q to raise the temperature from T_1

\) to T_2. Since volume is constant, the heat added is related to the change in internal energy:

$$Q = \Delta U = n C_v (T_2 - T_1)$$

Given the proportionality $T_2 = 3 T_1$, the heat supplied becomes:

$$Q = n C_v (3 T_1 - T_1) = 2 n C_v T_1$$

This quantifies the energy required to achieve the tripling of pressure (and temperature).

Analyzing the Process Step-by-Step

Initial Conditions

Suppose initially the gas has:
- Pressure: P_1
- Temperature: T_1
- Volume: V
- Number of moles: n

Applying the ideal gas law:

$$P_1 V = n R T_1$$

Final Conditions

After heating:
- Pressure: $P_2 = 3 P_1$
- Temperature: $T_2 = 3 T_1$
- Volume: V (unchanged)
- Moles: n (unchanged)

Applying the ideal gas law again:

$$P_2 V = n R T_2$$
$$3 P_1 V = n R \times 3 T_1$$

which confirms the proportionality between pressure and temperature.

Implications of the Process on Molecular Energy and Thermodynamic Quantities

Change in Internal Energy

Given the initial and final states:

$$ \Delta U = n C_v (T_2 - T_1) = n \times \frac{5}{2} R (3 T_1 - T_1) = n \times \frac{5}{2} R \times 2 T_1 = 5 R T_1 n $$

This indicates that the internal energy increases by a factor proportional to the initial temperature, reflecting the added energy.

Work Done by the System

In a constant volume process, the work done by the gas:

$$ W = P \Delta V $$

is zero because $\Delta V = 0$. All the heat supplied goes into increasing internal energy, aligning with the first law of thermodynamics:

$$ \Delta U = Q - W \Rightarrow Q = \Delta U $$

Additional Considerations and Real-World Implications

Vibrational Mode Excitation and Deviations from Ideal Behavior

At higher temperatures, vibrational modes become thermally accessible, increasing the degrees of freedom from 5 to 7 (including vibrational modes). This increment affects the heat capacity:

- At moderate temperatures: $C_v \approx \frac{5}{2} R$

- At high temperatures: $C_v \to \frac{7}{2} R$

This impacts the energy input required for a given temperature change and introduces non-trivial considerations for real gases, especially under extreme conditions.

Practical Relevance in Engineering and Physical Chemistry

Understanding such heating processes is vital in fields like:
- Aerospace engineering, where gases are subjected to rapid heating at constant volume.
- Chemical reactors, where controlling temperature and pressure is crucial.
- Thermodynamic cycle analysis, such as in the Carnot cycle or Otto cycle, where idealized models help optimize energy efficiency.

Conclusion: The Significance of the Pressure Tripling

The scenario of a diatomic ideal gas being heated at constant volume until its pressure triples encapsulates core principles of thermodynamics and molecular physics. The direct proportionality between pressure and temperature in an ideal gas simplifies analysis, but the nuanced internal energy changes due to molecular degrees of freedom add depth to the understanding.

This process underscores the importance of molecular structure in thermodynamic behavior. While the ideal gas law provides a first-order approximation, real-world applications must account for vibrational modes, intermolecular interactions, and deviations from ideality at high pressures or temperatures.

In essence, this exploration reveals how energy transfer at the microscopic level manifests as macroscopic measurable quantities—pressure, temperature, and internal energy—and demonstrates the elegant simplicity and complexity inherent in thermodynamic systems. Whether in laboratory experiments or industrial applications, grasping these fundamental relations enhances our ability to manipulate and optimize gaseous systems across various scientific and engineering domains.

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