(c) The Angular Velocity Of A Disc Of Mass 3 Kg And Radius 0.6 M Changes From 10 Rev/s To 15 Rev/s In

(c) The Angular Velocity Of A Disc Of Mass 3 Kg And Radius 0.6 M Changes From 10 Rev/s To 15 Rev/s In this article, we explore the fascinating dynamics of rotational motion, focusing on how the angular velocity of a disc varies under different conditions. Understanding the change in angular velocity is crucial for engineers, physicists, and students studying rotational mechanics, as it provides insights into how objects spin, accelerate, and conserve energy during rotation.

In this comprehensive guide, we will analyze the problem involving a disc with a mass of 3 kg and a radius of 0.6 meters, which experiences a change in its angular velocity from 10 revolutions per second (rev/s) to 15 rev/s. We will delve into the concepts of angular velocity, angular acceleration, moment of inertia, and rotational kinetic energy, providing clear explanations and practical examples.

Understanding Angular Velocity and Its Significance

What Is Angular Velocity?

Angular velocity (ω) is a measure of how quickly an object rotates or spins around a fixed axis. It is defined as the rate of change of angular displacement with respect to time and is usually expressed in radians per second (rad/s).

The relationship between revolutions per second (rev/s) and radians per second is given by:

$$
\omega = 2\pi \times (\text{rev/s})
$$

where:
- 1 revolution = 2π radians

Importance of Angular Velocity in Rotational Mechanics

Angular velocity is essential in various applications:
- Designing rotating machinery like turbines and engines
- Understanding the dynamics of spinning objects
- Calculating rotational kinetic energy
- Analyzing angular acceleration and torque

Initial and Final Conditions of the Disc

Given Data

The problem provides the following information:
- Mass of the disc, $m = 3\,kg$
- Radius of the disc, $r = 0.6\,m$
- Initial angular velocity, $\omega_i = 10\,rev/s$
- Final angular velocity, $\omega_f = 15\,rev/s$

Converting Rev/s to Rad/s

To perform calculations in SI units, convert the initial and final angular velocities:

$$
\omega_i = 10\,rev/s \times 2\pi\,rad/rev = 20\pi\,rad/s \approx 62.83\,rad/s
$$
$$
\omega_f = 15\,rev/s \times 2\pi\,rad/rev = 30\pi\,rad/s \approx 94.25\,rad/s
$$

Calculating the Change in Angular Velocity

Angular Acceleration (α)

The angular acceleration describes how quickly the disc's angular velocity changes over time:

$$
\alpha = \frac{\omega_f - \omega_i}{t}
$$

However, since the problem does not specify the time duration, we focus instead on energy and torque considerations, or we assume a certain time interval for more detailed analysis.

Determining the Work Done or Energy Change

The change in the disc's rotational kinetic energy can be calculated as:

$$
\Delta KE = \frac{1}{2} I (\omega_f^2 - \omega_i^2)
$$

where I is the moment of inertia of the disc.

Moment of Inertia of the Disc

Formula for a Solid Disc

The moment of inertia for a solid disc about its central axis is given by:

$$
I = \frac{1}{2} m r^2
$$

Substituting the given values:

$$
I = \frac{1}{2} \times 3\,kg \times (0.6\,m)^2 = \frac{1}{2} \times 3 \times 0.36 = 0.54\,kg\,m^2
$$

Significance of Moment of Inertia

The moment of inertia quantifies how much torque is needed for a given angular acceleration. A larger I means the object resists changes in its rotational speed more strongly.

Calculating the Change in Rotational Kinetic Energy

Using the earlier energy formula:

$$
\Delta KE = \frac{1}{2} \times 0.54\,kg\,m^2 \times \left((94.25)^2 - (62.83)^2 \right)
$$

Calculations:

$$
(94.25)^2 \approx 8884.56
$$

$$
(62.83)^2 \approx 3947.84
$$

$$
\Delta KE = 0.27 \times (8884.56 - 3947.84) = 0.27 \times 4936.72 \approx 1332.52\,J
$$

\]

Interpretation: The disc gains approximately 1332.52 joules of rotational kinetic energy as its angular velocity increases from 10 rev/s to 15 rev/s.

Torque Required for the Change in Angular Velocity

Relationship Between Torque and Angular Acceleration

The torque (τ) necessary to produce a certain angular acceleration is given by:

$$
\tau = I \alpha
$$

If the time taken for the change in angular velocity, t, is known, one can compute α:

$$
\alpha = \frac{\omega_f - \omega_i}{t}
$$

and then:

$$
\tau = I \times \frac{\omega_f - \omega_i}{t}
$$

Note: Without the time interval, we can only analyze the torque in terms of the angular acceleration.

Practical Applications and Real-World Examples

Rotational Mechanics in Engineering

Understanding the change in angular velocity is fundamental in designing:
- Electric motors
- Wind turbines
- Automotive flywheels
- Gyroscopes and stabilization systems

Sports and Recreation

Rotational motion principles are used to optimize performance in activities like:
- Figure skating spins
- Baseball pitches
- Bicycle wheel dynamics

Physics Education and Demonstrations

Experiments involving discs spinning at different speeds help students grasp concepts of inertia, energy conservation, and torque.

Conclusion

The change in the angular velocity of a disc from 10 rev/s to 15 rev/s signifies a substantial increase in rotational kinetic energy, requiring a specific amount of torque and energy input. The calculations demonstrate the importance of the moment of inertia and angular acceleration in analyzing rotational dynamics. Understanding these principles not only enhances theoretical knowledge but also provides vital insights for practical engineering applications, sports science, and physics education.

Summary of Key Points

1. Angular velocity measures how fast an object spins around its axis, expressed in rad/s or rev/s.

2. Converting rev/s to rad/s involves multiplying by 2π.

3. The moment of inertia for a solid disc is $I = \frac{1}{2} m r^2$.

4. The change in rotational kinetic energy depends on the initial and final angular velocities and the moment of inertia.

5. Torque required for a change in angular velocity relates to angular acceleration and inertia.

By mastering these concepts, students and professionals can better analyze and design systems involving rotational motion, ensuring efficiency, safety, and innovation in various technological fields.

Frequently Asked Questions

What is the initial angular velocity of the disc in radians per second?

The initial angular velocity is 10 revolutions per second, which equals $10 \times 2\pi = 20\pi \approx 62.83$ rad/sec.

What is the final angular velocity of the disc in radians per second?

The final angular velocity is 15 revolutions per second, which equals $15 \times 2\pi = 30\pi \approx 94.25$ rad/sec.

How do you calculate the change in angular velocity of the disc?

The change in angular velocity $\Delta\omega = \omega_{final} - \omega_{initial} = 94.25$ rad/sec – 62.83 rad/sec ≈ 31.42 rad/sec.

What is the moment of inertia of the disc with mass 3 kg and radius 0.6 m?

For a solid disc, $I = (1/2) M R^2 = 0.5 \times 3\text{ kg} \times (0.6\text{ m})^2 = 0.5 \times 3 \times 0.36 = 0.54\text{ kg}\cdot\text{m}^2$.

How can we determine the angular acceleration of the disc during this change?

Using $\alpha = \Delta\omega / \Delta t$. If the time duration is known, divide the change in angular velocity (~31.42 rad/sec) by that time to find angular acceleration.

What is the work done on the disc during this change in angular velocity?

Work done, $W = \Delta K.E = (1/2)\, I\, (\omega_{final}^2 - \omega_{initial}^2) = 0.5 \times 0.54 \times (94.25^2 - 62.83^2)$ rad²/sec², which calculates to approximately $0.27 \times (8880 - 3948) \approx 0.27 \times 4932 \approx 1331$ Joules.

If a torque is applied to change the disc's angular velocity, how is it related to the angular acceleration?

Torque $\tau = I \times \alpha$. Knowing I and α allows calculation of the torque required to produce the change in angular velocity.

What practical applications involve changing the angular velocity of a disc like this?

Applications include rotating machinery, flywheels in engines, disc brakes, and gyroscopic devices where controlled changes in rotational speed are essential.

Additional Resources

The Angular Velocity Of A Disc Of Mass 3 Kg And Radius 0.6 M Changes From 10 Rev/s To 15 Rev/s In a detailed analysis, understanding the principles of rotational motion is essential. This problem encapsulates fundamental concepts like angular velocity, angular acceleration, and the relationship between linear and rotational quantities, providing insight into how objects behave when their rotational speeds change.

Understanding Angular Velocity and Rotational Dynamics

Before diving into the specifics of the problem, it’s crucial to clarify some core concepts:

- Angular Velocity (ω): The rate at which an object rotates about an axis, measured in radians per second (rad/s). It indicates how quickly the object spins.
- Revolutions per second (Rev/s): A common unit used to describe rotational speed, which can be converted to radians per second.
- Moment of Inertia (I): A measure of an object's resistance to changes in its rotation, depending on its mass distribution. For a disc, $I = (1/2)\, m r^2$.
- Angular Acceleration (α): The rate of change of angular velocity over time, measured in rad/s^2.
- Torque (τ): The rotational equivalent of force that causes angular acceleration, $\tau = I\alpha$.

Converting Units: From Rev/s to Radians per Second

Since the initial and final angular velocities are given in revolutions per second, conversion to radians per second is necessary for calculations involving angular acceleration or torque.

- Conversion factor: 1 revolution = 2π radians.

Therefore,

$$
\omega = \text{Rev/sec} \times 2\pi
$$

Applying this:

- Initial angular velocity,

$$
\omega_i = 10 \, \text{Rev/s} \times 2\pi = 20\pi \, \text{rad/s} \approx 62.83 \, \text{rad/s}
$$

- Final angular velocity,

$$
\omega_f = 15 \, \text{Rev/s} \times 2\pi = 30\pi \, \text{rad/s} \approx 94.25 \, \text{rad/s}
$$

Calculating Angular Acceleration

Suppose the change in angular velocity occurs over a certain period, denoted as t. To find the angular acceleration:

$$
\alpha = \frac{\omega_f - \omega_i}{t}
$$

- If time is known, this formula directly yields the angular acceleration.
- If time is unknown, further information such as torque or energy considerations is required.

Scenario 1: Known Time Duration

Let’s assume the change from 10 Rev/s to 15 Rev/s occurs over 5 seconds.

- Calculate angular acceleration:

$$
\alpha = \frac{94.25 - 62.83}{5} = \frac{31.42}{5} \approx 6.28 \, \text{rad/s}^2
$$

This indicates the disc's angular velocity increases at approximately 6.28 rad/s^2 during this interval.

Scenario 2: Unknown Time – Applying Law of Conservation of Energy

If the problem involves only the change in rotational speed without external torque or energy losses, the kinetic energy of rotation provides insight.

- Rotational Kinetic Energy:

$$
KE = \frac{1}{2} I \omega^2
$$

- Moment of Inertia for a disc:

$$
I = \frac{1}{2} m r^2
$$

Given:

- $m = 3 \, \text{kg}$
- $r = 0.6 \, \text{m}$

Calculate:

$$
I = \frac{1}{2} \times 3 \times (0.6)^2 = 1.5 \times 0.36 = 0.54 \, \text{kg·m}^2
$$

- Initial kinetic energy:

$$
KE_i = \frac{1}{2} \times 0.54 \times (62.83)^2 \approx 0.27 \times 3947.84 \approx 1066.4 \, \text{J}
$$

- Final kinetic energy:

$$
KE_f = \frac{1}{2} \times 0.54 \times (94.25)^2 \approx 0.27 \times 8882.56 \approx 2400.5 \, \text{J}
$$

Energy Increase:

$$
\Delta KE = KE_f - KE_i \approx 2400.5 - 1066.4 = 1334.1 \, \text{J}
$$

This energy could be supplied by an external torque, which, over the duration t, does work equal to this energy increase.

Calculating Torque and Power

If the change occurs over a known time, the torque involved can be calculated:

$$
\tau = I \alpha
$$

Using the angular acceleration from earlier:

$$
\tau = 0.54 \times 6.28 \approx 3.39 \, \text{Nm}
$$

The power supplied during this acceleration:

$$
P = \tau \times \omega_{avg}
$$

where

$$
\omega_{avg} = \frac{\omega_i + \omega_f}{2} \approx \frac{62.83 + 94.25}{2} = 78.54 \, \text{rad/s}
$$

Thus,

$$
P \approx 3.39 \times 78.54 \approx 266.4 \, \text{W}
$$

This means the external source must deliver approximately 266.4 Watts to accelerate the disc over 5 seconds.

Implications and Practical Considerations

Understanding how the angular velocity changes from 10 Rev/s to 15 Rev/s involves multiple factors:

- Energy input: The increase in rotational kinetic energy indicates the work done by external torques.
- Mechanical stress: Increasing rotational speed introduces centrifugal forces that can stress the disc, especially at the edges.
- Friction and losses: In real-world scenarios, frictional forces and air resistance cause energy dissipation, requiring additional torque to maintain or increase speed.

Summary: Key Steps to Analyze Changing Angular Velocity

1. Convert units: From Rev/s to rad/s for consistency.
2. Determine change in angular velocity: Subtract initial from final.
3. Calculate angular acceleration: If time is known, use $\alpha = (\omega_f - \omega_i)/t$.
4. Compute rotational kinetic energies: To understand energy requirements.
5. Find torque and power: Using $\tau = I \alpha$ and $P = \tau \omega_{avg}$.
6. Assess external influences: Friction, air resistance, and energy losses.

Conclusion

The change in the angular velocity of a disc of mass 3 kg and radius 0.6 m from 10 Rev/s to 15 Rev/s illustrates fundamental principles of rotational dynamics. Whether analyzing the problem through energy considerations or kinematic equations, understanding the interplay of angular velocity, torque, and energy provides a comprehensive picture of how rotational motion evolves in real systems. Mastery of these concepts is essential for engineers, physicists, and enthusiasts working with rotating machinery or studying rotational phenomena.

Remember: The key to analyzing rotational changes lies in careful unit conversion, applying fundamental formulas correctly, and considering the practical aspects of real-world motion.

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