n Is A Four-digit Positive Integer. Dividing n By 9, The Remainder Is 5. Dividing n By 7,

n Is A Four-digit Positive Integer. Dividing n By 9, The Remainder Is 5. Dividing n By 7, is a classic problem in number theory that involves understanding modular arithmetic and the properties of divisibility. Such problems are not only fundamental in mathematics but also have practical applications in computer science, cryptography, and problem-solving competitions. In this article, we will explore the problem in depth, analyze its conditions, and provide methods to find all possible values of the four-digit integer n satisfying these criteria.

Understanding the Problem and Its Conditions

Before diving into solutions, it is essential to clarify the problem statement and interpret what the conditions imply.

Restating the Conditions

- n is a four-digit positive integer, meaning $1000 \leq n \leq 9999$.
- When n is divided by 9, the remainder is 5, i.e., $n \equiv 5 \pmod{9}$.
- When n is divided by 7, the problem likely involves a similar condition, such as a specific remainder. Although the initial prompt cuts off, typical problems include conditions like $n \equiv r \pmod{7}$, where r is a known remainder.

For the purpose of this comprehensive guide, let's assume the full problem states:

"Dividing n by 7, the remainder is r."

Suppose further that the problem specifies r explicitly, say $r = 2$. Thus, the complete problem becomes:

> Find all four-digit positive integers n such that:
> - $n \equiv 5 \pmod{9}$,
> - $n \equiv 2 \pmod{7}$.

This is a classic application of the Chinese Remainder Theorem (CRT), which helps find solutions to systems of simultaneous congruences.

Applying Modular Arithmetic and the Chinese Remainder Theorem

The core approach to these types of problems involves solving systems of congruences using modular arithmetic.

System of Congruences

Given:
$$
\begin{cases}
n \equiv 5 \pmod{9} \\
n \equiv 2 \pmod{7}
\end{cases}
$$

Our goal is to find all integers n within the range $1000 \leq n \leq 9999$ satisfying both conditions simultaneously.

The Chinese Remainder Theorem (CRT)

The CRT states that if two moduli are coprime (which 9 and 7 are, since $\gcd(9,7) = 1$), then there exists a unique solution modulo the product of the moduli.

- The combined modulus $M = 9 \times 7 = 63$.
- There exists a unique solution $n \pmod{63}$.

The process involves:

1. Expressing one congruence in terms of the other.
2. Finding a particular solution n_0 modulo 63.
3. Generating all solutions within the specified range.

Step-by-Step Solution

Let's solve the system explicitly.

Step 1: Write the first congruence

$$
n \equiv 5 \pmod{9}
$$

which means:
$$
n = 5 + 9k, \quad \text{for some integer } k.
$$

Step 2: Substitute into the second congruence

$$
n \equiv 2 \pmod{7}
$$
becomes:
$$
5 + 9k \equiv 2 \pmod{7}.
$$

Simplify:
$$
9k \equiv 2 - 5 \pmod{7}
$$
$$
9k \equiv -3 \pmod{7}
$$

Since $-3 \equiv 4 \pmod{7}$ (because $-3 + 7 = 4$):
$$
9k \equiv 4 \pmod{7}.
$$

Note that $ 9 \equiv 2 \pmod{7} $, so:
$$
2k \equiv 4 \pmod{7}.
$$

Now, solve for k:
$$
2k \equiv 4 \pmod{7}
$$
$$
\text{Since } 2 \times 4 = 8 \equiv 1 \pmod{7} \text{, but that does not directly help.}
$$

Alternatively, find the modular inverse of 2 modulo 7:

- The inverse of 2 mod 7 is 4, because $2 \times 4 = 8 \equiv 1 \pmod{7}$.

Multiply both sides by 4:

$$
k \equiv 4 \times 4 \pmod{7}
$$

$$
k \equiv 16 \pmod{7}
$$
$$
k \equiv 2 \pmod{7}
$$

Thus, the general form of k:
$$
k = 2 + 7t, \quad t \in \mathbb{Z}.
$$

Step 3: Express n in terms of t

Recall:
$$
n = 5 + 9k
$$
Substitute $k = 2 + 7t$:

$$
n = 5 + 9(2 + 7t) = 5 + 18 + 63t = 23 + 63t.
$$

This means all solutions n satisfying both congruences are of the form:
$$
n = 23 + 63t,
$$
where t is an integer.

Determining All Valid Four-Digit Values of n

Since n must be a four-digit integer:

$$
1000 \leq n \leq 9999.
$$

Plugging into the formula:
$$
1000 \leq 23 + 63t \leq 9999.
$$

Subtract 23:
$$
977 \leq 63t \leq 9976.
$$

\]

Divide through by 63:
\[
\frac{977}{63} \leq t \leq \frac{9976}{63}.
\]

Calculate bounds:

- Lower bound:
\[
t \geq \frac{977}{63} \approx 15.503 \Rightarrow t \geq 16,
\]
since \(t \) is an integer.

- Upper bound:
\[
t \leq \frac{9976}{63} \approx 158.365 \Rightarrow t \leq 158.
\]

Therefore, the set of all valid \(t \) is:

\[
t \in \{16, 17, 18, \ldots, 158\}.
\]

Correspondingly, the values of \(n \) are:

\[
n = 23 + 63t,
\]
for each integer \(t \) from 16 to 158.

Final List of Solutions

The four-digit integers \(n \) satisfying the initial conditions are:

\[
\boxed{
n = 23 + 63t, \quad t = 16, 17, 18, \ldots, 158.
}
\]

Explicitly, this sequence starts at:
\[
n = 23 + 63 \times 16 = 23 + 1008 = 1031,
\]

and ends at:
$$
n = 23 + 63 \times 158 = 23 + 9954 = 9977.
$$

All these values satisfy:

- $1000 \leq n \leq 9999$,
- $n \equiv 5 \pmod{9}$,
- $n \equiv 2 \pmod{7}$.

Summary of the Approach and Key Takeaways

This problem exemplifies how to use modular arithmetic and the Chinese Remainder Theorem to solve systems of congruences involving multiple moduli.

Key steps include:

- Expressing one congruence in terms of a variable.
- Substituting into the other congruence.
- Solving for the variable using modular inverse.
- Generating all solutions within the specified numerical range.

Extensions and Variations

While this example assumes specific remainders, similar techniques apply when different remainders are provided. For instance:

- If the problem states $n \equiv r_1 \pmod{a}$ and $n \equiv r_2 \pmod{b}$, and a, b are coprime, the same method applies.
- For non-coprime moduli, more advanced methods involving the extended Euclidean algorithm are necessary.

Additionally, similar problems can be extended to larger systems involving three or more congruences, where

Frequently Asked Questions

What is the possible range of the four-digit integer n that leaves a remainder of 5 when divided by 9?

Since n is a four-digit number (1000 to 9999) and leaves a remainder of 5 when divided by 9, n can be expressed as $n = 9k + 5$, where $1000 \leq n \leq 9999$. Solving for k, the smallest k is when $9k + 5 \geq 1000$, giving $k \geq 111.11$, so $k \geq 112$. The largest k is when $9k + 5 \leq 9999$, giving $k \leq 1110$. Therefore, k ranges from 112 to 1110, and n ranges from $9112 + 5 = 1013$ to $91110 + 5 = 9995$.

If n leaves a remainder of 5 when divided by 9, what is the general form of n?

The general form of n is $n = 9k + 5$, where k is an integer such that n is between 1000 and 9999.

Given n leaves a remainder of 5 when divided by 9, what additional condition must n satisfy when divided by 7?

Since n is divisible by 7 with some remainder, the problem suggests that n divided by 7 leaves a specific remainder (not specified in the prompt). To find possible n values, we need to analyze $n \equiv r \pmod 7$. The key is to determine n such that both conditions are satisfied simultaneously.

How can we find all four-digit integers n that satisfy both conditions: $n \equiv 5 \pmod 9$ and $n \equiv r \pmod 7$?

We can use the Chinese Remainder Theorem or system of congruences to find all such n. First, express n as $n \equiv 5 \pmod 9$, then find n that also satisfies $n \equiv r \pmod 7$. By solving these simultaneous congruences, we can find all n within the four-digit range.

What is the significance of solving for n in these modular conditions in number theory?

Solving for n under these modular conditions helps in understanding the structure of numbers that satisfy multiple divisibility constraints, which is fundamental in number theory, cryptography, and solving Diophantine equations.

Can you provide an example of a four-digit number that satisfies $n \equiv 5 \pmod 9$ and, say, $n \equiv 3 \pmod 7$?

Yes. To find such n, solve the system: $n \equiv 5 \pmod 9$ and $n \equiv 3 \pmod 7$. Using the Chinese Remainder Theorem, the solution is $n \equiv 23 \pmod{63}$. The four-digit numbers are those where $n \equiv 23 \pmod{63}$ and $1000 \leq n \leq 9999$. For example, $n = 23 + 63k$. Setting $23 + 63k \geq 1000$ gives $k \geq 16$. The smallest such n is $23 + 6316 = 23 + 1008 = 1031$. Thus, 1031 is an example.

Additional Resources

Mathematical Puzzle: Unlocking the Secrets of a Four-Digit Integer

---

Introduction

Mathematics often presents us with intriguing puzzles that challenge our logical reasoning and problem-solving skills. One such captivating scenario involves a four-digit positive integer n, with specific modular properties. In this article, we'll delve deeply into the problem:

> "Let n be a four-digit positive integer. When divided by 9, the remainder is 5. When divided by 7, the remainder is ..."

Our goal is to explore these conditions thoroughly, analyze the possible solutions, and understand the underlying number theory principles that govern such problems. Whether you're a math enthusiast, a student preparing for competitive exams, or simply someone who appreciates analytical challenges, this comprehensive review will shed light on the fascinating world of modular arithmetic and problem-solving techniques.

Setting the Stage: Understanding the Problem

The Basic Conditions

Given that:

- n is a four-digit positive integer: $1000 \leq n \leq 9999$
- When divided by 9, the remainder is 5: $n \equiv 5 \pmod{9}$
- When divided by 7, the remainder is ... (the problem seems to be incomplete here, but based on typical structures, it's likely the second remainder is specified as well, perhaps $n \equiv r \pmod{7}$)

Assuming the problem is to find the possible values of n satisfying the above conditions, or perhaps to determine the value of n based on additional constraints, we need to analyze these modular equations systematically.

Modular Arithmetic: The Foundation

What is Modular Arithmetic?

Modular arithmetic involves integers and their remainders upon division by a fixed number (called the modulus). For example, $a \equiv b \pmod{m}$ means that a and b leave the same remainder when divided by m.

This concept is fundamental for solving problems involving divisibility and remainders, especially when multiple conditions are combined.

Analyzing the Conditions

Condition 1: $n \equiv 5 \pmod{9}$

This indicates that:

$$
n = 9k + 5
$$

for some integer k. Since n is a four-digit number:

$$
1000 \leq 9k + 5 \leq 9999
$$

which implies:

$$
995 \leq 9k \leq 9994
$$

Dividing throughout by 9:

$$
\frac{995}{9} \leq k \leq \frac{9994}{9}
$$

$$
110.55\ldots \leq k \leq 1110.44\ldots
$$

Since k is an integer, the possible values are:

$$
k \in \{111, 112, 113, \ldots, 1110\}
$$

This gives a range of potential n values satisfying the first condition:

$$
n = 9k + 5, \quad k \in [111, 1110]
$$

Incorporating the Second Condition

Suppose the second condition is:

> When divided by 7, the remainder is some value r.

For the sake of completeness, let's analyze the general case:

$$
n \equiv r \pmod{7}
$$

where r is an integer between 0 and 6.

Solving the System of Congruences

The simultaneous solution to:

$$
n \equiv 5 \pmod{9}
$$
$$
n \equiv r \pmod{7}
$$

can be approached using the Chinese Remainder Theorem (CRT), which provides a systematic way to find solutions when the moduli are coprime. Since 9 and 7 are coprime, the CRT guarantees a unique solution modulo $9 \times 7 = 63$.

Step 1: Express n in terms of the first congruence:

$$
n = 9k + 5
$$

Insert into the second congruence:

$$
9k + 5 \equiv r \pmod{7}
$$

which simplifies to:

$$
9k + 5 \equiv r \pmod{7}
$$

Since $9 \equiv 2 \pmod{7}$, rewrite as:

$$
2k + 5 \equiv r \pmod{7}
$$

or

$$
2k \equiv r - 5 \pmod{7}
$$

\]

Determining \(k \) for Each Possible Remainder \(r \)

Because \(r \) can be 0 through 6, let's analyze each case:

Remainder \(r \)	Equation: \(2k \equiv r - 5 \pmod{7} \)	\(r - 5 \) mod 7	Solving for \(k \)
0	\(2k \equiv -5 \equiv 2 \pmod{7} \)	2	\(2k \equiv 2 \pmod{7} \)
1	\(2k \equiv -4 \equiv 3 \pmod{7} \)	3	\(2k \equiv 3 \pmod{7} \)
2	\(2k \equiv -3 \equiv 4 \pmod{7} \)	4	\(2k \equiv 4 \pmod{7} \)
3	\(2k \equiv -2 \equiv 5 \pmod{7} \)	5	\(2k \equiv 5 \pmod{7} \)
4	\(2k \equiv -1 \equiv 6 \pmod{7} \)	6	\(2k \equiv 6 \pmod{7} \)
5	\(2k \equiv 0 \pmod{7} \)	0	\(2k \equiv 0 \pmod{7} \)
6	\(2k \equiv 1 \pmod{7} \)	1	\(2k \equiv 1 \pmod{7} \)

Solving for \(k \)

Since 2 and 7 are coprime, 2 has a multiplicative inverse modulo 7, which is 4 because:

\[
2 \times 4 = 8 \equiv 1 \pmod{7}
\]

Thus, the solutions for \(k \) are:

\[
k \equiv (r - 5) \times 4 \pmod{7}
\]

Calculating each:

1. \(r=0 \):

\[
k \equiv 2 \times 4 = 8 \equiv 1 \pmod{7}
\]

2. \(r=1 \):

\[
k \equiv 3 \times 4 = 12 \equiv 5 \pmod{7}
\]

3. \(r=2 \):

\[

$$
k \equiv 4 \times 4 = 16 \equiv 2 \pmod{7}
$$

4. $r=3$:

$$
k \equiv 5 \times 4 = 20 \equiv 6 \pmod{7}
$$

5. $r=4$:

$$
k \equiv 6 \times 4 = 24 \equiv 3 \pmod{7}
$$

6. $r=5$:

$$
k \equiv 0 \times 4 = 0 \pmod{7}
$$

7. $r=6$:

$$
k \equiv 1 \times 4 = 4 \pmod{7}
$$

Expressing n in terms of r

Recall:

$$
n = 9k + 5
$$

and the solutions for k are:

$$
k \equiv c_r \pmod{7}
$$

with c_r as the values computed above.

Since $k \equiv c_r \pmod{7}$, all solutions for k are:

$$
k = c_r + 7t, \quad t \in \mathbb{Z}
$$

Correspondingly, n becomes:

$$
n = 9(c_r + 7t) + 5 = 9c_r + 63t + 5
$$

The key is to find all n within the four-digit range:

$$
1000 \leq n \leq 9999
$$

Final Solution Set for n

N Is A Four Digit Positive Integer Dividing N By 9 The Remainder Is 5 Dividing N By 7

N Is A Four Digit Positive Integer Dividing N By 9 The Remainder Is 5 Dividing N By 7

Related Articles

- A Ball Falls Past A Window Of Height H=1.4 M In A Time T=0.16 S. How High Above The Top Of The Window
- A Company Manufactures Aluminum Mailboxes In The Shape Of A Box With A Half-cylinder Top. The Company
- 56 POINTSStep 1: Analyze The Graph.Daredevil Danny Takes A Practice Jump As Shown On The Graph Below.

Back to Home