(i) Find The Gradient At The Point (1, 2) On The Curve Given By: I+ry + Y2 = 12 22 - Y2 (ii) Find The

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Understanding how to find the gradient (or slope) at a specific point on a curve is a fundamental aspect of calculus, particularly when dealing with implicit functions. In this article, we will explore a detailed method to determine the gradient at a given point on an implicitly defined curve, specifically focusing on the curve described by the equation I + ry + Y2 = 12 22 - Y2. Additionally, we will discuss how to approach the second part of the problem, which involves further analysis or related calculations. Whether you're a student preparing for exams or a mathematician refreshing your concepts, this guide will provide clear steps and explanations.

Understanding the Problem Statement

Before diving into the solution, it is crucial to interpret the given equation correctly and clarify any notation ambiguities.

Deciphering the Equation

The equation provided appears to be:

> I + ry + Y2 = 12 22 - Y2

At first glance, certain symbols and numbers seem inconsistent or misformatted. Common issues include:

- The letter "I" could be a typo or misinterpretation of "1".
- "ry" might be a product of variables r and y or a typo.
- "Y2" and "Y2" could be "y^2" (y squared).
- The numbers "12 22" could be "12" and "22" or "1222".

Given the context, a plausible interpretation is that the original equation intended is:

Equation 1:
$$ 1 + r y + y^2 = 12 $$
or perhaps

$$ x + r y + y^2 = 12 $$

Similarly, the second part "22 - Y2" might be "22 - y^2".

If we assume the standard form is:

$$ x + y + y^2 = 12 $$

or

$$ x + y^2 = 12 $$

then the point (1, 2) satisfies the relation.

For clarity, let's proceed assuming the curve is defined by:

Equation:
$$ x + y + y^2 = 12 $$

which is a common implicit relation involving x and y.

Part 1: Finding the Gradient at the Point (1, 2)

The goal is to find the slope of the tangent to the curve at the specified point. Since the equation involves both x and y in an implicit way, we will employ implicit differentiation.

Step 1: Write down the implicit equation

Assuming the simplified form:

$$ x + y + y^2 = 12 $$

Step 2: Differentiate both sides with respect to x

Because y is a function of x (i.e., y = y(x)), we differentiate each term accordingly:

- $\frac{d}{dx} [x] = 1$

- $\frac{d}{dx} [y] = \frac{dy}{dx}$
- $\frac{d}{dx} [y^2] = 2 y \frac{dy}{dx}$
- $\frac{d}{dx} [12] = 0$

Putting it all together:

$$ 1 + \frac{dy}{dx} + 2 y \frac{dy}{dx} = 0 $$

Step 3: Solve for $\frac{dy}{dx}$

Factor $\frac{dy}{dx}$:

$$ \frac{dy}{dx} (1 + 2 y) + 1 = 0 $$

Rearranged:

$$ \frac{dy}{dx} (1 + 2 y) = -1 $$

Therefore,

$$ \frac{dy}{dx} = \frac{-1}{1 + 2 y} $$

Step 4: Substitute the point (1, 2) into the derivative

At the point (x, y) = (1, 2):

$$ \frac{dy}{dx} = \frac{-1}{1 + 2 \times 2} = \frac{-1}{1 + 4} = \frac{-1}{5} $$

Result:

The gradient of the tangent to the curve at (1, 2) is $-\frac{1}{5}$.

This negative slope indicates the tangent line is decreasing at that point.

Part 2: Further Analysis or Related

Calculations

While the problem statement cuts off after the first part, it indicates "ii) Find the," suggesting a second task, possibly:

- The equation of the tangent line at the point (1, 2),
- The normal line at that point,
- The second derivative (curvature),
- Or an application involving the gradient.

Let's explore a common follow-up: finding the equation of the tangent line at the point (1, 2).

Finding the Equation of the Tangent Line

Given:

- The point: $(x_0, y_0) = (1, 2)$
- The gradient: $m = -\frac{1}{5}$

The point-slope form of the line:

$$ y - y_0 = m (x - x_0) $$

Substituting:

$$ y - 2 = -\frac{1}{5} (x - 1) $$

Or, equivalently:

$$ y = -\frac{1}{5} (x - 1) + 2 $$

This line represents the tangent to the curve at the given point.

Summary and Key Takeaways

- Implicit differentiation is essential when dealing with equations where x and y are intertwined.
- The derivative $\frac{dy}{dx}$ can be found by differentiating each term and solving for $\frac{dy}{dx}$.
- Substituting the point into the derivative yields the slope (gradient) at that point.
- The calculated gradient at (1, 2) for the assumed curve $x + y + y^2 = 12$ is $-\frac{1}{5}$.

- The equation of the tangent line can be written using the point-slope form, providing a geometric interpretation of the slope at that point.

Additional Tips for Implicit Differentiation

- Always verify the equation and clarify notation before differentiating.
- Use the chain rule carefully when differentiating terms involving y.
- Remember that $\frac{dy}{dx}$ is a function of y, which may need substitution after differentiation.
- Practice differentiating various implicit functions to become comfortable with the process.

Conclusion

Finding the gradient at a specific point on an implicit curve involves understanding the relationship between x and y and applying implicit differentiation diligently. Once the derivative $\frac{dy}{dx}$ is obtained, substituting the point's coordinates provides the slope of the tangent line at that point. This foundational technique in calculus opens the door to analyzing the behavior of complex curves, understanding tangent and normal lines, and exploring curvature—all vital skills for students and professionals alike.

If the original problem includes additional parts or specific functions, the same principles can be applied to extend the analysis further. Mastery of implicit differentiation and related concepts is essential for tackling advanced calculus problems effectively.

Frequently Asked Questions

How do I find the gradient at the point (1, 2) on the curve given by I + ry + y^2 = 12?

To find the gradient at the point (1, 2), differentiate the given curve implicitly with respect to x, then substitute x=1 and y=2 into the resulting derivative to get the gradient.

What is the first step in differentiating the equation I + ry + y^2 = 12 for finding the gradient?

The first step is to differentiate both sides of the equation implicitly with respect to x, applying the chain rule to terms involving y, since y is a function of x.

How do I differentiate the term ry in the given equation?

Differentiate ry with respect to x using the product rule: d(ry)/dx = r dy/dx + y dr/dx. If r is a constant, then it simplifies to r dy/dx.

What is the significance of substituting the point (1, 2) into the derivative?

Substituting the point (1, 2) into the derived expression for dy/dx allows you to compute the exact gradient of the curve at that specific point.

How do I handle the term y^2 when differentiating implicitly?

Differentiate y^2 with respect to x using the chain rule: d(y^2)/dx = 2y dy/dx, since y is a function of x.

What is the next step after finding dy/dx in the implicit differentiation process?

After differentiating and substituting the point coordinates, solve for dy/dx to determine the gradient at the given point on the curve.

What additional information is needed to fully solve for the gradient if r is not specified?

You need the value of r (a constant or function) to complete the differentiation process and compute the numerical value of the gradient at the point.

Additional Resources

Find The Gradient At The Point (1, 2) On The Curve Given By: I + ry + y^2 = 12
22 - y^2

Introduction

In the realm of calculus and analytical geometry, understanding the behavior of curves at specific points is fundamental. The gradient, or slope, of a curve at a given point reveals critical insights into its tangential direction and rate of change. This article delves into a comprehensive analysis of the problem: Find the gradient at the point (1, 2) on the curve given by the equation $I + ry + y^2 = 12\ 22 - y^2$. Although the original statement appears to contain typographical or formatting ambiguities, our focus is to interpret and resolve these to facilitate an accurate calculation.

This detailed exploration aims to serve as a thorough review suitable for educational, academic, and professional contexts. We will analyze the given equation, interpret its components, clarify potential ambiguities, and then proceed with the differentiation process to find the gradient at the specified point. Additionally, we will discuss related concepts, such as implicit differentiation, the significance of the gradient, and potential applications.

Clarification and Interpretation of the Given Equation

Understanding the Equation Components

The provided equation appears to be:

> $I + ry + y^2 = 12\ 22 - y^2$

At first glance, the notation seems inconsistent with standard mathematical syntax. Possible interpretations include:

- Typographical errors or formatting issues: The symbol 'I' might be intended as the imaginary unit i, or perhaps a variable 'I' (though less common). The term 'ry' suggests the product of some variable r and y. The segment '$12\ 22 - y^2$' could be a misprint of $12 \times 22 - y^2$.

- Potential intended form: Considering standard forms, perhaps the original equation is:

$$
I + r y + y^2 = 12 \times 22 - y^2
$$

which simplifies to

$$
I + r y + y^2 = 264 - y^2
$$

or

$$
I + r y + y^2 + y^2 = 264
$$

which simplifies further to:

$$
I + r y + 2 y^2 = 264
$$

Alternatively, if 'I' represents an imaginary unit, the equation might involve complex variables, but this seems unlikely given the context.

Assumptions for the Purpose of Calculation

Given the ambiguities, the most plausible intended equation for the curve—especially in the context of finding the gradient—is:

$$
I + r y + y^2 = 12 \times 22 - y^2
$$

which simplifies to:

$$
I + r y + y^2 = 264 - y^2
$$

Rearranged, this becomes:

$$
I + r y + y^2 + y^2 = 264
$$

$$
I + r y + 2 y^2 = 264
$$

Supposing I is a constant (possibly zero or some known value), or perhaps an initial term, the core variable-dependent part seems to involve $r y + 2 y^2$.

Alternatively, considering the standard notation of derivatives and the context, it's more probable that the original equation was intended as:

$$
x + r y + y^2 = \text{constant}
$$

or similar.

For clarity, we will proceed with the assumption:

The curve is given by:

\[
x + r y + y^2 = C
\]

where \(x \) and \(y \) are variables, and \(r \), \(C \) are constants.

Given the point (1, 2), and the probable relation, our goal is to find the gradient \(\frac{dy}{dx} \) at that point.

Establishing the Equation for the Curve

Hypothesized Equation

Based on typical calculus problems, a plausible form is:

\[
x + a y + y^2 = K
\]

where:

- \(x \) and \(y \) are variables,
- \(a \) is a constant coefficient,
- \(K \) is a constant.

Given the original ambiguous statement, for the sake of this analysis, assume the curve is:

\[
x + a y + y^2 = D
\]

and at the point (1, 2), the equation is satisfied.

Determining Constants

Plugging the point \((x, y) = (1, 2) \):

\[
1 + a \times 2 + (2)^2 = D
\]
\[
1 + 2a + 4 = D
\]
\[

$$
2a + 5 = D
$$

Without more specifics, we cannot determine a or D exactly. However, since the goal is to find the gradient at the point, the key is to differentiate the general form with respect to x.

Differentiation and Finding the Gradient

Implicit Differentiation of the General Curve

Assuming the curve is defined implicitly as:

$$
x + a y + y^2 = D
$$

we differentiate both sides with respect to x:

$$
\frac{d}{dx} (x) + a \frac{d}{dx} (y) + \frac{d}{dx} (y^2) = 0
$$

which yields:

$$
1 + a \frac{dy}{dx} + 2 y \frac{dy}{dx} = 0
$$

Rearranged to solve for $\frac{dy}{dx}$:

$$
a \frac{dy}{dx} + 2 y \frac{dy}{dx} = -1
$$

$$
\left(a + 2 y \right) \frac{dy}{dx} = -1
$$

$$
\frac{dy}{dx} = \frac{-1}{a + 2 y}
$$

Gradient at the Specific Point (1, 2)

At $y = 2$, the gradient becomes:

$$
\left. \frac{dy}{dx} \right|_{(1, 2)} = \frac{-1}{a + 4}
$$

\]

Recall that a relates to the specific form of the original equation; without explicit values, the gradient depends on a.

If the original equation is known or specified, the value of a can be plugged in to compute the exact gradient.

Practical Calculation with Assumed Constants

Suppose the original curve is:

$$
x + 3 y + y^2 = 14
$$

which satisfies the point $(1, 2)$:

$$
1 + 3 \times 2 + 4 = 1 + 6 + 4 = 11
$$

Since this sums to 11, not 14, adjust D. Let's assume:

$$
x + 3 y + y^2 = 11
$$

then, at $y = 2$:

$$
a = 3
$$

and the gradient:

$$
\frac{dy}{dx} = \frac{-1}{3 + 4} = \frac{-1}{7}
$$

Thus, the gradient at (1, 2) is $-\frac{1}{7}$.

Broader Implications and Applications

Significance of Gradient Calculation

Determining the gradient at a point on a curve is pivotal in multiple domains:

- Physics: Understanding the rate of change of a physical quantity with respect to another.
- Engineering: Designing slopes or gradients in infrastructure.
- Economics: Calculating marginal changes in functions.

Implicit Differentiation as a Tool

Implicit differentiation allows the derivation of derivatives when variables are intertwined in complex equations. Mastery of this technique is fundamental for advanced calculus, especially in multivariable calculus and differential equations.

Noteworthy Considerations

- Domain restrictions: The denominator in the gradient formula must not be zero.
- Multiple solutions: Curves may have points where the gradient is undefined or zero, indicating cusps or stationary points.
- Higher-order derivatives: For curvature analysis, second derivatives are pertinent.

Conclusion

The process of finding the gradient at a specific point on an implicitly defined curve hinges on correctly interpreting the original equation, applying appropriate differentiation techniques, and evaluating at the given coordinates. Given the ambiguities in the initial problem statement, our assumptions led us to a plausible form, enabling a clear calculation.

Key takeaways:

- The implicit differentiation method is essential for handling equations where y is not explicitly expressed as a function of x.
- The gradient at a point provides insight into the local behavior of the curve.
- Clarifying the original equation’s form is critical; assumptions must be validated against context and known data.

This analytical approach exemplifies the rigorous reasoning essential in advanced mathematics, ensuring precise understanding and application of calculus principles. Future work may involve verifying the original equation’s form and extending the analysis to include second derivatives, curvature, and other geometric properties.

References

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- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Thomas, G. B., & Finney, R.
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