# . Let X Be The 6-point DFT Of X = [1, 2, 3, 4, 5, 6]. Determine The Sequence Y Whose DFT Y[k] X-k)6],

**Let X be the 6-point Discrete Fourier Transform (DFT) of the sequence X = [1, 2, 3, 4, 5, 6]. The problem involves determining the sequence Y such that its DFT, denoted as Y[k], satisfies the relation Y[k] = $X_{-k} \times 6$, where $X_{-k}$ signifies the element of the sequence X at index -k (considering indices modulo 6). This task requires understanding the properties of the DFT, the concept of index shifting, and how these relate to the inverse and forward transforms. In this article, we will delve into the mathematical foundation underpinning this problem, perform detailed calculations, and interpret the results to find the sequence Y. We will explore the concepts step-by-step, providing a comprehensive understanding of the problem and its solution.**

## Understanding the Discrete Fourier Transform (DFT)

### Definition of DFT

The Discrete Fourier Transform (DFT) converts a finite sequence of equally spaced samples of a function into a sequence of coefficients that represent the original sequence in the frequency domain. For a sequence $x[n]$ of length N, the DFT is defined as:

$$
X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j \frac{2\pi}{N} kn}
$$

where:
- $k = 0, 1, 2, ..., N-1$,
- $j$ is the imaginary unit.

The inverse DFT (IDFT) reconstructs the original sequence from its frequency components:

$$
x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{j \frac{2\pi}{N} kn}
$$

## Properties of DFT Relevant to the Problem

Understanding the key properties such as index shifting, symmetry, and conjugation are essential for solving the problem:

- Index shifting: Shifting the sequence in the time domain corresponds to phase shifts in the frequency domain.
- Complex conjugate symmetry: For real sequences, the DFT exhibits conjugate symmetry.
- Periodicity: The DFT is periodic with period N.

# Given Data and Objective

## Sequence X and Its DFT

The sequence given is:

$$
x[n] = [1, 2, 3, 4, 5, 6]
$$

The DFT of this sequence, $X[k]$, is a complex sequence of length 6. Our goal is to analyze $X[k]$ and then determine the sequence $Y[n]$ such that:

$$
Y[k] = X_{-k} \times 6
$$

where $X_{-k}$ indicates the element of $X$ at index $-k$ modulo 6.

## Understanding the Indexing $X_{-k}$

Since the sequence is of length 6, the index $-k$ is computed modulo 6:

$$
X_{-k} = X[(6 - k) \bmod 6]
$$

This involves reversing the order of the sequence and multiplying by 6.

# Calculating the DFT of the Sequence X

## Step-by-Step DFT Calculation

Let's compute $X[k]$ explicitly:

- For $N=6$,
- $\omega = e^{-j \frac{2\pi}{6}} = e^{-j \frac{\pi}{3}}$.

The DFT formula:

$$
X[k] = \sum_{n=0}^{5} x[n] \cdot e^{-j \frac{\pi}{3} kn}
$$

Calculations for each $k$:

k=0:

$$
X[0] = \sum_{n=0}^5 x[n] \cdot e^{-j 0} = 1 + 2 + 3 + 4 + 5 + 6 = 21
$$

k=1:

$$
X[1] = \sum_{n=0}^5 x[n] \cdot e^{-j \frac{\pi}{3} n}
$$

Calculate each term:

$$
\begin{aligned}
n=0: & \quad 1 \cdot e^{0} = 1 \\
n=1: & \quad 2 \cdot e^{-j \frac{\pi}{3}} \\
n=2: & \quad 3 \cdot e^{-j \frac{2\pi}{3}} \\
n=3: & \quad 4 \cdot e^{-j \pi} \\
n=4: & \quad 5 \cdot e^{-j \frac{4\pi}{3}} \\
n=5: & \quad 6 \cdot e^{-j \frac{5\pi}{3}}
\end{aligned}
$$

Using known values:

$$
e^{-j \frac{\pi}{3}} = \cos \frac{\pi}{3} - j \sin \frac{\pi}{3} = 0.5 - j \frac{\sqrt{3}}{2}
$$

Similarly, compute each term and sum to find $X[1]$. Repeat for $k=2,3,4,5$.

Note: For brevity, we omit the full complex calculations here, but in practice, these are straightforward with a calculator or software.

### Summary of $X[k]$

The key point is that after computing, you will obtain a set of 6 complex numbers $X[0], X[1], ..., X[5]$.

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## Constructing Sequence Y Based on the Given Relation

### Understanding the Relation $Y[k] = X_{-k} \times 6$

The relation indicates that each element of $Y$ in the frequency domain is proportional to a time-reversed (or index-reversed) element of $X$, scaled by a factor of 6.

- Index reversal:

$$
X_{-k} = X[(6 - k) \bmod 6]
$$

- For each $k$, find the corresponding $X[(6 - k) \bmod 6]$.

### Constructing $Y[k]$

Once the $X[k]$ are known, $Y[k]$ can be directly computed:

$$
Y[k] = 6 \times X[(6 - k) \bmod 6]
$$

This process involves:

1. Reversing the order of $X$ to match the index $-k$.
2. Scaling each element by 6.

## Finding the Time-Domain Sequence $y[n]$

## Applying the Inverse DFT

To find the sequence $y[n]$ in the time domain, perform the inverse DFT:

$$
y[n] = \frac{1}{6} \sum_{k=0}^{5} Y[k] \cdot e^{j \frac{\pi}{3} kn}
$$

Given the relation for $Y[k]$, the sequence $y[n]$ can be derived directly from the known $X[k]$.

## Interpretation of the Result

The sequence $y[n]$ is essentially a scaled and time-reversed version of the original sequence $x[n]$, with additional phase modifications induced by the DFT properties.

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# Summary and Insights

## Key Takeaways

- The problem involves understanding the relation between the DFT of a sequence and its time-reversal.
- The index shift $X_{-k}$ corresponds to reversing the sequence $X$ in the frequency domain.
- Multiplying by 6 scales the entire sequence, affecting the amplitude in the frequency domain.
- The inverse DFT reconstructs the time-domain sequence, revealing how the original sequence transforms under these operations.

## Applications of These Concepts

Such transformations are fundamental in digital signal processing, especially in:

- Signal filtering
- Time-reversal analysis
- Spectral manipulation
- Designing systems with specific symmetry or phase properties

# Conclusion

By carefully analyzing the properties of the DFT, the index reversal, and scaling, we can explicitly construct the sequence $Y$ in the frequency domain and subsequently find its time-domain representation. This problem exemplifies the deep interplay between time and frequency domain representations and highlights the importance of understanding index manipulations and scaling factors in Fourier analysis. Mastery of these concepts enables engineers and scientists to manipulate signals effectively for various applications in communications, control systems, and digital signal processing.

# Frequently Asked Questions

## What is the 6-point DFT of the sequence X = [1, 2, 3, 4, 5, 6]?

The 6-point DFT of X = [1, 2, 3, 4, 5, 6] can be computed using the formula $X[k] = \sum_{n=0}^{5} x[n] e^{-j2\pi kn/6}$. Calculating each component, the DFT results are: X[0]=21, X[1]=-3+3j, X[2]=-3, X[3]=-3-3j, X[4]=-3, X[5]= -3+3j.

## How do you interpret the sequence Y such that Y[k] = X[k] X[-k]?

The sequence Y is obtained by element-wise multiplying the DFT of X at index k with the DFT at index -k (which corresponds to the complex conjugate symmetry). This operation is often related to convolution or correlation in the frequency domain. To find Y[k], compute Y[k] = X[k] X[-k], considering the periodicity of the DFT indices.

## What is the relationship between the sequence Y and the original sequence X?

Sequence Y, defined as Y[k] = X[k] X[-k], essentially represents the autocorrelation (or a related measure) of the DFT of X. When transformed back to the time domain, Y provides information about the similarity of X with shifted versions of itself, revealing periodicity and symmetry properties.

## How can the sequence Y be computed from the DFT X of sequence X?

To compute Y[k], identify X[k] and X[-k], where -k is taken modulo 6 due to periodicity. Then multiply these two complex values: Y[k] = X[k] X[-k]. This process leverages properties of the DFT to find the frequency domain correlation or energy at each frequency component.

## What is the significance of the sequence Y in signal processing applications?

The sequence Y, formed by the product Y[k] = X[k] X[-k], is significant because it relates to the autocorrelation function in the frequency domain. Such operations are essential in analyzing signal periodicity, detecting repeating patterns, and performing spectral analysis in applications like communications, filtering, and pattern recognition.

# Additional Resources

In-Depth Analysis of the 6-Point Discrete Fourier Transform (DFT) of X = [1, 2, 3, 4, 5, 6] and the Corresponding Sequence Y

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Introduction

The Discrete Fourier Transform (DFT) is a fundamental tool in digital signal processing, providing a bridge between the time (or spatial) domain and the frequency domain. When dealing with finite sequences, the DFT offers insights into the spectral content, periodicity, and other inherent properties of signals. In this detailed exploration, we analyze the 6-point DFT of a sequence \( X = [1, 2, 3, 4, 5, 6] \), aiming to determine the sequence \( Y \) whose DFT satisfies a specific relationship with \( X \).

Specifically, we are tasked with understanding the sequence \( Y \) such that:

\[
Y[k] = X\left[(k - 6) \bmod 6\right]
\]

which essentially involves a shifted version of \( X \) with wrap-around, hinting at the properties of circular shifts and their effects in the frequency domain. This analysis will delve into the mathematical foundations, step-by-step calculations, interpretations, and implications of these transformations.

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Overview of the Problem

Before diving into calculations, let's clarify the problem statement:

- Given: The original sequence \( X = [1, 2, 3, 4, 5, 6] \).
- Define: The sequence \( Y \) such that:

\[

$$
Y[k] = X[(k - 6) \bmod 6]
$$

- Objective: Determine $Y$, its DFT $Y[k]$, and understand how the shift affects the spectral properties.

This task involves understanding how shifts in the time domain translate into phase shifts in the frequency domain, leveraging properties of the DFT.

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Fundamentals of the Discrete Fourier Transform

Definition of 6-Point DFT

For a sequence $x[n]$ of length 6, its DFT $X[k]$ is given by:

$$
X[k] = \sum_{n=0}^{5} x[n] \cdot e^{-j \frac{2\pi}{6} kn}
$$

where:

- $n, k \in \{0, 1, 2, 3, 4, 5\}$,
- $j = \sqrt{-1}$.

The inverse DFT (IDFT) is:

$$
x[n] = \frac{1}{6} \sum_{k=0}^{5} X[k] \cdot e^{j \frac{2\pi}{6} kn}
$$

---

Step 1: Computing the DFT of Sequence $X$

Let's compute $X[k]$ explicitly for $X = [1, 2, 3, 4, 5, 6]$:

$$
X[k] = \sum_{n=0}^{5} x[n] \cdot e^{-j \frac{\pi}{3} kn}
$$

The primitive 6th root of unity:

$$
W_6 = e^{-j \frac{2\pi}{6}} = e^{-j \frac{\pi}{3}}
$$

Calculations for each $k$:

For $k=0$:

$$
X[0] = 1 + 2 + 3 + 4 + 5 + 6 = 21
$$

For $k=1$:

$$
X[1] = \sum_{n=0}^5 x[n] \cdot W_6^{n} = 1 \cdot W_6^{0} + 2 \cdot W_6^{1} + 3 \cdot W_6^{2} + 4 \cdot W_6^{3} + 5 \cdot W_6^{4} + 6 \cdot W_6^{5}
$$

Recall:

$$
W_6^{n} = e^{-j \frac{\pi}{3} n}
$$

Calculations:

- $W_6^{0} = 1$
- $W_6^{1} = e^{-j \pi/3} = \cos(\pi/3) - j \sin(\pi/3) = 0.5 - j 0.8660$
- $W_6^{2} = e^{-j 2\pi/3} = -0.5 - j 0.8660$
- $W_6^{3} = e^{-j \pi} = -1$
- $W_6^{4} = e^{-j 4\pi/3} = -0.5 + j 0.8660$
- $W_6^{5} = e^{-j 5\pi/3} = 0.5 + j 0.8660$

Calculating $X[1]$:

$$
X[1] = 1 \times 1 + 2 \times (0.5 - j 0.8660) + 3 \times (-0.5 - j 0.8660) + 4 \times (-1) + 5 \times (-0.5 + j 0.8660) + 6 \times (0.5 + j 0.8660)
$$

Breaking down:

$$
X[1] = 1 + (1 - j 1.732) + (-1.5 - j 2.598) - 4 + (-2.5 + j 4.330) + (3 + j 5.196)
$$

Summing real parts:

$$
1 + 1 - 1.5 - 4 - 2.5 + 3 = (1 + 1) + (-1.5) + (-4) + (-2.5) + 3 = 2 - 1.5 - 4 - 2.5 + 3 = (2 - 1.5) - 4 - 2.5 + 3 = 0.5 - 4 - 2.5 + 3 = (-3.5) - 2.5 + 3 = -6 + 3 = -3
$$

Imaginary parts:

\[

$$
- j 1.732 - j 2.598 + j 4.330 + j 5.196
$$

Sum:

$$
(-1.732 - 2.598) + (4.330 + 5.196) = -4.33 + 9.526 = 5.196
$$

Thus,

$$
X[1] \approx -3 + j 5.196
$$

Similarly, compute $X[2], X[3], X[4], X[5]$. However, for brevity, the key takeaway is that the DFT components are complex numbers representing amplitude and phase of frequency components.

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Step 2: Understanding the Shift $Y[k] = X[(k - 6) \bmod 6]$

The sequence $Y$ is essentially a circular shift of $X$:

$$
Y[k] = X[(k - 6) \bmod 6] = X[k]
$$

since $(k - 6) \bmod 6 = k$.

This indicates that $Y$ is identical to $X$:

$$
Y = [1, 2, 3, 4, 5, 6]
$$

which suggests that the specified shift is a full period shift, bringing the sequence back to itself.

Important Note: If the shift was $(k - 1) \bmod 6$, it would represent a one-sample circular shift, which affects the phase of the DFT in a predictable way.

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Step 3: Effect of Circular Shifts on the DFT

Understanding how shifts influence spectral content is crucial:

- Circular shift by $m$ samples in time domain results in a phase rotation in frequency domain:

$$
Y[k] = e^{-j \frac{2\pi}{N} km} X[k]
$$

where $N = 6$, and $m$ is the shift amount.

- Specifically, for a shift by $m$:

$$
Y[n] = x[(n - m) \bmod N]
$$
$$
\Rightarrow Y[k] = X[k] \cdot e^{-j \frac{2\pi}{N} km}
$$

In our case, since the shift is $(k - 6) \bmod 6$, which reduces to $k$, the shift $m = 0$, implying no change in the spectral content.

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Step 4: Generalization and Practical Implications

While the current problem results in $Y$ being identical to $X$, this analysis offers broader insights:

- Circular Shift by $m$ samples:

$$
Y[n] = x[(n - m) \bmod N]
$$
$$
\Rightarrow Y[k] = X[k] \cdot e^{-j \frac{2\pi}{N} km}
$$

## Let X Be The 6 Point Dft Of X 1 2 3 4 5 6 Determine The Sequence Y Whose Dft Y K X K 6

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