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Understanding vector operations is fundamental in physics, engineering, and mathematics. Among these operations, the vector cross product (also known as the vectorial product) plays a crucial role in determining perpendicular vectors, calculating torque, magnetic force, and in many applications involving three-dimensional space. This article provides a comprehensive guide to calculating the vector cross product of given vectors, focusing on the specific problem involving vectors A and 7, as well as multiplying the result by 5. We will explore the concepts step-by-step, ensuring clarity and depth for learners, students, and professionals alike.

Introduction to Vectors and the Cross Product

Before diving into the specific calculations, it is essential to understand what vectors are and how the cross product operates.

What Are Vectors?

Vectors are quantities that possess both magnitude and direction. They are represented in coordinate space using components along the axes, typically denoted with i, j, and k for the x, y, and z axes, respectively.

For example:

- Vector A = 2i + 5j - 2k

- Vector B = 1i - 4j + 2k

These components specify the vector's direction and length in three-dimensional space.

The Cross Product (Vectorial Product)

The cross product of two vectors A and B, denoted as A × B, results in a new vector that is perpendicular to both A and B. Its magnitude is given by:

|A × B| = |A| |B| sin(□)

where □ is the angle between A and B, and |A| and |B| are the magnitudes of the vectors.

The direction of A × B is determined by the right-hand rule:

- Point your right hand's fingers in the direction of A.

- Curl them towards B.

- Your thumb points in the direction of the cross product.

Mathematically, for vectors A = (A□, A□, A□) and B = (B□, B□, B□), the cross product is:

A × B = (A□B□ - A□B□)i - (A□B□ - A□B□)j + (A□B□ - A□B□)k

Given Vectors and Problem Statement

The problem provides the following vectors:

- Vector A = 2i + 5j - 2k
- Vector B = 1i - 4j + 2k

The tasks are:

1. Calculate the vector cross product of A and B, then multiply the result by 5 (i.e., compute (A × B) × 5).
2. Further, interpret or analyze the result as required.

Step-by-Step Calculation of the Cross Product

Step 1: Write the vectors in component form

- A = (2, 5, -2)
- B = (1, -4, 2)

Step 2: Apply the cross product formula

Using the determinant method:

\[

A \times B =

\begin{vmatrix}

i & j & k \\

A_1 & A_2 & A_3 \\

B_1 & B_2 & B_3 \\
\end{vmatrix}
\]

which expands to:

$$
A \times B = i (A_2 B_3 - A_3 B_2) - j (A_1 B_3 - A_3 B_1) + k (A_1 B_2 - A_2 B_1)
$$

Substituting the components:

$$
A \times B = i (5 \times 2 - (-2) \times -4) - j (2 \times 2 - (-2) \times 1) + k (2 \times -4 - 5 \times 1)
$$

Calculating each component:

- i component:

$$
5 \times 2 = 10
$$
$$
(-2) \times -4 = 8
$$
$$
i \text{ component} = 10 - 8 = 2
$$

- j component:

$$
2 \times 2 = 4
$$

$$
(-2) \times 1 = -2
$$

$$
j \text{ component} = -(4 - (-2)) = -(4 + 2) = -6
$$

- k component:

$$
2 \times -4 = -8
$$

$$
5 \times 1 = 5
$$

$$
k \text{ component} = -8 - 5 = -13
$$

Resultant cross product vector:

$$
A \times B = 2i - 6j - 13k
$$

Multiplying the Cross Product by 5

The next step involves multiplying the resulting vector by 5:

$$
(A \times B) \times 5 = 5 \times (2i - 6j - 13k) = (5 \times 2)i + (5 \times -6)j + (5 \times -13)k
$$

Calculations:

- $5 \times 2 = 10$
- $5 \times -6 = -30$
- $5 \times -13 = -65$

Final vector:

$$
\boxed{
\textbf{Result} = 10i - 30j - 65k
}
$$

Interpretation and Applications of the Result

The computed vector, 10i - 30j - 65k, has multiple implications depending on the context:

- It is perpendicular to both original vectors A and B.

- Its magnitude indicates the area of the parallelogram spanned by A and B, scaled by a factor of 5.
- The vector's components can be used in physics to calculate torque, magnetic forces, or rotational quantities.

Optimizations and Key Points in Cross Product Calculations

Understanding how to efficiently compute the cross product is vital for students, engineers, and scientists. Here are key points:

1. Component-wise calculation using determinants simplifies the process.
2. Right-hand rule helps verify the direction of the resultant vector.
3. Scaling vectors before or after the cross product is straightforward—multiply the entire vector after computation.
4. Use of vector software tools (like MATLAB, WolframAlpha, or Python libraries) can automate these calculations for complex vectors.

Summary of the Process

To summarize, the steps involved in calculating the vector cross product and subsequent scaling are:

1. Convert vectors into component form.
2. Apply the determinant formula for the cross product.
3. Compute each component carefully.
4. Multiply the resulting vector by the scalar 5.

5. Interpret the result in the context of your problem.

Conclusion

Calculating the vector cross product is an essential skill in vector algebra, with applications across physics, engineering, and mathematics. In this specific case, the cross product of vectors A and B resulted in a vector that, when scaled by 5, provides insights into the spatial relationships between the original vectors. Mastery of these calculations enhances understanding of three-dimensional vector operations and prepares learners to tackle more complex problems involving vector calculus.

Additional Resources for Learning Vector Calculus

- Khan Academy: Vector cross product tutorials
- MIT OpenCourseWare: Linear algebra and vector calculus courses
- WolframAlpha: Step-by-step vector calculations
- Books: "Vector Calculus" by Jerrold E. Marsden and Anthony J. Tromba

By practicing these calculations and understanding their applications, students and professionals can develop a strong grasp of multidimensional vector operations, crucial for solving real-world problems in science and engineering.

Keywords for SEO Optimization:

- Vector cross product calculation
- How to compute vectorial product
- Cross product of vectors example
- Vector algebra tutorial
- Mathematical operations with vectors
- 3D vector calculations
- Physics vector operations
- Engineering vector applications
- Scalar multiplication of vectors
- Vector mathematics for students

Frequently Asked Questions

How do you compute the vector cross product of vectors A = 2i + 5j - 2k and B = 1 - 4j + 2k?

To compute the cross product A × B, set up the determinant with unit vectors i, j, k on the first row, components of A on the second, and components of B on the third. Calculate the determinant to find the resulting vector.

What is the step-by-step process to calculate the vector product of A = 2i + 5j - 2k and B = 1 - 4j + 2k?

First, write the vectors in component form: A = (2, 5, -2), B = (1, -4, 2). Then, compute the determinant of the matrix:

|i j k|

|2 5 -2|

|1 -4 2|. Evaluate this determinant to find the cross product.

What is the value of the vector cross product A × B for the given vectors?

The cross product A × B is calculated as follows:

$A \times B = ((5)(2) - (-2)(-4)) i - ((2)(2) - (-2)(1)) j + ((2)(-4) - (5)(1)) k$

$= (10 - 8) i - (4 - (-2)) j + (-8 - 5) k$

$= 2i - 6j - 13k.$

Why is the vector cross product important in physics and engineering?

The vector cross product is used to find a vector perpendicular to two given vectors, representing quantities like torque, magnetic force, and angular momentum, which are essential in physics and engineering analyses.

Additional Resources

Vector Cross Product: An In-Depth Exploration of Vectors A and B

Introduction to the Vector Cross Product

The vector cross product, also known as the vector product, is a fundamental operation in vector algebra with profound applications across physics, engineering, and computer graphics. It provides a way to generate a third vector that is perpendicular to two given vectors in three-dimensional space, encapsulating both magnitude and directional information.

Imagine you have two vectors, each representing some physical quantity such as force, velocity, or magnetic field. The cross product allows you to find a new vector that is orthogonal to both, offering insights into properties like torque, angular momentum, and magnetic force.

In this article, we will examine a specific problem involving two vectors:

$$
\mathbf{A} = 2\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}
$$

$$
\mathbf{B} = 1\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}
$$

Our goal is to compute the vector cross product $\mathbf{A} \times \mathbf{B}$, providing a comprehensive explanation of each step, the underlying mathematics, and practical interpretations.

Understanding the Vectors: Components and Significance

Before diving into calculations, let's clarify what these vectors represent and their components.

- Vector $\mathbf{A}$:

$$
\mathbf{A} = 2\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}
$$

- Components: $A_x = 2$, $A_y = 5$, $A_z = -2$

- Vector $\mathbf{B}$:

$$
\mathbf{B} = 1\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}
$$

- Components: $B_x = 1$, $B_y = -4$, $B_z = 2$

Understanding these components is crucial because the cross product involves determinants constructed from these values.

The Mathematical Foundation of the Cross Product

The cross product of two vectors $\mathbf{A}$ and $\mathbf{B}$, denoted as $\mathbf{A} \times \mathbf{B}$, is defined as a vector:

$$
\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}|\sin\theta\, \mathbf{n}
$$

where:

- $|\mathbf{A}|$ and $|\mathbf{B}|$ are the magnitudes of the vectors,
- θ is the angle between $\mathbf{A}$ and $\mathbf{B}$,
- $\mathbf{n}$ is the unit vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

However, computationally, the cross product is most straightforwardly calculated using the determinant of a matrix constructed from the unit vectors and the components of $\mathbf{A}$ and $\mathbf{B}$:

\[

\mathbf{A} \times \mathbf{B} =

\begin{vmatrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\

A_x & A_y & A_z \\

B_x & B_y & B_z \\

\end{vmatrix}

\]

This determinant expands as:

\[

\mathbf{A} \times \mathbf{B} = \mathbf{i}(A_y B_z - A_z B_y) - \mathbf{j}(A_x B_z - A_z B_x) + \mathbf{k}(A_x B_y - A_y B_x)

\]

Step-by-Step Calculation of \(\mathbf{A} \times \mathbf{B}\)

Let's now meticulously perform this calculation.

Step 1: Write the components

\[

A_x = 2,\quad A_y = 5,\quad A_z = -2

\]

\[

B_x = 1,\quad B_y = -4,\quad B_z = 2

\]

Step 2: Compute the determinant components

- For the $\mathbf{i}$ component:

$$
A_y B_z - A_z B_y = (5)(2) - (-2)(-4) = 10 - 8 = 2
$$

- For the $\mathbf{j}$ component:

$$
A_x B_z - A_z B_x = (2)(2) - (-2)(1) = 4 + 2 = 6
$$

- For the $\mathbf{k}$ component:

$$
A_x B_y - A_y B_x = (2)(-4) - (5)(1) = -8 - 5 = -13
$$

Step 3: Assemble the cross product vector

Using the expanded form:

$$
\mathbf{A} \times \mathbf{B} = \mathbf{i}(2) - \mathbf{j}(6) + \mathbf{k}(-13)
$$

Or, explicitly,

\[
\boxed{
\mathbf{A} \times \mathbf{B} = 2\mathbf{i} - 6\mathbf{j} - 13\mathbf{k}
}
\]

Interpreting the Result

The resulting vector:

\[
\boxed{
\mathbf{A} \times \mathbf{B} = 2\mathbf{i} - 6\mathbf{j} - 13\mathbf{k}
}
\]

has several important properties:

- Perpendicularity: It is orthogonal to both \(\mathbf{A}\) and \(\mathbf{B}\), which can be verified by confirming the dot products:

\[
\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0
\]

\[

\mathbf{B} \cdot (\mathbf{A} \times \mathbf{B}) = 0

\]

These zero dot products confirm the orthogonality.

- Magnitude: The magnitude of \(\mathbf{A} \times \mathbf{B}\) is given by:

\[

|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin \theta

\]

Computing the magnitude:

\[

|\mathbf{A} \times \mathbf{B}| = \sqrt{(2)^2 + (-6)^2 + (-13)^2} = \sqrt{4 + 36 + 169} = \sqrt{209}

\approx 14.456

\]

- Physical Significance: In physics, this vector could represent torque generated by forces \(\mathbf{A}\) and \(\mathbf{B}\), or the normal vector to a plane defined by these two vectors, which is critical in calculating areas and orientations.

Practical Applications of the Cross Product

Understanding the cross product's computation is vital across various disciplines:

1. Physics:

- Calculating torque ($\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$)
- Determining magnetic force ($\mathbf{F} = q \mathbf{v} \times \mathbf{B}$)
- Finding angular momentum ($\mathbf{L} = \mathbf{r} \times \mathbf{p}$)

2. Engineering:

- Analyzing moments and rotations
- Computing the normal vector to surfaces for structural analysis

3. Computer Graphics:

- Calculating surface normals for rendering
- Determining the orientation of objects

4. Mathematics and Geometry:

- Computing areas of parallelograms and triangles
- Finding perpendicular vectors for coordinate transformations

---

Summary and Final Thoughts

The calculation of $\mathbf{A} \times \mathbf{B}$ exemplifies the power and elegance of vector algebra. By employing determinants and component-wise operations, we derive a vector that succinctly encodes perpendicularity, magnitude, and orientation.

Final result:

\[

\boxed{

\mathbf{A} \times \mathbf{B} = 2\mathbf{i} - 6\mathbf{j} - 13\mathbf{k}

}

\]

This vector not only confirms the mathematical operation's correctness but also provides meaningful insights into the spatial relationship between \(\mathbf{A}\) and \(\mathbf{B}\).

Closing Remarks

Mastering the cross product is essential for anyone delving into physics, engineering, or advanced mathematics. Its ability to reveal the orthogonal direction and magnitude of interacting vectors makes it an indispensable tool for analyzing three-dimensional phenomena. Whether calculating forces, moments, or normal vectors, the cross product remains a cornerstone of vector calculus, bridging abstract mathematics with tangible real-world applications.

B Given The Vectors A 2i 5j 2k And 7 1 4j 2k I Calculate The Vectorial Product X 5 Ii

B Given The Vectors A 2i 5j 2k And 7 1 4j 2k I Calculate The Vectorial Product X 5 Ii

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