. The Curve Y=x^4 +px+q Has A Point Of Inflexion (5/6), 19/36), Where P And Q Are Constants.(a) Find

The Curve Y = x^4 + px + q Has a Point of Inflection (5/6), 19/36), Where p and q Are Constants. (a) Find

Understanding the properties of polynomial functions is fundamental in calculus and algebra. In particular, analyzing the behavior of quartic functions such as y = x^4 + px + q provides insights into their critical points, inflection points, and how the coefficients p and q influence the overall shape of the curve. Given that the curve y = x^4 + px + q has a point of inflection at (5/6, 19/36), our goal is to determine the values of the constants p and q. This involves leveraging calculus concepts such as derivatives and the conditions for inflection points.

In this article, we will explore the step-by-step process to find p and q, starting with the key concepts involved, then formulating the necessary equations, and finally solving them to obtain the desired constants. Whether you're a student preparing for exams or an enthusiast interested in polynomial behavior, understanding this problem deepens your grasp of calculus applications.

Understanding Inflection Points in Polynomial Functions

What is an Inflection Point?

An inflection point on a curve is a point where the curve changes concavity. That is, the second derivative of the function changes sign at that point. This is different from a maximum or minimum, which are critical points where the first derivative is zero, but the concavity remains the same.

Conditions for an Inflection Point

For a function y = f(x), a point x = a is an inflection point if:

- f''(a) = 0 or f''(a) is undefined
- The second derivative changes sign at x = a

In our case, the function is y = x^4 + px + q, which is smooth and differentiable everywhere, so the key condition simplifies to setting the second derivative to zero at the inflection point.

Given Data and Objectives

The problem states:
- The curve $y = x^4 + px + q$ has a point of inflection at $(5/6, 19/36)$.
- p and q are constants to be determined.

Our goals:
- Find the values of p and q that satisfy these conditions.

Step 1: Derivatives of the Function

To find p and q, we need to analyze the derivatives:

First Derivative (dy/dx)

Given $y = x^4 + px + q$,

$$
dy/dx = 4x^3 + p
$$

Second Derivative (d^2y/dx^2)

Differentiating again:

$$
d^2y/dx^2 = 12x^2
$$

Note that the second derivative depends only on x, which simplifies the condition for inflection points.

Step 2: Applying Inflection Point Conditions

Since $(5/6, 19/36)$ is an inflection point:
- The second derivative at $x = 5/6$ must be zero:

$$
d^2y/dx^2 \bigg|_{x=5/6} = 0
$$

- The point must lie on the curve:

$$
y(5/6) = 19/36
$$

Analyzing the Second Derivative

$$
d^2y/dx^2 = 12x^2
$$

At $x = 5/6$:

$$
12 \times \left(\frac{5}{6}\right)^2 = 12 \times \frac{25}{36} = 12 \times \frac{25}{36} = \frac{12 \times 25}{36} = \frac{300}{36} = \frac{25}{3}
$$

Since this is not zero, the second derivative is positive at $x = 5/6$, indicating the curve is concave upward there. However, for a true inflection point, the second derivative must be zero or change sign.

But wait, the second derivative of the quartic $y = x^4 + px + q$ is $12x^2$, which is always non-negative and only zero at $x = 0$. Since the problem states the inflection point is at $x = 5/6$, we need to clarify the nature of the inflection point: in polynomial functions, a change in concavity occurs only at points where the second derivative changes sign, which for $y = x^4 + px + q$ only occurs at $x=0$, because $12x^2 \geq 0$ everywhere.

Important Note:

The second derivative is always positive except at $x=0$, where it is zero. So the point at $x=5/6$, where the second derivative is positive, cannot be an inflection point unless the problem refers to a different context or the function is more complex.

Re-examining the problem, it's likely that the problem involves the third derivative or that the inflection point is at $x=5/6$ with $y=19/36$, and the change of concavity occurs at some other point, or perhaps the problem is slightly different.

However, given the problem as stated, the second derivative is $12x^2$, which is always non-negative. So, the only potential inflection point is at $x=0$, where second derivative=0, but the problem specifies the inflection point at $x=5/6$.

Step 3: Correct Approach Based on the Given Data

Given this apparent inconsistency, the most logical interpretation is:

- The point $(5/6, 19/36)$ lies on the curve.
- The inflection point occurs at $(5/6, 19/36)$, meaning the second derivative at $x=5/6$ is zero or the second derivative changes sign at this x-value.

Since the second derivative is $12x^2$, which is positive everywhere except at $x=0$, it only equals zero at $x=0$. Therefore, the inflection point at $x=5/6$ suggests a different approach: perhaps the problem involves the third derivative or the conditions are different.

Step 4: Alternative Analysis — Using the First and Second Derivatives

Given the nature of the problem, a more suitable approach is:

- Use the fact that at the inflection point ($x=5/6$):
- The point lies on the curve: $y = x^4 + px + q$
- The second derivative is zero or change sign.

But since $d^2y/dx^2 = 12x^2$, which is positive at $x=5/6$, the second derivative is not zero, but the convexity does not change unless the third derivative also plays a role.

Therefore, the key is to check the third derivative:

$$
\frac{d^3 y}{dx^3} = 24x
$$

At $x=5/6$:

$$
24 \times \frac{5}{6} = 20 \neq 0
$$

Thus, the third derivative is non-zero, indicating the concavity does not change at $x=5/6$, which conflicts with the standard definition of an inflection point.

Step 5: Final Resolution — Likely the Intended Problem

Given the initial confusion, the most logical interpretation is:

- The polynomial is $y = x^4 + px + q$
- The inflection point is at $(5/6, 19/36)$
- The second derivative at this point is zero, indicating a point of inflection.

Since $d^2y/dx^2 = 12x^2$, setting this equal to zero gives $x=0$, not $x=5/6$, which conflicts with the provided point.

Therefore, perhaps the problem assumes the function $y = x^4 + px + q$ has an inflection point at x=5/6, and the second derivative at that point is zero. But as per the derivatives, this cannot happen unless the function is different.

Conclusion: Clarification and Step-by-Step Solution

Despite the potential inconsistencies, the core approach to solving for p and q remains:

1. Use the point (5/6, 19/36) to set up the equation y = x^4 + px + q.
2. Use the conditions for an inflection point, which involve derivatives.

Assuming the problem intends for us to find p and q given that (5/6, 19/36) lies on the curve, and the inflection point is at this x-coordinate, then:

- The point's coordinates satisfy the function:
\[
19/36 = \left(\frac{5}{6}\right)^4 + p \times \frac{5}{6} + q
\]

Calculating:
\[
\left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = \frac{25}{36} \times \frac{25}{36} = \frac{625}{1296}
\]

So,
\[
19/36 = \frac{625}{1296} + \frac{5

Frequently Asked Questions

How do you find the inflection point of the curve y = x^4 + px + q?

To find the inflection point, differentiate y twice to find the second derivative, set it to zero, and solve for x. The inflection point occurs where the second derivative changes sign, indicating a change in concavity.

Given the inflection point (5/6, 19/36), how can we use this to find the constants p and q?

Substitute x = 5/6 and y = 19/36 into the original equation y = x^4 + px + q to get one equation. Then, differentiate y to find the first and second derivatives, set the second derivative to zero at x = 5/6, and use this to find p and q.

What is the first step in solving for p and q given the inflection point on the curve?

Calculate the first and second derivatives of $y = x^4 + px + q$. Then, substitute $x = 5/6$ into the second derivative and set it equal to zero to find a relationship between p and q.

How do you verify that the point (5/6, 19/36) lies on the curve $y = x^4 + px + q$?

Plug $x = 5/6$ and $y = 19/36$ into the original equation $y = x^4 + px + q$. If the equation holds true after substituting the known point, it confirms the point lies on the curve.

What is the significance of the second derivative being zero at the inflection point?

The second derivative being zero at the point indicates a change in concavity of the curve, which characterizes an inflection point. This condition helps in determining the constants p and q.

Can you outline the steps to find p and q for the curve with the given inflection point?

Yes. First, write down the derivatives, set the second derivative to zero at $x = 5/6$ to find a relation between p and q. Next, substitute the inflection point coordinates into the original equation to get a second equation. Solve these two equations simultaneously to find p and q.

Additional Resources

Curve Analysis: Investigating the Inflection Point of the Quartic Function $Y = x^4 + px + q$

Introduction

In the realm of advanced calculus and mathematical analysis, understanding the behavior of polynomial functions is fundamental. These functions, especially quartic (degree four) polynomials, exhibit a rich variety of features such as minima, maxima, points of inflection, and curvature changes. Today, we delve into an intriguing problem: analyzing the curve $Y = x^4 + px + q$, where p and q are constants, particularly focusing on its inflection point located at $(\frac{5}{6}, \frac{19}{36})$.

This exploration is not only academically stimulating but also crucial for applications in physics, engineering, and computer science, where understanding the geometry and concavity of functions informs design and analysis.

Understanding the Function $Y = x^4 + px + q$

The given function is a quartic polynomial with a specific structure: it contains the leading term x^4, which dominates the behavior at large $|x|$, and linear terms involving constants p and q. The general form:

$$
Y = x^4 + px + q
$$

is relatively straightforward but rich in features, especially when analyzing its derivatives to find critical points and inflection points.

The Significance of Inflection Points

In calculus, an inflection point is a point on the curve where the concavity changes—meaning the second derivative shifts sign. These points are pivotal in understanding the shape and behavior of a graph, providing insights into the function's curvature and potential applications such as optimization.

Key characteristics of an inflection point:
- The second derivative Y'' equals zero at the inflection point.
- The concavity of the function changes around this point.

Given the point's coordinates $(\frac{5}{6}, \frac{19}{36})$, our goal is to determine the constants p and q that satisfy the conditions imposed by this inflection point.

Step 1: Calculating Derivatives

To analyze the inflection point, we first compute the derivatives of Y:

- First derivative (slope):

$$
Y' = \frac{dY}{dx} = 4x^3 + p
$$

- Second derivative (concavity):

$$
Y'' = \frac{d^2Y}{dx^2} = 12x^2
$$

Notice that the second derivative depends solely on x and is independent of p and q. This is a crucial insight because it simplifies the analysis of concavity changes.

Step 2: Conditions for the Inflection Point

Since the inflection point occurs at $\left(\frac{5}{6}, \frac{19}{36}\right)$, the following conditions hold:

1. The point lies on the curve:

$$
Y\left(\frac{5}{6}\right) = \frac{19}{36}
$$

2. The second derivative at that point is zero:

$$
Y''\left(\frac{5}{6}\right) = 0
$$

Step 3: Analyzing the Second Derivative

Using the second derivative:

$$
Y'' = 12x^2
$$

At $x = \frac{5}{6}$:

$$
Y''\left(\frac{5}{6}\right) = 12 \left(\frac{5}{6}\right)^2 = 12 \times \frac{25}{36} = 12 \times \frac{25}{36} = \frac{12 \times 25}{36} = \frac{300}{36} = \frac{25}{3}
$$

Since $\frac{25}{3} \neq 0$, the second derivative at $x=\frac{5}{6}$ is not zero. This indicates a need to revisit the initial assumption: the phrase "has a point of inflection at $\left(\frac{5}{6}, \frac{19}{36}\right)$" suggests that the inflection point is at that coordinate, but perhaps the function's structure or conditions need reassessment.

Clarification: Is the inflection point at $x=\frac{5}{6}$?

Given the second derivative $Y'' = 12x^2$, it is zero only at $x=0$. Therefore, the inflection point occurs at $x=0$, not at $x=\frac{5}{6}$.

However, the problem states the inflection point is at $\left(\frac{5}{6}, \frac{19}{36}\right)$.

This suggests that perhaps the problem is asking us to find p and q such that the curve passes through $\left(\frac{5}{6}, \frac{19}{36}\right)$ and that this point is an inflection point only if the second derivative there equals zero. But based on the derivative, it's not possible unless the function's form is different.

Reconciling the Problem Statement

Given the above, let's revisit the derivatives:

- For the second derivative:

$$
Y'' = 12x^2
$$

which is zero only at $x=0$. Therefore, to have an inflection point at $x=\frac{5}{6}$, the second derivative must be zero at that point, which is impossible unless the function has a different form or we are considering a more general context.

Possible interpretation:

- The problem might be asking us to find p and q such that the curve passes through $\left(\frac{5}{6}, \frac{19}{36}\right)$ and has an inflection point at $\left(\frac{5}{6}, \frac{19}{36}\right)$.

- Since the second derivative at $x=\frac{5}{6}$ is $\frac{25}{3}$, not zero, then the point is not an inflection point unless the function is adjusted or unless the problem is about a different point.

Revised Approach:

Assuming that the original problem is to find p and q such that:

- The curve passes through $\left(\frac{5}{6}, \frac{19}{36}\right)$,
- The second derivative at that point is zero (which is not possible with current $Y''=12x^2$),
- Or the first derivative at that point is zero, indicating a potential extremum.

Given the derivative:

$$
Y' = 4x^3 + p
$$

At $x = \frac{5}{6}$, the slope:

$$
Y'\left(\frac{5}{6}\right) = 4 \left(\frac{5}{6}\right)^3 + p = 4 \times \frac{125}{216} + p = \frac{500}{216} + p = \frac{125}{54} + p
$$

Similarly, the point lies on the curve:

$$

$$
Y = x^4 + px + q
$$

At $x=\frac{5}{6}$:

$$
Y\left(\frac{5}{6}\right) = \left(\frac{5}{6}\right)^4 + p \times \frac{5}{6} + q = \frac{625}{1296} + \frac{5p}{6} + q
$$

Set equal to $\frac{19}{36}$:

$$
\frac{625}{1296} + \frac{5p}{6} + q = \frac{19}{36}
$$

Step 4: Formulating Equations to Find p and q

Now, we have two equations:

1. For the point on the curve:

$$
\frac{625}{1296} + \frac{5p}{6} + q = \frac{19}{36}
$$

2. For the slope (if considering a stationary point, i.e., extremum):

$$
Y'\left(\frac{5}{6}\right) = 0 \Rightarrow \frac{125}{54} + p = 0 \Rightarrow p = -\frac{125}{54}
$$

Now, substitute p into the first equation to find q.

Step 5: Solving for p and q

- Calculate p:

$$
p = -\frac{125}{54}
$$

- Substitute into the first equation:

$$
\frac{625}{1296} + \frac{5}{6} \times \left(-\frac{125}{54}\right) + q = \frac{19}{36}
$$

Compute:

$$
\frac{5}{6} \times -\frac{125}{54} = -\frac{625}{324}
$$

Now, rewrite all fractions with denominator 1296 (since $1296 = 36^4$) to combine easily.

- Convert \(\frac{625}{

[The Curve Y X 4 Px Q Has A Point Of Inflexion 5 6 19 36 Where P And Q Are Constants A Find]

The Curve Y X 4 Px Q Has A Point Of Inflexion 5 6 19 36 Where P And Q Are Constants A Find

Related Articles

- 1. A Department Is Looking For An Entry-level Cashier. One Of The Job Applicants Is A Cashier With 30
- A Current Of 6.09 A In A Solenoid Of Length 14.8 Cm Creates A 0.203 T Magnetic Field At The Center Of
- 10. Ari And Caleb Are Best Friends. One Morning, Ari Sees Caleb In The Hallway At School And Says Hi.

Back to Home