(a) Find The Remainder When F (x) Is Divided By G(x) If F (x) = 2x5 3x4 X3 2x2 X 8 And G(x) = X 10 In

(a) Find The Remainder When F (x) Is Divided By G(x) If F (x) = 2x^5 + 3x^4 + x^3 + 2x^2 + x + 8 And G(x) = x + 10

Introduction

Polynomial division is a fundamental concept in algebra that involves dividing one polynomial by another, much like long division with numbers. When dividing a polynomial $F(x)$ by a divisor $G(x)$, the result can be expressed as:

$$
F(x) = Q(x) \times G(x) + R(x)
$$

where:

- $Q(x)$ is the quotient polynomial,
- $R(x)$ is the remainder polynomial.

In cases where the divisor $G(x)$ is linear, such as $x + a$, the Remainder Theorem simplifies the process of finding the remainder: the remainder is simply $F(-a)$.

In this article, we will explore how to find the remainder when dividing the polynomial $F(x) = 2x^5 + 3x^4 + x^3 + 2x^2 + x + 8$ by $G(x) = x + 10$.

Understanding the Problem

Polynomial Definitions

- Dividend Polynomial $F(x)$:

$$
F(x) = 2x^5 + 3x^4 + x^3 + 2x^2 + x + 8
$$

- Divisor Polynomial $G(x)$:

$$
G(x) = x + 10
$$

Since $G(x)$ is linear, the Remainder Theorem applies, which states:

> The remainder when $F(x)$ is divided by $x + a$ is $F(-a)$.

In our case, $a = 10$, so the remainder is $F(-10)$.

Applying the Remainder Theorem

Step 1: Identify a

Given $G(x) = x + 10$, the root of $G(x)$ is:

$$
x = -10
$$

Step 2: Evaluate $F(-10)$

To find the remainder, plug $x = -10$ into $F(x)$:

$$
F(-10) = 2(-10)^5 + 3(-10)^4 + (-10)^3 + 2(-10)^2 + (-10) + 8
$$

Let's compute each term carefully.

Calculating $F(-10)$

Step 3: Compute each term

1. $2(-10)^5$:

$$
(-10)^5 = (-10) \times (-10) \times (-10) \times (-10) \times (-10)
$$

Calculate step by step:

- $(-10) \times (-10) = 100$
- $100 \times (-10) = -1000$
- $-1000 \times (-10) = 10,000$
- $10,000 \times (-10) = -100,000$

So,

$$
(-10)^5 = -100,000
$$

Therefore,

$$
2 \times (-100,000) = -200,000
$$

2. $3(-10)^4$:

Calculate $(-10)^4$:

- $(-10)^4 = ((-10)^2)^2$

- $(-10)^2 = 100$

- $100^2 = 10,000$

Thus,

$$
(-10)^4 = 10,000
$$

Now,

$$
3 \times 10,000 = 30,000
$$

3. $(-10)^3$:

$$
(-10)^3 = (-10) \times (-10) \times (-10)
$$

- $(-10) \times (-10) = 100$

- $100 \times (-10) = -1,000$

So,

$$
(-10)^3 = -1,000
$$

4. $2(-10)^2$:

- $(-10)^2 = 100$

- $2 \times 100 = 200$

5. -10:

This is straightforward, equals -10.

6. Constant term: 8

Summing all terms

Now, combine all computed values:

$$
F(-10) = -200,000 + 30,000 - 1,000 + 200 - 10 + 8
$$

Step-by-step:

- $-200,000 + 30,000 = -170,000$
- $-170,000 - 1,000 = -171,000$
- $-171,000 + 200 = -170,800$
- $-170,800 - 10 = -170,810$
- $-170,810 + 8 = -170,802$

Final Remainder

$$
\boxed{
\text{Remainder} = -170,802
}
$$

This means when dividing $F(x)$ by $G(x) = x + 10$, the remainder is $-170,802$.

Additional Methods for Polynomial Remainder Calculation

While the Remainder Theorem offers a quick solution for linear divisors, other methods are useful for more complex divisors.

Polynomial Long Division

- Dividing $F(x)$ by $G(x)$ involves repeatedly subtracting multiples of $G(x)$ from $F(x)$ to reduce the degree.

Synthetic Division

- A simplified form of polynomial division used when dividing by linear factors, especially when the divisor is of the form $x - c$.

- In our case, since $G(x) = x + 10$, rewrite as $x - (-10)$, which makes synthetic division applicable using $c = -10$.

Using Synthetic Division to Find the Remainder

Step 1: Set up synthetic division with $c = -10$

Coefficients of $F(x)$:

$$
2, 3, 1, 2, 1, 8
$$

(Note: The polynomial is written as $2x^5 + 3x^4 + x^3 + 2x^2 + x + 8$. The coefficients are in order of decreasing power.)

Step 2: Perform synthetic division

Coefficients	2	3	1	2	1	8
-10		-20	170	-1,700	17,000	-170,000

The process involves:

- Bring down the leading coefficient: 2
- Multiply by $c = -10$: $2 \times -10 = -20$
- Add to next coefficient: $3 + (-20) = -17$
- Multiply: $-17 \times -10 = 170$
- Add: $1 + 170 = 171$
- Multiply: $171 \times -10 = -1,710$

- Add: $2 + (-1,710) = -1,708$
- Multiply: $-1,708 \times -10 = 17,080$
- Add: $1 + 17,080 = 17,081$
- Multiply: $17,081 \times -10 = -170,810$
- Add: $8 + (-170,810) = -170,802$

The final value, $-170,802$, is the remainder, consistent with previous calculations.

Summary and Key Takeaways

- The Remainder Theorem provides a quick way to find the remainder when dividing by a linear polynomial: evaluate $f(-a)$ where $g(x) = x + a$.
- For our example, $g(x) = x + 10$, so the remainder is $f(-10) = -170,802$.
- Synthetic division is an alternative method that is faster and less error-prone for linear divisors, confirming the remainder as $-170,802$.
- Polynomial division is essential in algebra for simplifying expressions, solving polynomial equations, and factoring.

Practical Applications

Understanding how to find remainders has applications across various fields:

- Algebraic simplification
- Factor theorem applications
- Polynomial interpolation
- Cryptography algorithms
- Error detection in coding theory

Conclusion

Finding the remainder when dividing polynomials is a vital skill in algebra. In our example, using the Remainder Theorem allowed us to efficiently compute that the remainder when dividing $f(x) = 2x^5 + 3x^4 + x^3 + 2x^2 + x + 8$ by $g(x) = x + 10$ is $-170,802$. Whether through

Frequently Asked Questions

How do I find the remainder when a polynomial f(x) is divided by a linear divisor g(x) = x - 10?

You can find the remainder by evaluating f(10) using the Remainder Theorem, since for linear divisors x - a, the remainder is f(a).

What is the Remainder Theorem and how does it apply to divide f(x) by g(x) = x - 10?

The Remainder Theorem states that when dividing a polynomial f(x) by x - a, the remainder is equal to f(a). So, for g(x) = x - 10, the remainder is f(10).

Given $F(x) = 2x^5 - 3x^4 + x^3 + 2x^2 - 8$ and $G(x) = x - 10$, how do I compute the remainder?

Calculate F(10): $F(10) = 2(10)^5 - 3(10)^4 + (10)^3 + 2(10)^2 - 8$.

Can synthetic division be used to find the remainder of F(x) divided by x - 10?

Yes, synthetic division simplifies the process of evaluating the polynomial at $x = 10$, which directly gives the remainder in this case.

What is the value of F(10) for $F(x) = 2x^5 - 3x^4 + x^3 + 2x^2 - 8$?

$F(10) = 2(100000) - 3(10000) + 1000 + 2(100) - 8 = 200000 - 30000 + 1000 + 200 - 8 = 170,192$.

Is the calculation of F(10) the same as finding the remainder when dividing F(x) by x - 10?

Yes, by the Remainder Theorem, evaluating F(10) gives the exact remainder.

What are the steps to find the remainder when dividing F(x) by G(x) = x - 10?

Step 1: Recognize $G(x) = x - 10$. Step 2: Evaluate F(10). The result is the remainder.

Why is the Remainder Theorem useful in polynomial division problems like this?

It allows you to quickly find the remainder without performing full polynomial division, saving time and effort.

Additional Resources

Find The Remainder When F(x) Is Divided By G(x) If $F(x) = 2x^5 + 3x^4 + x^3 + 2x^2 + x + 8$ and $G(x) = x + 10$ is a common problem in algebra that involves polynomial division and the Remainder Theorem. Whether you're a student brushing up on core concepts or a math enthusiast tackling polynomial problems, understanding how to find the remainder efficiently is essential. This guide will provide a comprehensive breakdown of the process, step by step, ensuring you grasp the underlying principles and can apply them confidently in similar problems.

Understanding the Problem

When asked to find the remainder when a polynomial F(x) is divided by another polynomial G(x), it typically involves understanding polynomial division concepts and the Remainder Theorem.

Given:
- $F(x) = 2x^5 + 3x^4 + x^3 + 2x^2 + x + 8$
- $G(x) = x + 10$

The goal is to find the remainder R when F(x) is divided by G(x).

The Key Concept: The Remainder Theorem

What is the Remainder Theorem?

The Remainder Theorem states that for any polynomial $F(x)$ divided by a linear divisor of the form $(x - a)$, the remainder is simply $F(a)$.

In mathematical terms:
- If $G(x) = x - a$, then Remainder $= F(a)$.

Applying the Remainder Theorem to Our Problem

Since $G(x) = x + 10$, we can rewrite it as:

$x + 10 = 0$

which implies:

$x = -10$

Thus, the remainder when $F(x)$ is divided by $G(x)$ is $F(-10)$.

Step-by-Step Process to Find the Remainder

1. Recognize the divisor as a linear polynomial

Our divisor $G(x) = x + 10$ fits neatly into the Remainder Theorem framework because it's linear.

2. Find the value of x that makes $G(x)$ zero

Set $G(x) = 0$:

$x + 10 = 0$

$x = -10$

3. Calculate $F(-10)$

Now, substitute $x = -10$ into $F(x)$:

$F(-10) = 2(-10)^5 + 3(-10)^4 + (-10)^3 + 2(-10)^2 + (-10) + 8$

Let's evaluate each term carefully.

Calculating $F(-10)$: Detailed Breakdown

Step 1: Compute powers of -10

- $(-10)^5 = -10 -10 -10 -10 -10$

Since $(-10)^5$ is an odd power, the result is negative:

$(-10)^5 = -100000$

- $(-10)^4 = 10,000$ (since even power makes it positive)

- $(-10)^3 = -1000$

- $(-10)^2 = 100$

Step 2: Plug in the computed powers into F(-10)

F(-10) = 2 (-100000) + 3 (10000) + (-1000) + 2 (100) + (-10) + 8

Step 3: Calculate each term

- 2 (-100000) = -200000
- 3 10000 = 30000
- (-1000) remains -1000
- 2 100 = 200
- (-10) remains -10
- 8 remains 8

Step 4: Sum all terms

Now, sum these step by step:

- -200000 + 30000 = -170000
- -170000 + (-1000) = -171000
- -171000 + 200 = -170800
- -170800 + (-10) = -170810
- -170810 + 8 = -170802

Therefore, F(-10) = -170802.

Final Result: The Remainder

According to the Remainder Theorem, the remainder when dividing F(x) by G(x) is:

Remainder = F(-10) = -170802

Additional Insights and Tips

Why does this method work?

- The Remainder Theorem simplifies the division process by reducing it to a simple evaluation problem.
- Instead of performing polynomial long division, which can be time-consuming, evaluating F at x = -10 gives the remainder directly.

When is this method applicable?

- When dividing by linear polynomials of the form (x - a).
- For higher-degree divisors, polynomial division or synthetic division methods are used, but the Remainder Theorem still provides insight into the remainder for linear divisors.

Practice Problems

1. Find the remainder when F(x) = x^3 - 4x^2 + 7x - 3 is divided by x - 2.
2. If G(x) = 3x + 5, what is the remainder when F(x) = 5x^4 - x^3 + 2x - 1 is divided by G(x)?
3. Use synthetic division to divide F(x) = 2x^3 + 3x^2 + x + 5 by x + 4 and find the remainder.

Summary

- To find the remainder when a polynomial F(x) is divided by a linear polynomial G(x) = x + c, identify the root of G(x) (which is x = -c).
- Evaluate F at that root: Remainder = F(-c).
- The process is straightforward and significantly faster than polynomial long division.

Final Thoughts

Mastering polynomial division problems, especially those involving finding remainders, is fundamental in algebra. The Remainder Theorem offers a quick and elegant way to determine the remainder for division by linear polynomials. Through understanding and applying this principle, problem-solving becomes more efficient, boosting your confidence in tackling advanced algebraic challenges.

Always remember: the key is recognizing when the divisor fits the form (x - a). Once you see that, evaluating F(a) instantly gives you the remainder, saving you time and effort. Keep practicing with different polynomials to solidify your understanding and become proficient in polynomial division techniques.

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