. Find Two Numbers Such That If 7 Is Added To The Larger Number, The Value Obtained Is Four Times The

Find Two Numbers Such That If 7 Is Added To The Larger Number, The Value Obtained Is Four Times The Smaller Number

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Introduction

Understanding how to formulate and solve algebraic word problems is a fundamental skill in mathematics. One common type of problem involves finding two numbers based on given conditions involving addition, multiplication, or other operations. In this article, we explore a specific problem: finding two numbers where, if 7 is added to the larger number, the result equals four times the smaller number. We will analyze the problem, establish the necessary equations, and solve it systematically, providing clear explanations at each step.

Understanding the Problem

Restating the Problem

The problem can be summarized as follows:
- There are two unknown numbers, which we will label as x and y.
- One of these numbers is larger; without loss of generality, assume x is the larger number and y the smaller.
- When we add 7 to the larger number x, the resulting value should be four times the smaller number y.

Interpreting the Conditions

The key conditions are:
- Larger number: x
- Smaller number: y
- Condition: $x + 7 = 4y$

Note that the problem does not explicitly specify whether the larger number is greater than the smaller, but this assumption is logical given the phrasing. If necessary, we can verify this assumption later.

Setting Up the Equations

Defining Variables

Let:
- x = larger number
- y = smaller number

Based on the problem statement, the main equation is:
$$x + 7 = 4y$$

Since the problem involves two unknowns, we need a second equation to solve for both x and y.

Additional Conditions or Assumptions

The problem as stated only provides one equation. To find a unique solution, we need additional information or assumptions. Common approaches include:

- Assuming the two numbers are integers
- Assuming the two numbers are related in some other way (e.g., the difference between them)
- Or, considering that the larger number is greater than the smaller (i.e., $x > y$)

Suppose we assume that the larger number is exactly greater than the smaller by some fixed amount, or we explore possible solutions systematically.

Introducing an Additional Equation

For the purpose of solving, let's assume the two numbers are related via their difference:

Assumption: The larger number exceeds the smaller by a certain amount d:

$$x = y + d$$

where $d > 0$.

Alternatively, if no specific relation is given, we might consider that the two numbers are positive integers and explore integer solutions.

Solving the Equation

Express x in terms of y

From the main equation:
$$x + 7 = 4y$$
$$\Rightarrow x = 4y - 7$$

Since x is the larger number, and y is the smaller, we need:
$$x > y$$
$$4y - 7 > y$$
$$3y > 7$$
$$y > \frac{7}{3}$$

If y is an integer, then:
$$y \geq 3$$

Now, for each integer $y \geq 3$, find the corresponding x:

$$x = 4y - 7$$

Check if x is larger than y:

$$x - y = (4y - 7) - y = 3y - 7$$

For $y \geq 3$,
- When $y=3$: $x=4(3)-7=12-7=5$,
difference: $5 - 3=2$, and $x=5$ is larger than $y=3$.

- When $y=4$: $x=4(4)-7=16-7=9$,
difference: $9 - 4=5$.

- When $y=5$: $x=20-7=13$,
difference: $13-5=8$.

And so on.

In all these cases, $x > y$, satisfying the assumption.

Examples of Solutions

- For $y=3$: $x=5$, solution: (5, 3)
- For $y=4$: $x=9$, solution: (9, 4)
- For $y=5$: $x=13$, solution: (13, 5)
- For $y=6$: $x=17$, solution: (17, 6)

Each pair satisfies the main condition: when 7 is added to the larger number x, the result equals four times the smaller number y.

Verify with $y=3$, $x=5$:
$$ x + 7 = 5 + 7 = 12 $$
$$ 4y = 4 \times 3 = 12 $$
Matches the condition.

Verify with $y=4$, $x=9$:
$$ 9 + 7=16 $$
$$ 4 \times 4=16 $$
Again, condition satisfied.

General Solution and Analysis

Summary of the Solutions

The solutions form an infinite set of pairs where:

$$ y \geq 3 \quad \text{and} \quad x=4y-7 $$

with the condition that $x > y$, which is automatically satisfied for $y \geq 3$.

Interpreting the Results

This analysis shows that the problem has infinitely many solutions under the assumptions made, especially for integer solutions where $y \geq 3$.

If the problem intended only specific solutions, additional constraints such as the sum of the numbers or the difference could be introduced to narrow down the options.

Conclusion

Finding two numbers based on a condition involving addition and multiplication requires establishing equations and analyzing their solutions systematically. In this case, by assuming x is the larger number and formulating the primary equation $x + 7 = 4y$, we derived expressions for x in terms of y. Constraints such as the numbers being integers and the order $x > y$ helped us determine the range of feasible solutions.

The key takeaway is that mathematical problems often require assumptions or additional information to reach a unique solution. Here, the solutions include pairs like (5, 3), (9, 4), (13, 5), etc., all satisfying the original condition.

In conclusion, this problem illustrates the importance of translating word problems into algebraic equations, analyzing the constraints, and systematically exploring the solution space. Whether for academic purposes or real-world applications, mastering such problem-solving techniques is essential for developing strong mathematical reasoning skills.

Frequently Asked Questions

How do I set up an equation to find two numbers where adding 7 to the larger gives four times the smaller?

Let the smaller number be x and the larger be y. The equation is $y + 7 = 4x$, with the condition $y > x$.

What steps are involved in solving for the two numbers in this problem?

First, express one variable in terms of the other using the equation $y + 7 = 4x$, then substitute and solve for the variables step-by-step.

Can this problem be solved using substitution or elimination methods?

Yes, substitution is typically used here: express y in terms of x ($y = 4x - 7$) and then find specific values that satisfy the conditions.

Are there multiple solutions to this problem?

Yes, as long as the larger number y is greater than the smaller x, multiple solutions can exist depending on the values chosen.

What constraints should I consider when solving this problem?

Since the problem involves numbers and adding 7 to the larger number results in four times the smaller, ensure that the larger number is indeed greater than the smaller and that both are real numbers.

How can I verify the solutions once I find the two numbers?

Substitute the values back into the original condition $y + 7 = 4x$ to check if the equality holds true.

Is this type of problem common in algebra exercises?

Yes, problems involving setting up equations based on word descriptions and solving for variables are common in algebra practice and help develop problem-solving skills.

Additional Resources

Find Two Numbers Such That If 7 Is Added To The Larger Number, The Value Obtained Is Four Times The

Introduction

Mathematics often presents intriguing problems that challenge our reasoning and problem-solving skills. Among these, algebraic word problems stand out due to their blend of real-world context and abstract thinking. One such problem is: "Find two numbers such that if 7 is added to the larger number, the resulting value is four times the smaller number." This problem, while seemingly straightforward, requires careful analysis, formulation of equations, and logical reasoning to arrive at the solution. In this comprehensive article, we will delve into the problem's intricacies, explore various approaches to its solution, and analyze the underlying mathematical principles, all while maintaining an engaging and educational tone.

Understanding the Problem

Restating the Problem

The core of the problem involves two unknown numbers, which we will denote as x and y. The problem specifies a relationship involving these two numbers:

- When 7 is added to the larger of the two numbers, the result equals four times the smaller number.

The problem inherently involves identifying which number is larger and establishing a relationship between the two. To proceed, clarity on which number is larger is essential to set up the correct equations.

Clarifying Ambiguities

One crucial aspect is the phrase "the larger number." The problem does not explicitly specify whether x or y is larger. Therefore, we must consider two scenarios:

1. Scenario 1: x is the larger number.
2. Scenario 2: y is the larger number.

By exploring both scenarios, we ensure that no potential solutions are overlooked, and we develop a comprehensive understanding of the problem.

Formulating Mathematical Equations

Establishing Variables

To systematically analyze the problem, assign variables:

- Let L denote the larger number.
- Let S denote the smaller number.

Since the problem involves the larger and smaller numbers, these variables should satisfy $L > S$.

Developing Equations

Given the problem statement, the key relationship is:

> "If 7 is added to the larger number, the value obtained is four times the smaller number."

This can be mathematically expressed as:

$$
L + 7 = 4S
$$

Additionally, because one number is larger, we have:

$$
L > S
$$

Now, depending on which variable we assign as larger, the problem transforms into two cases:

Case 1: $L = x$, $S = y$, with $x > y$

In this case, the equation is:

$$
x + 7 = 4 y
$$

Since $x > y$, this relationship constrains the possible values of x and y.

Expressing One Variable in Terms of the Other

Rearranged:

$$
x = 4 y - 7
$$

Because $x > y$, we get:

$$
4 y - 7 > y
$$

Solving for y:

$$
4 y - 7 > y
$$
$$
4 y - y > 7
$$
$$
3 y > 7
$$

$$
y > \frac{7}{3}
$$

Since we're dealing with numbers that could be integers or real numbers, this inequality indicates:

- The smaller number y must be greater than $2.\overline{3}$.

Possible Values and Constraints

- For integer solutions, y can be any integer greater than 2.33, i.e., $y \geq 3$.
- Correspondingly, $x = 4 y - 7$.

Let's explore some values:

y	$x = 4 y - 7$	Check if $x > y$
3	$4 \times 3 - 7 = 12 - 7 = 5$	5 > 3 (Yes)
4	$16 - 7 = 9$	9 > 4 (Yes)
5	$20 - 7 = 13$	13 > 5 (Yes)
6	$24 - 7 = 17$	17 > 6 (Yes)

Thus, for integer solutions, the pairs are:

- $(x, y) = (5, 3)$
- $(9, 4)$
- $(13, 5)$
- $(17, 6)$

And so on. These pairs satisfy the original equation and the condition that the larger number exceeds the smaller.

Case 2: $L = y$, $S = x$, with $y > x$

In the second scenario, the larger number is y, and the smaller is x. The key equation remains:

$$
L + 7 = 4 S
$$

or

$$
y + 7 = 4 x
$$

Given $y > x$, rearranged:

$$
y = 4 x - 7
$$

Since $y > x$:

$$
4 x - 7 > x
$$
$$
4 x - x > 7
$$
$$
3 x > 7
$$
$$
x > \frac{7}{3}
$$

Again, x must be greater than approximately 2.33.

Possible solutions:

x	$y = 4 x - 7$	Check if $y > x$
3	$12 - 7 = 5$	5 > 3 (Yes)
4	$16 - 7 = 9$	9 > 4 (Yes)
5	$20 - 7 = 13$	13 > 5 (Yes)

Corresponding pairs:

- $(x, y) = (3, 5)$
- $(4, 9)$
- $(5, 13)$

Note that in these solutions, the larger number is y, and the smaller is x, consistent with the scenario.

Analytical Summary of Solutions

From the two cases, the solutions are:

- Case 1:

\[
\boxed{
\text{Larger} = x = 4 y - 7,\quad \text{Smaller} = y,\quad y > \frac{7}{3}

}
\]

For integer solutions:

$$
y \geq 3,\quad x = 4 y - 7
$$

- Case 2:

$$
\boxed{
\text{Larger} = y = 4 x - 7,\quad \text{Smaller} = x,\quad x > \frac{7}{3}
}
$$

For integer solutions:

$$
x \geq 3,\quad y = 4 x - 7
$$

These parametric forms encompass infinitely many solutions, depending on the choice of integer y or x satisfying the inequalities.

Real-World Interpretation and Applications

Practical Contexts

Problems like these have real-world analogs in various domains:

- Financial Planning: Determining investment amounts where increasing one account by a fixed sum results in a multiple of another account.
- Engineering: Adjusting component values such that modifying one parameter yields a specific multiple related to another.
- Statistics: Finding data points with specific proportional relationships after adjustments.

Understanding these solutions facilitates decision-making processes that involve constraints and proportional relationships.

Educational Significance

Such problems serve as excellent teaching tools for:

- Developing algebraic modeling skills.
- Enhancing reasoning about inequalities.
- Reinforcing the importance of considering multiple cases.

Mathematical Principles Underlying the Problem

Equations and Inequalities

The problem demonstrates the power of formulating problems into equations and inequalities, allowing for systematic analysis and solution derivation.

Case Analysis

Considering multiple scenarios ensures comprehensive coverage of all solutions, especially when the problem involves unspecified relational properties (e.g., which number is larger).

Parameterization

Expressing solutions in terms of a parameter (e.g., $y \geq 3$) reveals the infinite nature of solutions and the underlying structure of the problem.

Extending the Problem: Variations and Challenges

Introducing Additional Conditions

- Different constants: What if the number added was 5 instead of 7?
- Different multiples: What if the resulting value was three times the smaller number?
- Multiple relationships: Incorporate relationships involving the sum or difference of the two numbers.

Solving for Non-Integer Solutions

The analysis can be extended to real numbers, allowing for a broader set of solutions. This involves relaxing the integer constraints and focusing on inequalities.

Graphical Representation

Plotting the equations $x = 4 y - 7$ and $y = 4 x - 7$ alongside the inequalities can provide visual insights into the solution sets, illustrating the feasible regions in the coordinate plane.

Conclusion

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