[By Hand] Sketch The Root Locus For Positive K For The Unity Feedback System With Open Loop Transfer

[By Hand] Sketch The Root Locus For Positive K For The Unity Feedback System With Open Loop Transfer

Introduction to Root Locus Method

The root locus technique is a graphical method used in control system design to analyze how the roots of the characteristic equation change with variations in a system parameter, typically the gain K. It provides insight into the stability and transient response of a feedback control system as the gain varies from zero to infinity. When dealing with a unity feedback system, the open loop transfer function $L(s)$ plays a crucial role in determining the root locus plot.

Understanding the System Setup

In the context of this discussion, consider a unity feedback system with an open loop transfer function $L(s)$. The system's characteristic equation is given by:

$$
1 + K L(s) = 0
$$

where $K > 0$. Our goal is to sketch the root locus as K varies from 0 to ∞, focusing on positive gain values.

Step 1: Determine the Open Loop Transfer Function $L(s)$

Before sketching, identify the open loop transfer function. Typically, it is expressed as a ratio of polynomials:

$$
L(s) = \frac{N(s)}{D(s)}
$$

where:
- $N(s)$ is the numerator polynomial.
- $D(s)$ is the denominator polynomial.

For example, suppose:

$$
L(s) = \frac{K}{s(s + a)}
$$

This is a common configuration involving integrator and pole at $-a$.

Step 2: Find Open Loop Poles and Zeros

Locate the poles and zeros of $L(s)$:

- Poles: Roots of $D(s) = 0$.
- Zeros: Roots of $N(s) = 0$.

For the example $L(s) = \frac{K}{s(s + a)}$:

- Poles at $s=0$ and $s=-a$,
- No zeros.

Plot these on the complex s-plane:

- Poles are marked as 'X'.
- Zeros as 'O' (if any).

In this example, two poles at the origin and $-a$.

Step 3: Determine Asymptotes and Centroid

Root locus branches for systems with more poles than zeros tend to infinity, and their asymptotes guide the path as $K \to \infty$.

- Number of poles n: count of total poles.
- Number of zeros m: count of total zeros.

For the example:

- $n=2$,
- $m=0$.

Asymptote angles:

$$
\theta_k = \frac{(2k+1)180^\circ}{n - m} \quad \text{for } k=0,1,\dots,(n - m - 1)
$$

which simplifies to:

$$
\theta_k = \frac{(2k+1)180^\circ}{2} = 90^\circ, 270^\circ

So, asymptotes at 90° and 270°.

Centroid:

$$
\sigma_{centroid} = \frac{\sum \text{poles} - \sum \text{zeros}}{n - m}
$$

For the example:

$$
\sigma_{centroid} = \frac{0 + (-a) - 0}{2} = -\frac{a}{2}
$$

This point is where the asymptotes intersect and guides the root locus at high gain.

Step 4: Identify Real Axis Segments

The root locus lies on the real axis in segments where the total number of poles and zeros to the right of the point is odd.

- Starting from the poles, check the number of poles and zeros to the right.
- Segments on the real axis where an odd number of poles and zeros are to the right are part of the root locus.

For the example:
- On the negative real axis, between $-a$ and 0, and to the left of $-a$, the root locus segments are determined based on counting poles and zeros.

Step 5: Sketch the Root Locus Paths

Now, construct the root locus:

1. At $K=0$: The roots are at the poles: $s=0$ and $s=-a$.
2. As K increases: The roots move along paths starting from these poles.
3. Poles move toward zeros or infinity:

- Since no zeros are present, the branches tend towards infinity along asymptotes.
- The asymptotes indicate the directions the poles move as $K \to \infty$.

4. Branch behavior:

- One branch originates at $s=0$ and moves towards infinity along the 90° asymptote.
- The other branch from $s=-a$ moves along the 270° asymptote to infinity.

Step 6: Mark Critical Points and Behavior

Identify key points such as:

- Breakaway points: where root paths come together on the real axis.
- Imaginary axis crossings: where the system transitions from stability to instability.

To find these points:

- Set the derivative of the characteristic equation with respect to s to zero, or
- Use standard root locus criteria to locate imaginary axis crossings.

In our example, the crossing occurs at a particular value of K, obtained by solving the characteristic equation for purely imaginary roots ($s=j\omega$).

Step 7: Final Sketch and Interpretation

After plotting all poles, zeros, asymptotes, and key points, draw the root locus branches:

- Start at the poles when $K=0$.
- Follow the paths along the real axis segments.
- Branches approach infinity along asymptotes.
- Note the behavior at large K, which influences stability.

This graphical representation allows for quick assessment of system stability:

- Stable: All roots in the left half-plane.
- Unstable: Roots in the right half-plane.

Practical Tips for Hand Sketching Root Locus

Understand the System's Poles and Zeros

Begin by accurately locating the poles and zeros on the complex plane, as these are the starting points for root locus paths.

Determine Asymptotes and Centroid

Calculate asymptote angles and centroid to understand the behavior at high gain.

Use Real Axis Rule

Identify segments on the real axis where the root locus exists based on the counting rule.

Plot Critical Points

Find breakaway points and imaginary axis crossing points analytically for precise sketching.

Connect the Dots

Draw smooth paths from poles to zeros or to infinity, following the rules and asymptote directions.

Interpret the Results

Use the sketch to assess stability and transient response for various values of K.

Conclusion

Sketching the root locus by hand for a positive gain K in a unity feedback system involves understanding the open loop transfer function, locating poles and zeros, calculating asymptotes and centroid, and carefully plotting the paths of the roots as the gain varies. This graphical method provides an intuitive and effective way to analyze the stability and dynamic behavior of control systems, making it a fundamental skill for control engineers. Mastery of this technique, through practice and understanding of the underlying principles, enables designers to predict system performance and make informed adjustments to achieve desired specifications.

Frequently Asked Questions

What is the significance of sketching the root locus for positive K in a unity feedback system with an open loop transfer function?

Sketching the root locus for positive K helps determine the system's stability and transient response characteristics as the gain varies, enabling designers to ensure desired performance and stability margins in a unity feedback configuration.

How do you identify the points where the root locus begins and ends when sketching for positive K?

The root locus begins at the open loop poles and ends at the open loop zeros (including zeros at infinity). For positive K, these points are identified by plotting the pole-zero configuration of the open loop transfer function.

What are the key steps involved in sketching the root locus for a unity feedback system with positive gain?

Key steps include: 1) Determine the open loop transfer function, 2) find the poles and zeros, 3) locate the real-axis segments of the locus, 4) determine the asymptotes and their angles, 5) identify breakaway points, and 6) analyze the stability regions as K varies.

How does increasing positive K affect the location of the root locus in a unity feedback system?

Increasing positive K causes the closed-loop poles to move along the root locus branches, typically moving toward zeros or infinity, which can lead to enhanced or diminished stability depending on their trajectory, and may eventually cause the system to become unstable if poles cross into the right-half plane.

What role do asymptotes play in sketching the root locus for positive K, and how are they determined?

Asymptotes indicate the paths of the root locus branches that go to infinity as K approaches infinity. They are determined by calculating the angles based on the difference between the number of poles and zeros, and their intersection point (centroid) is found using the average of the pole and zero locations.

Additional Resources

Root Locus Sketching for Positive K in Unity Feedback Systems with Open Loop Transfer: An Expert Insight

---

Introduction

In control systems engineering, the root locus method stands as a fundamental graphical technique used to analyze and design feedback controllers. When dealing with unity feedback systems, understanding how the system poles move in the complex plane as the system gain varies is crucial for assessing stability and transient response. This article delves into the detailed process of sketching the root locus by hand for positive gain K, specifically focusing on systems with a given open loop transfer function.

The essence of this discussion revolves around the open loop transfer function $L(s) = K \cdot G(s)$, where $G(s)$ is a known transfer function. By examining the locus of the closed-loop poles as K varies from zero to infinity, engineers can predict system behavior, ensure stability, and fine-tune system performance.

Fundamental Concepts of Root Locus

What is Root Locus?

Root locus is a graphical method that illustrates the possible locations of the closed-loop poles as the gain K varies along the positive real axis. It provides insights into system stability, transient behavior, and response characteristics without solving the characteristic equation explicitly for each value of K.

Why Focus on Positive K?

In most practical control applications, the gain K is positive, representing physical parameters such as proportional control gains. Negative gains generally lead to different stability considerations and are less common in standard feedback systems.

Step-by-Step Approach to Sketching Root Locus by Hand

Creating an accurate root locus plot involves methodical steps. Each step builds upon the previous to accurately depict pole movements and asymptotic behaviors.

Step 1: Identify the Open Loop Transfer Function $L(s)$

Begin with the open loop transfer function:

$$
L(s) = K \times G(s)
$$

where $G(s)$ is a rational function expressed as:

$$
G(s) = \frac{N(s)}{D(s)}
$$

with numerator polynomial $N(s)$ and denominator polynomial $D(s)$.

Example:

Suppose:

$$
G(s) = \frac{1}{s(s + 2)}
$$

then:

$$
L(s) = \frac{K}{s(s + 2)}
$$

Step 2: Determine the Open Loop Poles and Zeros

Locate the poles (roots of $D(s)$) and zeros (roots of $N(s)$) of $L(s)$.

- Poles: Values of s where $D(s) = 0$.
- Zeros: Values of s where $N(s) = 0$.

In the example:

- Poles at $s=0$ and $s=-2$.
- No zeros (since numerator is 1).

Step 3: Plot Poles and Zeros on the Complex Plane

Mark the poles with 'X' and zeros with 'O'. The root locus will originate from the poles and move towards zeros or infinity.

Step 4: Determine the Asymptotes (for Unmatched Poles and Zeros)

If the number of poles P exceeds the number of zeros Z, the root locus branches tend to infinity along asymptotes.

- Number of asymptotes: $P - Z$.
- Angles of asymptotes:

$$
\theta_a = \frac{(2k + 1)180^\circ}{P - Z}, \quad k=0,1,\dots, P - Z - 1
$$

- Centroid of asymptotes:

$$
\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{P - Z}
$$

Example:

Poles at 0 and -2, zero zeros.

$$
\sigma_a = \frac{(0) + (-2)}{2} = -1
$$

Number of asymptotes: 2 (since 2 poles, no zeros).

Angles:

$$
\theta_a = \frac{(2k+1)180^\circ}{2} = 90^\circ, 270^\circ
$$

so asymptotes at $+90^\circ$ and -90°, emanating from $\sigma_a = -1$.

Step 5: Find the Breakaway and Break-in Points

- Breakaway points occur where multiple root locus branches leave or meet on the real axis, typically between poles on the real axis.
- Find points where the characteristic equation's root locus crosses the real axis by solving:

$$
1 + K G(s) = 0
$$

or equivalently,

$$
\frac{1}{K} = - G(s)
$$

- For the real axis segments, the root locus exists where the sum of angles of poles and zeros satisfies specific conditions.

Method:

- For points on the real axis, the total angle contributed by poles and zeros must be an odd multiple of 180°.

Step 6: Determine the Real Axis Segments

The root locus exists on real-axis segments where:

$$
\text{Number of poles and zeros to the right of the point} = \text{odd}
$$

- For example, between $s=-\infty$ and -2: check the number of poles/zeros to the right.
- Between -2 and 0: check similarly.

In the example:

- To the right of $-\infty$, all poles are to the right, but since both poles are on the real axis, the locus exists on segments where the number of poles to the right is odd.

Step 7: Sketch the Root Locus Branches

Using the above information, sketch the paths:

- Starting from poles.
- Moving towards zeros or infinity along asymptotes.
- Passing through breakaway points if applicable.
- Respect the real-axis segments where the locus exists.

Step 8: Determine the Behavior at Infinity

- For large K, the poles tend to the asymptotes.
- The branches tend to infinity along these asymptotes, moving away from the centroid.

Practical Example: Sketching for a Specific System

Let's consider a practical example with the open loop transfer function:

$$
L(s) = \frac{K}{s(s+2)+K}
$$

which simplifies to:

$$
1 + \frac{K}{s(s+2)} = 0
$$

or

$$
s(s+2) + K = 0
$$

The characteristic equation:

$$
s^2 + 2s + K = 0
$$

- Poles are at:

$$
s = -1 \pm \sqrt{1 - K}
$$

- For $K<1$, roots are real and distinct.
- For $K=1$, roots are repeated at -1.
- For $K>1$, roots are complex conjugates.

By plotting how roots move as K varies, the root locus can be sketched by examining these cases.

Additional Tips for Hand Sketching

- Use symmetry: Root locus is symmetric about the real axis.
- Identify critical points: Breakaway points, zeros, poles at infinity.
- Apply the angle criterion: The sum of angles for points on the locus must satisfy the angle condition:

$$
\angle G(s) = (2k+1)180^\circ
$$

- Estimate the shape: Use the asymptote directions, crossing points, and the known behavior near poles and zeros.

Conclusion

Mastering by hand root locus sketching is a vital skill for control engineers. It combines analytical reasoning with graphical intuition, enabling precise predictions of how a system's stability and transient response evolve with gain K. While computational tools assist in complex cases, understanding the foundational steps—identifying poles and zeros, asymptotes, breakaway points, and real axis segments—empowers engineers to design robust control systems confidently.

The process outlined herein provides a comprehensive framework to approach root locus analysis systematically. With practice, one can quickly generate accurate hand sketches that inform critical control decisions, ultimately leading to more stable, efficient, and responsive systems.

References

- Ogata, K. (2010). Modern Control Engineering. Prentice Hall.
- Dorf, R. C., & Bishop, R. H. (2011). Modern Control Systems. Pearson.
- Nise, N. S. (2015). Control Systems Engineering. Wiley.

Note: The above approach emphasizes clarity and thoroughness, designed for control engineers, students, or professionals seeking a detailed understanding of hand-drawing root locus for positive gain in unity feedback systems.

By Hand Sketch The Root Locus For Positive K For The Unity Feedback System With Open Loop Transfer

By Hand Sketch The Root Locus For Positive K For The Unity Feedback System With Open Loop Transfer

Related Articles

- When I Stand 20 Feet Away From My Home, The Angle Of Elevation From The Ground To The Top Of My Home
- A Clothing Designer Determines That The Number Of Shirts She Can Sell Is Given By The Formula S = 4x2
- When Societies Use Trained Teachers To Pass On Specialized Knowledge That Adults Will Need For Their

Back to Home